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Cluster synchronization for delayed complex networks via periodically intermittent pinning control

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ABSTRACT

In this paper, we investigate the cluster synchronization for linearly coupled networks with constant time delay by pinning periodically intermittent controllers. The network topology can be directed. The time delay is assumed to be less than the control width. At first, we give some criteria for cluster synchronization under constant pinning control strategy. Furthermore, by applying the adaptive approach, we design a centralized adaptive algorithm on the intermittent control gain, and also prove its validity rigorously. Finally, numerical simulations are given to demonstrate the correctness of obtained theoretical results.

1. Introduction

The study of synchronization [1] of complex networks, including chaotic systems and neural networks, has aroused a great deal of interest among researchers from various fields, such as communication, engineering, physical science, mathematics, and sociology. In the real world, synchronization of coupled oscillators can not only explain many natural phenomena [2], but also have many applications, such as image processing [3], secure communication [4], etc. Over the past decades, several different regimes of synchronization have been investigated, for example, complete synchronization [5], generalized synchronization [6], projective synchronization [7], phase synchronization [8], lag synchronization [9], and anticipating synchronization [10].

In general, N linearly coupled identical systems with external control can be described as

$$\dot{x}_i(t)=f(x_i(t))+\sum_{j=1}^{N}a_{ij}\Gamma x_j(t)+u_i,$$

where $x_i(t)=(x_i^1(t),\ldots,x_i^n(t))^T\in R^n$ is the n-dimensional state variable of the i th node, $i=1,2,\ldots,N$; the first term $f(x_i(t)):R^n\to R^n$ is a continuous function, which denotes the dynamical behavior of each uncoupled node; the second term $\sum_{j=1}^{N}a_{ij}\Gamma x_j(t)$ represents the linear and local coupling among pairs of connected nodes, where the positive definite matrix $\Gamma=\mathrm{diag}(\gamma_1,\ldots,\gamma_n)$ denotes the inner coupling matrix, and the network topology is represented by the outer coupling matrix $A=(a_{ij})\in R^{N\times N}$, if $a_{ij}\neq 0$, then node i's dynamics is impacted by node j, in other words, node i receives direct information from node j; the third term u_i is the external control, if controllers are added on only a small fraction of network nodes, then it is called the pinning control problem, see [11–13] and references therein. For example, [11] has proved that only one controller under certain conditions can realize the pinning control; [12] has proposed some pinning schemes to select pinned nodes by investigating the relationship among pinning synchronization, network topology, and the coupling strength; while [13] has studied the second-order leader-following consensus problem of nonlinear multi-agent systems with general network topologies.

If $\|x_i(t)-x_j(t)\|\to 0$ as $t\to+\infty$, $i,j=1,\ldots,N$, we say that the complete synchronization is realized. Until now, many approaches have been derived for complete synchronization, see [11–18] and references therein. Especially, when $f(\cdot)=0$, then the synchronization problem becomes the consensus problem, see [19–23] and references therein.

On the other hand, if the set of nodes can be divided into m nonempty clusters, that is,

$$\{1,2,\ldots,N\}=C_1\cup C_2\cup\cdots\cup C_m, \tag{1}$$

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where

$$C_1=\{1,\ldots,r_1\},C_2=\{r_1+1,\ldots,r_2\},\ldots,C_m=\{r_{m-1}+1,\ldots,N\},\tag{2}$$

such that coupled nodes in the same cluster can be synchronized, but there is no synchronization among different clusters, then the network is said to realize the cluster synchronization. In particular, when the number of cluster $m=1$, the cluster synchronization becomes the complete synchronization.

Cluster synchronization is considered to be significant in biological sciences [24] and communication engineering [25]. In the past few years, cluster synchronization has been widely studied by many researchers, see [26–34] and references therein.

Recently, periodically intermittent control, which is composed of "work time" (with control) and "rest time" (without control) in a period, has drawn increasing interests in many fields, including process control, ecosystem management, synchronizing chaotic systems, and communication. Compared with the continuous control method, the intermittent control method is advantageous in its efficiency and practicability, see [35–38] and references therein. For example, the exponential synchronization of coupled chaotic systems with intermittent linear state feedback control and constant time delay is investigated by means of Lyapunov function and differential inequality method in [35,36]; [37] investigates the quasi-synchronization for coupled chaotic systems with time delay in the presence of parameter mismatches by using intermittent linear state feedback control; and [38] investigates the periodically intermittent control of complex dynamical networks with time-varying delays.

As we know, time delays are playing important role in some fields. It is also known that time delays may cause undesirable dynamical behaviors such as oscillation and instability. There have been many works dealing with synchronization problem with delay, see [35–43] and references therein. For example, [40] investigates the complete synchronization problem for linearly coupled networks with delay by pinning a simple aperiodically intermittent controller and gives the corresponding criteria dependent on and independent of the time delay; [41] and [42] investigate the synchronization of delayed networks with no restriction on the time delay; [43] proposes a new lemma to deal with the increase coefficient estimation for delayed differential equations.

