Neurocomputing 160 (2015) 281–286

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Brief Paper

Novel criteria for finite-time stabilization and guaranteed cost control of delayed neural networks

P. Niamsup [a], K. Ratchagit [b], V.N. Phat [c,*]

[a] Department of Mathematics, Chiang Mai University, Chiang Mai 50200, Thailand
[b] Department of Mathematics, Maejo University, Chiang Mai 50290, Thailand
[c] Institute of Mathematics, VAST, 18 Hoang Quoc Viet Road, Hanoi 10307, Vietnam

ARTICLE INFO

Article history:
Received 24 July 2014
Received in revised form
3 December 2014
Accepted 12 February 2015
Communicated by H. Jiang
Available online 24 February 2015

Keywords:
Finite-time stabilization
Neural networks
Guaranteed cost control
Interval time-varying delays
Lyapunov function
Linear matrix inequalities

ABSTRACT

In this paper, the problem of robust finite-time stabilization with guaranteed cost control for a class of delayed neural networks is considered. The time delay is a continuous function belonging to a given interval, but not necessary to be differentiable. We develop a general framework for finite-time stabilization with guaranteed cost control based on the Lyapunov functional method and new generalized Jensen integral inequality. Novel criteria for the existence of guaranteed cost controllers are established in terms of linear matrix inequalities (LMIs). The proposed conditions allow us to design the state feedback controllers which robustly stabilize the closed-loop system in the finite time. A numerical example is given to illustrate the efficiency of the proposed method.

1. Introduction

The concept of finite-time stability (FTS), introduced by Dorato [1], plays an important role in stability theory of dynamical systems. A system is said to be finite-time stable if its state does not exceed a certain threshold during a specified time interval. Compared with the Lyapunov stability, finite-time stability concerns the boundedness of system during a fixed finite-time interval. The problem of FTS has been revisited using linear matrix inequality technique, which allows us to find feasible conditions guaranteeing FTS. A lot of interesting results on finite-time stability and stabilization in the context of linear time-delay systems have been obtained (see, e.g., [2–4] and the references therein). In many practical systems, it is desirable to design the system which is not only finite-time stable but can also guarantee an adequate level of system performance. One approach to this problem is the guaranteed cost control [5–8] in which a fixed quadratic Lyapunov function is used to derive an upper bound on the closed-loop value of an integral quadratic cost function.

Guaranteed cost control problem has the advantage of providing an upper bound on a given system performance index and thus the system performance degradation incurred by the uncertainties or time delays is guaranteed to be less than this bound. The Lyapunov–Krasovskii functional technique has been among the popular and effective tool in the design of guaranteed cost controls for neural networks with time delay. Nevertheless, despite such diversity of results available, most existing work either assumed that the time delays are constant or differentiable. Although, in some cases, delay-dependent sufficient conditions for stability of neural networks with time-varying delays were considered in [9–14], the approach used there cannot be applied to systems with interval, non-differentiable time-varying delays. On the other hand, due to the widespread use in control systems of digital computers that employ finite-precision arithmetic, the signals often need to be quantized before the manipulation of feedback. However, as far as we know, few results are reported on the finite-time stabilization of neural networks with guaranteed cost control feedback, especially of neural networks with interval, non-differentiable time-varying delays.

In this paper, we consider problem of the finite-time stabilization with guaranteed cost control for delayed neural networks. Such systems can be regarded as a special class of functional differential equations, namely dynamical nonlinear time-delay systems [15]. We show how to design guaranteed cost feedback controllers to robustly finite-time stabilizes the closed-loop delay neural networks by using the method of Lyapunov–Krasovskii functionals. The novel features of this research are (i) the neural

* Corresponding author. Tel.: +84 437563474; fax: +84 437564303.
E-mail addresses: piyapong.n@cmu.ac.th (P. Niamsup), kreangkri@mju.ac.th (K. Ratchagit), vnphat@math.ac.vn (V.N. Phat).

http://dx.doi.org/10.1016/j.neucom.2015.02.030
0925-2312/

network under consideration subjected to interval time-varying delays; (ii) nonlinear cost function is considered as a delay performance measure for the closed-loop system; (iii) using new bounding estimation technique by generalized Jenzen integral inequality, a set of simple Lyapunov–Krasovskii functionals is constructed to solve LMI conditions. The LMI stabilizing conditions are easily determined by utilizing MATLABs LMI Control Toolbox [16].

