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Dynamic neural network-based robust observers for uncertain nonlinear systems☆

H.T. Dinh [a,*], R. Kamalapurkar [b], S. Bhasin [c], W.E. Dixon [b]

[a] Department of Mechanical Engineering, University of Transport and Communications, Viet Nam
[b] Department of Mechanical and Aerospace Engineering, University of Florida, Gainesville, FL 32611-6250, USA
[c] Department of Electrical Engineering, Indian Institute of Technology, Delhi, India

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ABSTRACT

A dynamic neural network (DNN) based robust observer for uncertain nonlinear systems is developed. The observer structure consists of a DNN to estimate the system dynamics on-line, a dynamic filter to estimate the unmeasurable state and a sliding mode feedback term to account for modeling errors and exogenous disturbances. The observed states are proven to asymptotically converge to the system states of high-order uncertain nonlinear systems through Lyapunov-based analysis. Simulations and experiments on a two-link robot manipulator are performed to show the effectiveness of the proposed method in comparison to several other state estimation methods.

1. Introduction

Full state feedback is not available in many practical systems. In the absence of sensors, the requirement of full-state feedback for the controller is typically fulfilled by using *ad hoc* numerical differentiation techniques, which are sensitive to noise, leading to unusable state estimates. Observers are an alternative method to numerical methods. Several nonlinear observers are available in literature to estimate unmeasurable states. For instance, sliding mode observers were designed for nonlinear systems in Canudas De Wit and Slotine (1991), Mohamed, Karim, and Safya (2010) and Slotine, Hedrick, and Misawa (1986) based on an assumption that exact model knowledge of the dynamics is available. Model-based observers are also developed in Lee and Khalil (1997) and Shin and Lee (1999) which require a high-gain to guarantee estimation error regulation. The observers introduced in Astolfi, Ortega, and Venkatraman (2010) and Lotfi and Namvar (2010) are both applied for Lagrangian dynamic systems to estimate the velocity. Global exponential convergence to the true velocity is obtained in Astolfi et al. (2010), and a global asymptotic result is proven in Lotfi and Namvar (2010). The result in Astolfi et al. (2010) is based on the immersion and invariance approach to reconstruct the unmeasurable state. The use of this approach requires the solution of a partial differential equation. Given the challenge of finding such a solution, an approximation technique is employed that introduces error in the estimation, the effects of which are dominated by high-gain terms introduced in the observer dynamics. In Lotfi and Namvar (2010), the system dynamics must be expressed in a non-minimal model and feedback from force sensors are used to develop a velocity estimate. In Adhyaru (2012), a constrained optimal observer is developed for a nonlinear system under the assumption of exact model knowledge, where a nonquadratic performance cost function is used to impose magnitude constraints on an observer gain matrix.

The design of robust observers for uncertain nonlinear systems is considered in Davila, Fridman, and Levant (2005), Dawson, Qu, and Carroll (1992), Vasiljevic and Khalil (2008) and Xian, de Queiroz, Dawson, and McIntyre (2004). In Davila et al. (2005), a second-order sliding mode observer for uncertain systems using a super-twisting algorithm is developed, where a nominal model of the system is assumed to be available and estimation errors are proven to converge in finite-time to a bounded set around the origin. In Dawson et al. (1992), the developed observer guarantees that the state estimates exponentially converge to the actual state, if there exists a vector function satisfying a complex set of matching conditions. An asymptotic velocity observer is developed

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* Corresponding author. Tel.: +84 979721484.
E-mail addresses: huyendtt214@gmail.com, huyentdinh@utc.edu.vn (H.T. Dinh), rkamalapurkar@ufl.edu (R. Kamalapurkar), sbhasin@ee.iitd.ac.in (S. Bhasin), wdixon@ufl.edu (W.E. Dixon).

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in Xian et al. (2004) for general second-order systems; however, all nonlinear uncertainties in the system are damped out by a sliding-mode term resulting in high-frequency state estimates. In Vasiljevic and Khalil (2008), a high-gain derivative estimator is developed to estimate the derivative(s) of a signal in the presence of measurement noise. In the absence of noise, the derivative estimation error asymptotically converges as the observer gain grows to infinity. In contrast, the result developed in this paper yields asymptotic convergence with bounded gains.

Neural networks (NN) and fuzzy logic systems provide an effective approximation method that facilitates new observer designs, improving and complementing the base of conventional observer design approaches. For example, the approaches in Boulkroune, Tadjine, MSaad, and Farza (2008), Choi and Farrell (1999), Kim and Lewis (1999), Park and Park (2003) and Vargas and Hemerly (2001) use the universal approximation property in adaptive observer designs. However, estimation errors in Boulkroune et al. (2008), Choi and Farrell (1999), Kim and Lewis (1999), Park and Park (2003) and Vargas and Hemerly (2001) are only guaranteed to be bounded due to function reconstruction errors resulting from the NN or fuzzy system.

The challenge to obtain asymptotic estimation stems from the fact that to robustly account for disturbances, feedback of the unmeasurable error and its estimate are required. Typically, feedback of the unmeasurable error is obtained by taking the derivative of the measurable state and manipulating the resulting dynamics (e.g., this is the approach used in methods such as Kim & Lewis, 1999 and Xian et al., 2004). However, such an approach provides a linear feedback term of the unmeasurable state. Hence, a sliding mode term could not be simply added to the NN structure of the result in Kim and Lewis (1999), for example, to yield an asymptotic result, because it would require the signum of the unmeasurable state. It is unclear how such a nonlinear function of the unmeasurable state can be injected in the closed-loop error system using traditional methods. Likewise, it is not clear how to simply add an NN-based feedforward estimation of the nonlinearities in results such as Xian et al. (2004) because of the need to inject nonlinear functions of the unmeasurable state.

