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Global exponential almost periodicity of a delayed memristor-based neural networks

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HIGHLIGHTS

- Use the Filippov solution to study the dynamics of delayed memristor-based neural networks.
- Prove the existence and uniqueness of almost periodic solution of the neural network under some conditions.
- Obtain the global exponential stability of the almost periodic solution.
- Prove the existence and stability of periodic solution of delayed neural networks with periodic memristor.

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ABSTRACT

In this paper, the existence, uniqueness and stability of almost periodic solution for a class of delayed memristor-based neural networks are studied. By using a new Lyapunov function method, the neural network that has a unique almost periodic solution, which is globally exponentially stable is proved. Moreover, the obtained conclusion on the almost periodic solution is applied to prove the existence and stability of periodic solution (or equilibrium point) for delayed memristor-based neural networks with periodic coefficients (or constant coefficients). The obtained results are helpful to design the global exponential stability of almost periodic oscillatory memristor-based neural networks. Three numerical examples and simulations are also given to show the feasibility of our results.

1. Introduction

Periodic oscillation in the neural networks is an interesting dynamical behavior, as many biological and cognitive activities (e.g., heartbeat, respiration, mastication, locomotion, and memorization) require repetition. It has been found applications in associative memories (Nishikawa, Lai, & Hoppensteadt, 2004), pattern recognition (Chen, Wang, & Liu, 2000; Wang, 1995), learning theory (Ruiz, Owens, & Townley, 1998; Townley et al., 2000), and robot motion control (Jin & Zacksenhouse, 2003). The studies of the periodic oscillation of various neural networks such as the Hopfield network, cellular neural networks, and bidirectional associative memories are all reported in the literature (see, for instance, Cao & Chen, 2004; Cao & Wang, 2005; Chen & Wang, 2004, 2005; Huang, Cao, & Ho, 2006; Tan & Tan, 2009 and references therein). Almost periodic functions, with a superior spatial structure, are a generalization of periodic functions. Meanwhile, in practice, almost periodic phenomenon is more common than periodic phenomenon (see, for instance, Levitan & Zhikov, 1982). For example, upon considering long-term dynamical behaviors, the periodic parameters of the neural networks often turn out to experience certain perturbations, that is, parameters become periodic up to a small error. So, almost periodic oscillatory behavior is considered to be more accordant with reality. There have been a great number of results on almost periodic oscillation of the neural networks with or without delay (see, for instance, Allegretto, Papini, & Forti, 2010; Cao, Chen, & Huang, 2005; Huang & Cao, 2003, 2009; Huang et al., 2006; Jiang, Zhang, & Teng, 2005; Levitan & Zhikov, 1982; Qin, Xue, & Wang, 2013; Wang, 2010; Wang, Lu, & Chen, 2009 and Xiang & Cao, 2009a, 2009b).

Memristor is a new circuit elements which possesses many properties of resistors and shares the same unit of measurement (ohm) and offers a nonvolatile memory storage with in a simple device structure attractive for potential applications. Since Chua's work (Chua, 1971) in 1971, a series of properties of the memristor are described, the usefulness of which in the modeling and

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understanding of various physical systems is shown (see Chua, 1971; Corinto, Ascoli, & Gilli, 2011; Itoh & Chua, 2008, 2009; Merrikh-Bayat & Shouraki, 2011a, 2011b; Mullins, 2009; Pershin & Di Ventra, 2010a, 2010b, 2011; Petras, 2010; Strukov, Snider, Stewart, & Williams, 2008; Ventra, Pershin, & Chua, 2009).

Among of them two properties of the memristor attracted much attention, which are its memory characteristic and its nanometer dimensions. The memory property and latching capability enable us to think about new methods for nano-computing, with the nanometer scale device providing a very high density and is less power hungry. From the previous work it follows that the memristor exhibits features just as the neurons in the human brain have.

It is well known that a neural networks can be implemented by circuits such as that the Hopfield neural network model can be implemented in a circuit where the self feedback connection weights are implemented by resistors. Suppose that we use memristors instead of resistors in the circuits, then we can build a new model, that is a memristor-based neural networks. Especially, it has been shown that memristors have been proposed to work as synaptic weights in artificial neural networks. In Hu and Wang (2010), a piecewise-linear mathematical model of the memristor is first given to characterize its feature of pinched hysteresis and a recurrent neural network model with time delays based on this model is then proposed due to the natural implementation of learning the weights (e.g., Hebbian learning). Such a model is basically a state-dependent nonlinear switching dynamical system. Soon afterwards many scholars dedicated to study dynamical behaviors of the memristor-based neural networks model. (see, for instance, Chen, Zeng, & Jiang, 2014; Hu & Wang, 2010; Pershin & Di Ventra, 2010a; Wu, Wen, & Zeng, 2012; Wu & Zeng, 2012, 2013; Wu, Zeng, Zhu, & Zhang, 2011; Zhang, Shen, & Sun, 2012; Zhang, Shen, & Wang, 2013). However, to the best of our knowledge, there exists no result on the almost periodic solution of delayed memristor-based neural networks (DMNN). The main purpose of this paper is to give the conditions for the existence and exponential stability of the almost periodic solutions for a DMNN. By applying new Lyapunov function techniques, we derive some new sufficient conditions ensuring the existence, uniqueness and exponential stability of the almost periodic solution, which are new and they complement previously known results. Moreover, the obtained conclusion on the almost periodic solution is applied to prove the existence and stability of periodic solution (or equilibrium point) for delayed memristor-based neural networks with periodic coefficients (or constant coefficients).

