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Brief Papers

Containment for linear multi-agent systems with exogenous disturbances

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ABSTRACT

This paper investigates containment for linear multi-agent systems (LMAS) with exogenous disturbances under fixed topologies. Both state feedback and output feedback control protocols are proposed, under which all the followers will asymptotically converge to the convex hull spanned by the leaders. Using tools from matrix, graph and Lyapunov stability theories, sufficient conditions for containment of linear multi-agent systems with exogenous disturbances are obtained by designing exogenous disturbance observers. Finally, numerical simulations are presented to illustrate the theoretical findings.

1. Introduction

In recent years, distributed cooperative control of multi-agent systems has attracted much attention for its broad potential applications in sensor networks, combat intelligence, surveillance, etc. [1]. Lots of works have been reported including formation control [2,3], flocking [4–6], leaderless consensus [7–9], leader-following consensus [10–12], containment [13–26], and so forth.

Consensus is one of the most fundamental problems in distributed cooperative control, which means that the state of the agents reaches an agreement on a common physical quantity of interest by implementing an appropriate consensus protocol based on the information from local neighbors. According to the number of leaders, consensus problem and its extensions can be roughly classified into leaderless consensus (there is no leader in the network), consensus tracking or leader-following consensus (there is only one leader in the network), containment control (multiple leaders case) [13].

Containment control for multi-agent systems with multiple leaders has been intensively investigated for the past several years. Under fixed topology, for guaranteeing that all the follower agents asymptotically converge into the convex hull formed by multiple stationary/moving leaders, a hybrid control protocol based on the stop-and-go strategy was proposed [14]. Distributed containment control for multi-agent systems with single/double-integrator dynamics was investigated in the presence of multiple stationary or dynamic leaders under fixed and switching directed topologies, respectively [15,16]. Distributed finite-time attitude containment control was studied for multiple rigid bodies [17]. Necessary and sufficient conditions were established for containment of multi-agent systems with stationary or dynamic leaders via both continuous and sampled control protocols [18]. Distributed containment control with multiple dynamic leaders was studied for double-integrator dynamics under the constraints that the velocities and the accelerations of the agents are not available [19]. Based on sample-data control protocol, necessary and sufficient containment conditions were obtained for second-order multi-agent systems without velocity measurement [20]. Impulsive containment for second-order multi-agent systems under a directed network topology was investigated [21]. And the containment was investigated for multi-agent systems with heterogeneous dynamics [22]. Distributed containment was studied for second-order multi-agent systems with inherent nonlinear dynamics [23], in which an adaptive containment protocol was proposed. Considering that linear system can model more natural dynamics in reality, the containment control for general linear multi-agent systems (LMAS) began to be discussed. A state feedback protocol was proposed by solving an algebraic Riccati equation [24]. Both output feedback and state feedback protocols were proposed for containment of linear

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multi-agent systems [25]. The semi-global containment control for general linear multi-agent systems with input saturation was studied [26].

However, external disturbances widely exist in real processes, which is a main source of instability and poor performance. Thereby, this paper focuses the containment for linear multi-agent systems (LMAS) with exogenous disturbances. The consensus of multi-agent systems with disturbances has been studied by many researchers [27–29]. The adaptive consensus problem of multi-agent systems with partly unknown parameters and bounded external disturbances is studied by adopting the model reference adaptive control method and an adaptive disturbance compensator in [27]. By proposing a distributed protocol using the neighbors' measured outputs, output consensus problem of directed networks of multiple high-order agents with external disturbances is studied [28]. The consensus of second-order multi-agent dynamical systems with exogenous disturbances is studied by designing disturbance-observer-based consensus protocol [29]. Motivated by [29,30], disturbance-observer-based containment protocol is used to compensate for the influence of the exogenous disturbances. When the state can be used, a state feedback control protocol is designed based on a state feedback disturbance-observer. While the state cannot be used, an output feedback control protocol is designed based on an output feedback disturbance-observer. The main contribution of this paper is as follows: (1) The plant in this paper is general linear dynamic, which can be used model more real plants. (2) Output feedback containment protocol based disturbance-observer is proposed in this paper, which is more difficult and more practical than the state feedback case. (3) Disturbance-observer is used for disturbance attenuation.

The rest of the paper is organized as follows. In Section 2, we state the model considered in the paper and give some basic definitions, lemmas and assumptions. In Section 3, both state feedback and output feedback containment protocol based disturbance-observer are proposed, and sufficient containment conditions are obtained. Numerical examples are given in Section 4. Finally, we conclude the paper in Section 5.

2. Preliminaries and model description

In this section, some notations and preliminaries are introduced. The following notations are used throughout this paper. I_n denotes the $n \times n$ identity matrix. For a matrix A (or a vector x), A^T (or x^T) represents the transpose of A (or x).