The approach used in this paper circumvents the challenge of injecting feedback to yield an asymptotic result by using nonlinear (sliding-mode) feedback of the measurable state, and then exploiting the recurrent nature of a dynamic neural network (DNN) structure to inject terms that cancel cross terms associated with the unmeasurable state. The approach is facilitated by using the filter structure inspired by Xian et al. (2004) and a novel stability analysis. The stability analysis is based on the idea of segregating the nonlinear uncertainties into terms which can be upper-bounded by constants and terms which can upper-bounded by states. The terms upper-bounded by states can be canceled by the linear feedback of the measurable errors, while the terms upper-bounded by constants are partially rejected by the sliding mode feedback (of the measurable state) and partially eliminated by the novel DNN-based weight update laws.

The contribution of this paper (and its preliminary version in Dinh, Kamalapurkar, Bhasin, & Dixon, 2011) is that the observer is designed for Nth order uncertain nonlinear systems, where the output of the Nth order system is assumed to be measurable up to $N-1$ derivatives. The on-line approximation of the unmeasurable uncertain nonlinearities via the DNN structure should heuristically improve the performance of methods that only use high-gain feedback. Asymptotic convergence of the estimated states to the real states is proven using a Lyapunov-based analysis. The developed observer can be used separately from the controller even if the relative degree between the control input and the output is arbitrary. Simulation and experiment results on a two-link robot manipulator indicate the effectiveness of the proposed observer when compared with the standard numerical central differentiation algorithm, along with the high-gain observer proposed in Vasiljevic and Khalil (2008) and the observer in Xian et al. (2004).

2. DNN-based observer development

Consider an Nth order control affine nonlinear system given in MIMO Brunovsky form as

$$\begin{aligned} \dot{x}_1 &= x_2, \\ &\vdots \\ \dot{x}_{N-1} &= x_N, \\ \dot{x}_N &= f(x) + G(x)u + d, \end{aligned} \tag{1}$$

where $x = \begin{bmatrix} x_1^T & x_2^T & \dots & x_N^T \end{bmatrix}^T \in \mathbb{R}^{Nn}$ is the generalized state of the system, $u \in \mathbb{R}^m$ is the control input, $f : \mathbb{R}^{Nn} \to \mathbb{R}^n$, $G : \mathbb{R}^{Nn} \to \mathbb{R}^{n\times m}$ are unknown continuous functions, $d \in \mathbb{R}^n$ is an external disturbance. The following assumptions about the system in (1) will be utilized in the observer development.

Assumption 1. The state x is bounded, i.e., $x_i \in \mathcal{L}_\infty$, $i = 1, 2, \dots, N$, and the state x_1 is measurable up to and including the $N-1$th derivative, i.e. x_i, $i = 1, 2, \dots, N-1$, are measurable.

The states x_i, $i = 1, 2, \dots, N-1$ are available from sensor feedback. However, the higher order state x_N is not used by the subsequent development because it is not typically included as available sensor measurements. The subsequent development does not require feedback of the state x_N. Motivation of this design choice is that it reduces the need for an additional sensor or additional signal processing that is typically not included in the stability analysis. For example, to control the trajectory of robotic manipulator, many results have been developed that only require output feedback (e.g., for the second order system, only position feedback is required). Such results are motivated by the facts that typical robotic systems do not include tachometers and numerical derivatives introduce additional noise. If sufficient sensing of x_N is available, then the developed observer could be simplified (e.g., the subsequently desired dynamic filter could be eliminated) or an alternate method could be used.

Assumption 2. The unknown functions f and G, and the control input u are C^1, and $u, \dot{u} \in \mathcal{L}_\infty$.

Assumption 3. The disturbance d is differentiable, and $d, \dot{d} \in \mathcal{L}_\infty$.

The universal approximation property states that given any continuous function $F : \mathbb{S} \to \mathbb{R}^n$, where $\mathbb{S}$ is a compact set, there exist ideal weights such that the output of the NN, $\hat{F}$ approximates F to an arbitrary accuracy (Hornick, 1991). Hence, the unknown functions f and G in (1) can be replaced by multi-layer NNs (MLNN) as

$$\begin{aligned} f(x) &= W_f^T \sigma_f \left(\sum_{j=1}^{N} V_{f_j}^T x_j \right) + \varepsilon_f(x), \\ g_i(x) &= W_{gi}^T \sigma_{gi} \left(\sum_{j=1}^{N} V_{gi_j}^T x_j \right) + \varepsilon_{gi}(x), \end{aligned} \tag{2}$$

where $W_f \in \mathbb{R}^{L_f+1\times n}$, $V_{f_j} \in \mathbb{R}^{n\times L_f}$ are unknown ideal constant weight matrices of the MLNN having L_f hidden layer neurons, g_i is the ith column of the matrix G, $W_{gi} \in \mathbb{R}^{L_{gi}+1\times n}$, $V_{gi_j} \in \mathbb{R}^{n\times L_{gi}}$ are unknown ideal constant weight matrices of the MLNN having L_{gi} hidden layer neurons, $i = 1 \dots m$, $j = 1, 2, \dots, N$, $\sigma_f : \mathbb{R}^{Nn} \to$