2. Preliminaries

The following notation will be used in this paper. R^+ denotes the set of all non-negative real numbers; R^n denotes the n-dimensional space with the scalar product $\langle x, y\rangle$ or x^Ty of two vectors x, y; $M^{n\times r}$ denotes the space of all matrices of $(n\times r)$-dimensions. A^T denotes the transpose of matrix A; A is symmetric if $A=A^T$; I denotes the identity matrix; $\lambda(A)$ denotes the set of all eigenvalues of A; $\lambda_{\max}(A)=\max\{\mathrm{Re}\,\lambda;\lambda\in\lambda(A)\}$. $x_t := \{x(t+s): s\in[-h,0]\}$, $\|x_t\| = \sup_{s\in[-h,0]}\|x(t+s)\|$; $C([0,T],R^n)$ denotes the set of all R^n-valued continuous functions on $[0,T]$; $L_2([0,T],R^m)$ denotes the set of all the R^m-valued square integrable functions on $[0,T]$; matrix A is called semi-positive definite $(A\geq 0)$ if $\langle Ax,x\rangle\geq 0$, for all $x\in R^n$; A is positive definite $(A>0)$ if $\langle Ax,x\rangle>0$ for all $x\neq 0$; $A>B$ means $A-B>0$. The notation diag $\{\ldots\}$ stands for a block-diagonal matrix. The symmetric term in a matrix is denoted by $*$.

Consider the following neural networks with interval time-varying delay:

$$\dot{x}(t) = -Ax(t)+W_0f(x(t))+W_1g(x(t-h(t)))+Bu(t)+W_2w(t), \quad t\in[0,T],$$
$$x(t)=\phi(t), \quad t\in[-h_2,0], \tag{2.1}$$

where $x(t)=[x_1(t),x_2(t),\ldots,x_n(t)]^T\in R^n$ is the state of the neural, $u(\cdot)\in L_2([0,T],R^m)$ is the control; $w(\cdot)\in L_2([0,T],R^r)$, n is the number of neurals, and $f(x(t))=[f_1(x_1(t)),f_2(x_2(t)),\ldots,f_n(x_n(t))]^T$ and $g(x(t))=[g_1(x_1(t)),g_2(x_2(t)),\ldots,g_n(x_n(t))]^T$ are the activation functions; $A=\mathrm{diag}\,(a_1,a_2,\ldots,a_n), a_i>0$ represents the self-feedback term; $B\in R^{n\times m}$ is the control input matrix; W_0 and W_1 denote the connection weights and the discretely delayed connection weights, respectively; W_2 denotes the connection disturbance. The delay function $h(t)$ is continuous and satisfies the condition

$$0\leq h_1\leq h(t)\leq h_2, \quad t\in[0,T].$$

The initial functions $\phi(t)\in C([-h_2,0],R^n)$ and the disturbance is continuous function satisfying

$$\exists d>0: \quad \int_0^T w^T(t)w(t)\,dt\leq d. \tag{2.2}$$

In this paper, we consider various activation functions and assume that the activation functions are Lipschitzian with the Lipschitz constants $k_i, l_i>0, f_i(0)=g_i(0)=0$:

$$|f_i(\xi_1)-f_i(\xi_2)|\leq k_i|\xi_1-\xi_2|, \quad i=1,2,\ldots,n, \quad \forall\xi_i\in R,$$
$$|g_i(\xi_1)-g_i(\xi_2)|\leq l_i|\xi_1-\xi_2|, \quad i=1,2,\ldots,n, \quad \forall\xi_i\in R, \tag{2.3}$$

Under the above assumptions on $h(\cdot), f(\cdot), g(\cdot)$ and the initial function $\phi(t)$, the system (2.1) has a unique solution $x(t,\phi)$ on $[0,T]$ (see, e.g., [15]). The performance index associated with the system (2.1) is the following function:

$$J=\int_0^T f^0(t,x(t),x(t-h(t)),u(t))\,dt, \tag{2.4}$$

where $f^0(t,x(t),x(t-h(t)),u(t)):[0,T]\times R^n\times R^n\times R^m\to R^+$ is a nonlinear continuous function that satisfies