Let $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ be a directed graph with a nonempty set of nodes $\mathcal{V}=(\upsilon_1,\upsilon_2,\ldots,\upsilon_{N+M})$, a set of edges $\mathcal{E}\subseteq\mathcal{V}\times\mathcal{V}$, and a weighted adjacent matrix $\mathcal{A}=[a_{ij}]$. An edge is denoted by (υ_i,υ_j) in a directed graph which means that vertex j can obtain information from vertex i, but not necessarily vice versa. a_{ij} represents the weight of the edge (υ_j,υ_i) and $a_{ij}>0 \Longleftrightarrow (\upsilon_j,\upsilon_i)\in\mathcal{E}$. Node υ_j is called the parent node, node υ_i is the child node, and υ_j is a neighbor of υ_i. The neighbor set $\mathcal{N}_i=\{\upsilon_j|(\upsilon_j,\upsilon_i)\in\mathcal{E}\}$. A graph is undirected if $(\upsilon_j,\upsilon_i)\in\mathcal{E}$ implies $(\upsilon_i,\upsilon_j)\in\mathcal{E}$. A directed path is a sequence of edges in a directed graph of the form $(\upsilon_1,\upsilon_2),(\upsilon_2,\upsilon_3),\ldots$, where $\upsilon_j\in\mathcal{V}$. The Laplacian matrix $\mathcal{L}=[l_{ij}]\in R^{(N+M)\times(N+M)}$ is defined as $l_{ii}=\sum_{j=1,j\neq i}^{N+M} a_{ij}$ and $l_{ij}=-a_{ij}, i\neq j$.

For a multi-agent system, an agent is called leader if the agent has no neighbor, and an agent is called follower if the agent has at least one neighbor. Assume that there are N follower agents, labeled as $1,2,\ldots,N$, and M leaders, labeled as $N+1,N+2,\ldots,N+M$. Denote the set of followers as $\mathcal{F}$ and the set of leaders as $\Re$. Noting that the leaders have no neighbors, $\mathcal{L}$ can be rewritten as

$$\begin{bmatrix}\mathcal{L}_1 & \mathcal{L}_2\\ 0 & 0\end{bmatrix}.$$

The dynamics of the ith agent are described by

$$\dot{x}_i(t)=Ax_i(t)+B(u_i(t)+\omega_i(t)),\\ y_i(t)=Cx_i(t)+D\omega_i(t), \tag{1}$$

where $x_i(t)\in R^n$, $u_i(t)\in R^m$, $\omega_i(t)\in R^m$ and $y_i(t)\in R^p$ are the state, control input, bounded exogenous disturbance and the output of the ith follower agent, respectively. And $A\in R^{n\times n}$, $B\in R^{n\times m}$, $C\in R^{p\times n}$, $D\in R^{p\times n}$ are constant matrices. Suppose that the disturbance $\omega_i(t)$, $i=1,\ldots,N$, is generated by the following linear exogenous system:

$$\dot{\xi}_i(t)=E\xi_i(t),\\ \omega_i(t)=F\xi_i(t), \tag{2}$$

where $\xi_i(t)\in R^l$ denotes the state of the exogenous system, $E\in R^{l\times l}$ and $F\in R^{m\times l}$ are the matrices of the disturbance system.

Definition 1 (*Li et al. [19]*). Let $\mathcal{C}$ be a subset of R^n, the set $\mathcal{C}$ is said to be convex if for any x and y in $\mathcal{C}$ and any $\alpha\in[0,1]$, the point $(1-\alpha)x+\alpha y\in\mathcal{C}$. The convex hull of a set of points $X=\{x_1,x_2,\ldots,x_n\}$ is the minimal convex set containing all points in X. We denote the convex hull of X as $\mathrm{Co}(X)$.

Assumption 1. Suppose that the edges between the followers are undirected, i.e., all the followers can access each other's information. Moreover, for each follower, there exists at least one leader that has a directed path to that follower.

Assumption 2. The matrix pair (A,B) is stabilizable.

Remark 1 (*Liu et al. [24]*). Under Assumption 2, there is a unique symmetric matrix $P>0$ that solves the following algebra Riccati equation:

$$A^TP+PA-PBB^TP+I=0. \tag{3}$$

Lemma 1 (*Li et al. [25]*). *Under Assumption 1, $\mathcal{L}_1$ is positive definite, each entry of $-\mathcal{L}_1^{-1}\mathcal{L}_2$ is nonnegative, and each row of $-\mathcal{L}_1^{-1}\mathcal{L}_2$ has a sum equal to one.*

3. Main results

In this section, the distributed containment control problem with exogenous disturbances will be solved in virtue of disturbance observers, i.e., to design distributed containment protocols to ensure that all the followers can be driven into the convex hull spanned by the leaders asymptotically.

3.1. Containment of LMAS with disturbances via state feedback control

In this subsection, distributed containment protocol is proposed to drive the followers into the convex hull spanned by the leaders via state feedback control. A disturbance observer is designed as follows:

$$\dot{\eta}_i=(E+HBF)(\eta_i-Hx_i)+H(Ax_i+Bu_i),\\ \hat{\xi}_i=\eta_i-Hx_i,\\ \hat{\omega}=F\hat{\xi}_i, \tag{4}$$

where $\eta_i\in R^{l\times l}$ is the internal state variable of the observer, $\hat{\xi}_i$ and $\hat{\omega}$ are the estimated values of ξ_i and ω_i, respectively. $H\in R^{l\times n}$ is the gain matrix of the observer. Denoting $e_i=\xi_i-\hat{\xi}_i$, from (2) and (4), we have

$$\dot{e}_i=(E+HBF)e_i. \tag{5}$$

Remark 2. Noting that

$$\dot{\hat{\xi}}_i=(E+HBF)\hat{\xi}_i-HBF\xi_i,$$