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# Adaptive fuzzy finite-time control for a class of switched nonlinear systems with unknown control coefficients

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ABSTRACT

In this paper, an adaptive fuzzy finite-time control scheme is proposed for a class of switched nonlinear systems with unknown control coefficients and nonlinearities via backstepping methodology. Fuzzy logic systems (FLSs) are employed to approximate the unknown continuous functions. Under some appropriate assumptions, it is proven that if the designed parameters in the controller and adaptive laws are suitably chosen, then all the signals (i.e., the system state, the parameter estimations and the control input) in the closed-loop system are bounded and the system state can converge to the origin in finite time. Effectiveness of the developed scheme is illustrated by a simulation example.


## 1. Introduction

Switched systems are a type of hybrid systems, which comprise a collection of subsystems equipped with a switching law orchestrating among these systems. In the past decades, switched systems have attracted more and more attention due to their significance in the modeling of many engineering applications, such as power systems, chemical processes, aircraft control systems, robot control systems, multi-agent systems and so on [1–4]. During the last decades, much research effort has been made on the problems of stability analysis or stabilization for switched nonlinear systems (see [5–9] and references therein).

As unknown nonlinearity is inherent in practical systems, the problem of stabilization for such systems is of practical interest and academically challenging. It has been proven that the fuzzy logic systems (FLSs) and neural networks (NNs) can approximate arbitrary nonlinear continuous functions to a given accuracy on a closed set [10,11]. Based on this idea, some results have been reported for nonlinear/switched nonlinear systems using adaptive neural/fuzzy control method [12–22]. It should be emphasized that only boundedness of state can be obtained in the aforementioned results, that is, the state of the closed-loop system converged to a small neighborhood of the origin.

Recently, finite-time stabilization has been widely investigated for various nonlinear systems due to its higher tracking precision and greater robustness to uncertainties [23,24]. Compared with the asymptotical stabilization, finite-time stabilization renders the trajectories of the closed-loop system converge to the equilibrium in finite settling time rather than in infinite settling time. It has been proved that some inherently nonlinear systems, which cannot be stabilized by any smooth feedback control method, may be stabilized by using finite-time control methods [26,27]. Up to now, a large number of studies have been done on finite-time stability and stabilization of nonlinear systems or switched nonlinear systems [25–39]. However, unknown nonlinearities in the systems are usually restricted or even unconsidered in the existing finite-time stabilization results. For example, in [26,31,36], the considered systems do not include unknown nonlinearities. In [27,32], under the restricted assumption that there exist known functions which can dominate the unknown nonlinearities in the nonlinear systems, finite-time controllers were constructed. In [29,30,38], unknown nonlinearities were allowed in the considered systems and finite-time controllers were constructed under some assumptions that the unknown nonlinearities were dominated by the products of unknown linear parameters and known functions, which is very restricted and some practical systems may not satisfy these conditions. In [35,37], some results on finite-time control for robotic manipulators by using neural network have been reported. It is worth pointing out that the control coefficients of the systems studied in the aforementioned papers are required to be either exactly known [26,27,29–31,36,39] or unknown but limited in a known constant interval [32]. Based on

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the above analysis, an interesting question naturally arises: to what extent can the aforementioned assumptions be relaxed so as to finite-time stabilize nonlinear systems with both unknown control coefficients and unknown nonlinearities? The purpose of this paper is to address this important question and to provide an alternative and yet efficient solution.

In this paper, we propose an adaptive finite-time control strategy for a class of switched nonlinear systems by using FLSs. FLSs are used to estimate the unknown continuous functions. Based on the backstepping technique and the common Lyapunov function method, a common continuous finite-time controller and adaptive laws are designed. The main contributions of this paper are twofold:

(1) Both unknown control coefficients and unknown nonlinearities are allowed in the considered switched nonlinear systems. The nonlinearities of the systems are more general than those in [27,29,30,32,38].
(2) Under some appropriate assumptions, some functions are introduced in the virtual controllers to deal with unknown control coefficients. By choosing suitable designed parameters in the controller and adaptive laws, the proposed scheme can guarantee that the state of the closed-loop system converges to the origin in finite-time and the parameter estimations are bounded, which is different from those in [11,14,16,22].

The remainder of this paper is organized as follows. In Section 2, the problem formulation and some preliminary results are given. The adaptive fuzzy finite-time stabilization results are derived in Section 3. Section 4 provides an example to illustrate the proposed results, and concluding remarks are given in Section 5.

*Notations*: $R_+$ denotes the set of all nonnegative real numbers; $R^n$ denotes the real $n$-dimensional space; $R_{odd}^{\geq 1} \triangleq \{q \in R: \ q \geq 1$ and $q$ is a ratio of odd integers$\}$; $R_{odd}^{+} \triangleq \{q \in R: \ q > 0$ and $q$ is a ratio of odd integers$\}$; $C^i$ denotes the set of all functions with continuous $i$th partial derivatives; $|a|$ denotes the absolute value of scalar $a$; $\|x\|$ denotes the Euclidean norm of a vector $x$; $\|x\|_1$ is used for the 1-norm of vector $x$, i.e., $\|x\|_1 = \sum_{i=1}^{n} |x_i|$ with $x_i$ being the $i$th element of $x$; $x \succcurlyeq 0$ means that all elements of vector $x$ are nonnegative; sign$(a)$ denotes the sign of a real number $a$, that is, sign$(a)=1$ if $a>0$, sign$(a)=-1$ if $a<0$ and sign$(a)=0$ if $a=0$; diag$\{\cdot\}$ stands for a block-diagonal matrix; $I$ represents the unit matrix.