The rest of the paper is organized as follows. A DMNN model is introduced and some necessary definitions are given in Section 2. A sufficient criterion ensuring the global existence and boundedness of any solutions, the existence and exponential stability of an almost periodic solution of the networks in Section 3. The applications of our main results which are the existence, uniqueness and stability of periodic solution and equilibrium point in Section 4. Three examples and simulations are obtained in Section 5. Finally, the paper is concluded in Section 6.

2. Model description and preliminaries

Referring to some relevant works in Chen et al. (2014); Hu and Wang (2010); Pershin and Di Ventra (2010a); Wu et al. (2012); Wu and Zeng (2012); Wu and Zeng (2013); Wu et al. (2011); Zhang et al. (2012) and Zhang et al. (2013) and which deal with the detailed construction of some general classes of memristor-based recurrent neural networks from the aspects of circuit analysis and memristor physical properties. Consider a class of DMNN model described by the following equation:

$$\frac{dx_i(t)}{dt} = -c_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t, x(t))f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(t, x(t-\tau_{ij}))g_j(x_j(t-\tau_{ij})) + I_i(t), \tag{2.1}$$

for $i = 1, 2, \dots, n$, where n corresponds to the number of units in a neural network; $x_i(t)$ denotes the state variable associated with the ith neuron; $f_{ij}(x_i(t))$ and $g_{ij}(x_j(t-\tau_{ij}))$ denote the output of the jth unit at time t and $t-\tau_{ij}$, respectively; $c_i(t) > 0$ represents the rate with which the ith unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs at time t; $I_i(t)$ denotes the external bias on the ith unit at time t; τ_{ij} corresponds to the transmission delay of the ith unit along the axon of the jth unit; $a_{ij}(t, x(t))$ and $b_{ij}(t, x(t-\tau_{ij}))$ are memristor-based weights, denote the strengths of the jth unit on the ith unit at time t and time $t-\tau_{ij}$, respectively, which are defined as follows

$$a_{ij}(t, x) = \begin{cases} \hat{a}_{ij}(t), & h_j(x) > T_j, \\ \check{a}_{ij}(t), & h_j(x) < T_j, \end{cases} \tag{2.2}$$

and

$$b_{ij}(t, x) = \begin{cases} \hat{b}_{ij}(t), & h_j(x) > T_j, \\ \check{b}_{ij}(t), & h_j(x) < T_j, \end{cases} \tag{2.3}$$

for $i, j = 1, 2, \dots, n$ and $t \in R$, $a_{ij}(t, x) = \hat{a}_{ij}(t)$ or $\check{a}_{ij}(t)$ when $h_j(x) = T_j$, where $h_j : R^n \to R (j = 1, 2, \dots, n)$ are threshold level functions, $T_j \in R (j = 1, 2, \dots, n)$ are threshold level, $\hat{a}_{ij}(t)$, $\check{a}_{ij}(t)$, $\hat{b}_{ij}(t)$ and $\check{b}_{ij}(t)$ are all continuous functions.

Remark 2.1. In the existing literature (see Chen et al., 2014; Hu & Wang, 2010; Pershin & Di Ventra, 2010a; Wu et al., 2012; Wu & Zeng, 2012, 2013; Wu et al., 2011; Zhang et al., 2012, 2013), consider only the case $h_j(x) = |x_j| (j = 1, \dots, n)$. From this point, we can see that DMNN (2.1) is of a more general form than ever.

Suppose $E \subset R^n$, then $x \to F(x)$ is called a set-valued map from E to R^n, if for each point $x \in E$, there exists a nonempty set $F(x) \subset R^n$. A set-valued map F with nonempty values is said to be upper semicontinuous at $x_0 \in E$, if for any open set N containing $F(x_0)$, there exists a neighborhood M of x_0 such that $F(M) \subset N$. The map $F(x)$ is said to have a closed (convex, compact) image if for each $x \in E$, $F(x)$ is closed (convex, compact). Let $C_\tau := C([-\tau, 0])$ denote a Banach space of all continuous functions $\varphi : [-\tau, 0] \to R$. Sometime, for $x \in R^n$, we write $x \in C_\tau$ means $x(s) \equiv x$ in $[-\tau, 0]$. For $\varphi \in C_\tau$, let $\|\varphi\|_c = \sup_{s\in[-\tau,0]} \|\varphi(s)\|$. Given the function $V : R^n \to R$, ∇V denotes the gradient of V and ∂V denotes the Clarke's generalized gradient of V.

The initial states associated with DMNN (2.1) are of the form

$$x_i(s) = \varphi_i(s), \quad s \in [-\tau, 0], \; i = 1, \dots, n \tag{2.4}$$

where $\tau = \max_{1\le i,j\le n}\{\tau_{ij}\}$.

Let $x_t \in C([-\tau, 0], R^n)$ be defined by $x_t(s) = x(t+s)$, $-\tau \le s \le 0$, and the initial states (2.4) can be rewritten as

$$x_0 = \varphi \in C_\tau := C([-\tau, 0], R^n). \tag{2.5}$$

Remark 2.2. From (2.2)–(2.3), $a_{ij}(t, x)$ and $b_{ij}(t, x)$ can be discontinuous in $x \in \Lambda_j$ if $h_j(x) = T_j$ have a solution set Λ_j and $\hat{b}_{ij}(t) \neq \check{b}_{ij}(t)$.