Based on the above analysis, in this paper, we will investigate the cluster synchronization of linearly coupled systems with delay by pinning periodically intermittent control as

$$\dot{x}_i(t)=f(x_i(t),x_i(t-\tau))+\sum_{j=1}^{N}a_{ij}\Gamma x_j(t)+u_i,\quad i=1,\ldots,N\tag{3}$$

To our best knowledge, there are few works to deal with this problem, for example, [44,45] investigate the cluster (lag) synchronization of complex networks via periodically intermittent control without delay.

Furthermore, synchronization problem under adaptive scheme [46] has been investigated for periodically intermittent pinning control without delay [44]. However, as for networks with delay, there are rare results about adaptive algorithm for periodically intermittent pinning control. Therefore, can we get similar theoretical results about the adaptive approach applied on periodically intermittent control strategy with delay?

The main contributions of this paper can be summarized as follows:

(1) A new result about the delayed differential equations (DDEs) is established with a more precise estimation for DDEs than previous works (like [40]).

(2) Under the above lemma, a new adaptive rule for DDEs with intermittent control is proposed and rigorously proved.

The rest of this paper is organized as follows. In Section 2, some necessary definitions, lemmas and notations are given. In Section 3, we investigate the cluster synchronization problem with an asymmetric matrix and intermittent control for static control gain. Some criteria for exponential cluster synchronization are obtained by the Lyapunov method. In Section 4, how to design the adaptive rule and apply it on the intermittent control is studied and its correctness is also rigorously proved. Numerical simulations to show the validity of obtained theoretical results are presented in Section 5. Finally, this paper is concluded in Section 6.

2. Preliminaries

In this section, we present some definitions, assumptions, lemmas and notations, which will be used throughout this paper.

Definition 1. Matrix $A=(a_{ij})_{i,j=1}^N$ is said to belong to class $A1$, denoted as $A\in A1$, if the following conditions are satisfied:

1. $a_{ij}\ge 0,\ i\ne j,\ a_{ii}=-\sum_{j=1,j\ne i}^N a_{ij},\ i=1,\ldots,N;$
2. A is irreducible.

If $A\in A1$ is symmetric, then we say that A belongs to class $A2$, denoted as $A\in A2$.

It is clear that if $A\in A1$, then $\sum_{j=1}^N a_{ij}=0$ for $i=1,\ldots,N$, which is called the zero row-sum property. To generalize this property, we have

Definition 2. Matrix $A=(a_{ij})\in R^{N_1\times N_2}$ is said to belong to class $A3$, denoted as $A\in A3$, if its each row-sum is zero, i.e., $\sum_{j=1}^{N_2}a_{ij}=0,\ i=1,\ldots,N_1$.

Now, using the above types of matrices, we can define a new type of coupling matrix A for the following cluster synchronization analysis.

Definition 3 (*Liu and Chen [44]*). Suppose $A\in R^{N\times N}$, the indexes $\{1,\ldots,N\}$ can be divided into m clusters defined by (1), and the following form holds:

$$A=\begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1m}\\ A_{21} & A_{22} & \cdots & A_{2m}\\ \cdots & \cdots & \cdots & \cdots\\ A_{m1} & A_{m2} & \cdots & A_{mm}\end{pmatrix},\tag{4}$$

where $A_{ij}\in R^{(r_i-r_{i-1})\times(r_j-r_{j-1})}$, $r_0=0$, r_i is defined by (2), $A_{ii}\in A1$ and $A_{ij}\in A3$, $i\ne j\in\{1,\ldots,m\}$. Then matrix A is said to belong to class $A4$, denoted as $A\in A4$.

Remark 1. In general, $a_{ij}>0(\text{or }<0)$, $i\ne j$ is regarded as the cooperative (or competitive) relationship between node i and node j. Thus, $A\in A4$ means that nodes in the same cluster only have cooperative relationship, while nodes belonging to different clusters can have both cooperative and competitive relationship. While in [32], the authors assume that $a_{ij}\ge 0,\ i\ne j$, that is to say, nodes in the network only have the cooperative relationships.

Definition 4. Network (3) with N nodes is said to realize cluster synchronization, if N nodes can be divided into m clusters as defined by (1) and (2), such that, for any node $i\in C_k$, $k=1,\ldots,m$,

$$\lim_{t\to+\infty}\|x_i(t)-s_k(t)\|=0\tag{5}$$

and

$$\lim_{t\to+\infty}\|s_k(t)-s_l(t)\|\ne 0,\ l\ne k\tag{6}$$