$$\exists Q_1,Q_2,H: f^0(t,x,y,u)\leq\langle Q_1x,x\rangle+\langle Q_2y,y\rangle+\langle Hu,u\rangle, \tag{2.5}$$

for all $(t,x,y,u)\in R^+\times R^n\times R^n\times R^m$ and $Q_1,Q_2\in R^{n\times n}, H\in R^{m\times m}$ are given symmetric positive definite matrices. The objective of this paper is to design a memoryless state feedback controller $u(t)=Kx(t)$ and a positive number J^* such that the resulting closed-loop system

$$\dot{x}(t)=-(A-BK)x(t)+W_0f(x(t))+W_1g(x(t-h(t)))+W_2w(t), \tag{2.6}$$

is finite-time stable for all disturbances $w(t)$ satisfying (2.2) and the value of the cost function (2.4) is bounded by J^*.

Definition 2.1 (*Dorato [1]*). For a given time $T>0$, numbers $c_2>c_1>0$, and R is a symmetric positive definite matrix, the unforced control system (2.1) $(u(t)=0)$ is robustly finite-time stable w.r.t (c_1,c_2,T,R) if the following relation holds for all disturbances $w(t)$ satisfying (2.2):

$$\sup_{-h_2\leq s\leq 0}\left\{\phi^T(s)R\phi(s),\dot{\phi}^T(s)R\dot{\phi}(s)\right\}\leq c_1\Longrightarrow x^T(t)Rx(t)<c_2, \quad \forall t\in[0,T].$$

Definition 2.2 (*Chang and Peng [5]*). For a given time $T>0$, numbers $c_2>c_1>0$, and R is a symmetric positive definite matrix, if there exist a memoryless state feedback control law $u^*(t)=Kx(t)$ and a positive number J^* such that the closed-loop system (2.6) is robustly finite-time stable w.r.t (c_1,c_2,T,R) and the cost function (2.4) satisfies $J(u^*)\leq J^*$, then the value J^* is a guaranteed cost value and the control $u^*(t)$ is a guaranteed cost controller.

We introduce the following technical well-known propositions, which will be used in the proof of our results.

Proposition 2.1 (*Schur complement lemma, Boyd [17]*). *Given constant matrices X, Y, Z with appropriate dimensions satisfying* $Y=Y^T>0, X=X^T$, *then* $X+Z^TY^{-1}Z<0$ *if and only if*

$$\begin{pmatrix} X & Z^T \\ Z & -Y\end{pmatrix}<0.$$

Proposition 2.2 (*Generalized Jensen inequality, Seuret and Gouaisbaut [18]*). *For a given symmetric matrix* $R>0$ *and any differentiable function* $\varphi:[a,b]\to R^n$, *the following inequality holds*

$$\int_a^b\dot{\varphi}^T(u)R\dot{\varphi}(u)\,du\geq\frac{1}{b-a}(\varphi(b)-\varphi(a))^TR(\varphi(b)-\varphi(a))+\frac{12}{b-a}\Omega^TR\Omega,$$

where $\Omega=(\varphi(b)+\varphi(a))/2-(1/(b-a))\int_a^b\varphi(u)\,du$.

3. Main result

In this section, we give a design of memoryless guaranteed feedback cost control for neural networks (2.1) with interval time-varying delays such that the closed-loop system is robustly

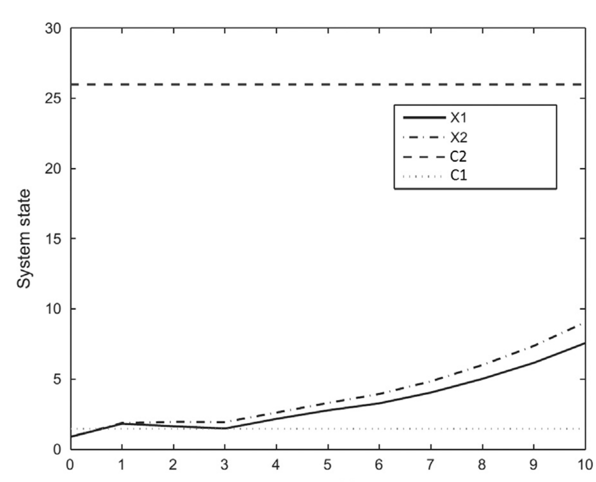

Fig. 1. The trajectories $x_1(t)$, and $x_2(t)$ of closed-loop system.