## 2. Problem formulation and preliminary results

### 2.1. Problem formulation

In this paper, we investigate the following uncertain switched nonlinear system:

$$\begin{aligned} \dot{x}_i(t) &= g_{i,\sigma(t)}(\overline{x}_i(t)) x_{i+1}^{p_i}(t) + f_{i,\sigma(t)}(x(t)), \quad i=1,2,\dots,n-1, \\ \dot{x}_n(t) &= g_{n,\sigma(t)}(x(t)) u^{p_n}(t) + f_{n,\sigma(t)}(x(t)), \end{aligned} \tag{1}$$

where $x=[x_1,\dots,x_n]^T \in R^n$ is the state, $u \in R$ is the system control input; $\overline{x}_i = [x_1,\dots,x_i]^T$; $p_i \in R_{odd}^{\geq 1}$, $i=1,2,\dots,n$; $\sigma(t):[0,+\infty) \to \underline{m} = \{1,2,\dots,m\}$ is a piecewise constant switching signal function with $m$ being the number of subsystems; $g_{i,k}(\overline{x}_i)$ and $f_{i,k}(\overline{x}_i)$ are unknown continuous functions, called control coefficients and nonlinearities of the system, respectively; moreover, $f_{i,k}(\overline{x}_i)$ satisfies $f_{i,k}(0)=0$, $i=1,2,\dots,n$, $\forall k \in \underline{m}$.

We assume that the state of switched nonlinear system (1) does not jump at the switching instants and the switching signal $\sigma(t)$ has a finite number of switching on every bounded time interval, which are common assumptions in the literature.

In the following, for the purpose of simplicity, the time variable $t$ will be omitted from the corresponding functions.

Now, the following assumptions are imposed on the unknown control coefficients and nonlinearities of the switched nonlinear system (1).

**Assumption 1** (*Liu [38]*). The signs of control coefficients $g_{i,k}(\overline{x}_i)$, $i=1,2,\dots,n$, $k \in \underline{m}$, are known, and for each $i$, the signs of $g_{i,k}(\overline{x}_i)$ are the same, $\forall k \in \underline{m}$. There exist unknown positive constants $c_1$, $c_2$ and known positive continuous functions $\underline{g}_i(\overline{x}_i)$, $\overline{g}_i(\overline{x}_i)$, $i=1,2,\dots,n$, such that, for any $t \geq 0$ and $x \in R^n$,

$$c_1 \underline{g}_i(\overline{x}_i) \leq |g_{i,k}(\overline{x}_i)| \leq c_2 \overline{g}_i(\overline{x}_i), \quad i=1,2,\dots,n. \tag{2}$$

**Remark 1.** Compared with the systems studied in [26,27,29–31,36], where all the control coefficients are exactly known (equal to 1), switched nonlinear system (1) allows the control coefficients to be unknown. Moreover, Assumption 1 implies that the known signs of $g_{i,k}(\overline{x}_i)$'s are nonzero and remain unchanged, which is reasonable because being away from zero is the controllable condition of switched nonlinear system (1).

**Assumption 2.** Given $p_i$ defined in (1) and a constant $\tau<0$, there exist known smooth functions $\varphi_{i,k,1}(\overline{x}_i) \geq 0$ satisfying $\varphi_{i,k,1}(0)=0$ and unknown continuous functions $\varphi_{i,k,2}(\overline{x}_i) \geq 0$ for $i=1,2,\dots,n$, $k \in \underline{m}$, such that

$$|f_{i,k}(x)| \leq \left| x_{i+1}^{p_i} \right|^{q_i} \sum_{j=1}^{i} |x_j|^{(r_i+\tau)(1-q_i)/r_j} \varphi_{i,k,1}(\overline{x}_i) \varphi_{i,k,2}(\overline{x}_i), \tag{3}$$

where $r_1=1$, $r_{i+1}=(r_i+\tau)/p_i>0$, $q_i \in [0,1)$.

By taking $r_i+\tau>0$ into consideration, it is easy to get $-1 \big/ (1+\sum_{g=1}^{n-1}(p_1 \cdots p_g)) < \tau < 0$. For simplicity, we let $\tau=-a/b$ with $a$ being an even integer and $b$ being an odd integer, i.e., $r_i \in R_{odd}^{+}$.

**Remark 2.** It should be emphasized that Assumption 2 encompasses the assumptions in existing results [32,38,39]. Similarly, when $\varphi_{i,k,2}(\overline{x}_i)$ is an unknown linear parameter, Assumption 2 becomes Assumption 3.2 in [38].Assumption 3 in [39] can be obtained when $q_i=0$.

In what follows, we review the finite-time stability definition and some useful lemmas.

We denote the solution of switched nonlinear system (1) by $x(t,x_0)$, where $x_0$ is the initial state.

**Definition 1** (*Moulay et al. [31]*). Switched nonlinear system (1) is finite-time stable under arbitrary switching if:

(1) system (1) is stable under arbitrary switching;
(2) for any $x_0 \in R^n$, there exists $0 \leq T(x_0) < \infty$ such that $x(t,x_0)=0$, $\forall t \geq T(x_0)$, where $T_0(x_0) = \inf\{t \geq 0: x(t,x_0)=0\}$ is a functional called the settling time of system (1).

**Lemma 1** (Liu and Huang [34]). *Consider switched nonlinear system (1). If all the subsystems of switched nonlinear system (1) share a continuous function $V:\Omega \to R$, called common finite-time Lyapunov function, which satisfies:*

(1) *$V$ is positive definite*;
(2) *there exist scalars $c>0$, $0<\alpha<1$ such that*
$\dot{V}(x) + c(V(x))^{\alpha} \leq 0, \quad x \in \Omega \setminus \{0\}$,