Neurocomputing 156 (2015) 52–59

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Synchronization of continuous-time Markovian jumping singular complex networks with mixed mode-dependent time delays ☆

Yuechao Ma, Yuqing Zheng *

College of Science, Yanshan University, Qinhuangdao Hebei 066004, PR China

ARTICLE INFO

Article history:
Received 6 August 2014
Received in revised form
12 November 2014
Accepted 1 January 2015
Communicated by Hongyi Li
Available online 9 January 2015

Keywords:
Globally synchronization
Markovian jump
Mixed time-delays
Singular complex networks
Mode-dependent

ABSTRACT

In this paper, the problem of globally synchronization is investigated for an array of N linearly coupled singular complex networks with Markovian jump and mixed time-delays. The complex network systems consist of m modes and the systems switch from one mode to another according to a Markovian chain with known transition probability. The time delays include discrete delays and distributed delays. The singular matrices in the consider systems are mode-dependent. By utilizing the Lyapunov–Krasovskii stability theory, linear matrix inequality (LMI) approach, stochastic analysis techniques and Kronecker product, some sufficient globally synchronization criteria are obtained. Several numerical examples are given to illustrate the feasibility and effectiveness of the proposed methods.

1. Introduction

In the past decades, the researches on the dynamics of complex networks have attracted extensive attention due to their ubiquity in the nature world. Synchronization is a collective behavior phenomenon which not only exists extensively in nature, but also plays an important role in theory and practice. It is widely believed that there exist many benefits of having synchronization in many fields, such as secure communication, modelling brain activity and pattern recognition phenomena [1–3]. In particular, one of the interesting phenomena in complex networks is the synchronization, which is an important research subject with the rapidly increasing research, and there are a lot of results [4–6].

Recently, Markovian jump systems have received many increasing research interests [7–9]. The reason that Markovian jump systems have been paid a great deal of attention is that they are often employed to model the abrupt phenomena such as random failures of the components and sudden environmental changes. Markovian jump systems are more complex than the systems without Markovian jump parameters. The systems without Markovian jump parameters have only one component in the state, but, Markovian jump systems are the hybrid systems with two components in the state. On the other hand, the singular Markovian jump systems with mode-dependent singular matrices have been attracting increasing attention [10]. This class of systems may have wide applications in practical systems. The singular Markovian jump systems with mode-dependent singular matrices may also have applications in other practical systems [11]. Therefore, it is interesting and challenging to study the singular Markovian jump systems with mode-dependent singular matrices.

Complex networks with Markovian jumping parameters are of great significance in modelling complex networks with finite network modes. Dynamics analysis problem of Markovian jumping systems (MJSs) has stirred initial research interests [13,32]. A great number of efforts have been made to investigate the issues of stability, stabilization, and filtering of MJSs [30]. At the same time, several literatures have been published concerning the synchronization analysis of the complex networks with Markovian jumping parameters. For example, in [12,14], the exponential synchronization problem of complex networks with Markovian jumping parameters and mixed delays is investigated. However, it is worth pointing out that, up to now, all the aforementioned results concerning dynamics analysis problems for delayed complex networks with or without Markovian jumping parameters have been applied to continuous-time models [15–20]. Both the discrete time delays and distributed time delays are concerned in the researches of the synchronization problems [21–26]. On the other hand, there has been a growing interest in singular systems for their extensive application in control theory, economics, circuits and other areas. Singular systems can be introduced to improve the traditional complex networks to describe the singular dynamic behaviors of nodes [27–31]. In [3], the synchronization problem for singular hybrid coupled networks with time-varying nonlinear perturbation is studied.

☆ Project supported by National Nature Science Foundation of China (No. 61273004) and the Natural Science Foundation of Hebei Province (No. F2014203085).

* Corresponding author. Tel.: +86 18230353937.
E-mail address: qing1503@hotmail.com (Y. Zheng).

http://dx.doi.org/10.1016/j.neucom.2015.01.001
0925-2312/© 2015 Elsevier B.V. All rights reserved.

Motivated by the above analysis, in this paper, we investigate the problem of globally synchronization for singular complex networks with Markovian jumping parameters as well as mode-dependent mixed time delays. Note that the mixed time delays comprise both the discrete and distributed delays that are dependent on the Markovian jumping mode. By utilized a novel Lyapunov–Krasovskii functional and the Kronecker product, the novel delay dependent synchronization conditions are derived in terms of linear matrix inequality (LMI). The LMI can be easily solved by using the available Matlab LMI toolbox [28].

The main contributions of this paper consist in the development of synchronization for complex networks. A new similar structure has been given, where the singular matrix E_{r_t} taken into consideration, while most previous works failed to do so. Meanwhile, the time-varying delays are mode-dependent. So, the conclusion of this paper can ensure that the system is stable in the random process and make the system have better anti-jamming. Consequently, the delay-dependent sufficient condition for stability is derived in terms of LMIs. The maximum values of delays that the network can tolerate depend not only on the size of the mixed time-varying delays, but also on the occurrence probability distribution of the stochastic discrete time-varying delay. Moreover, the main criterion derived in this paper is successfully extended to singular systems with Markovian jump parameters and stochastic coupling delay, and numerical examples are given to show the effectiveness of this application.

Notation: The notations are quite standard. Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times m}$ denote, respectively, the n-dimensional Euclidean space and the set of all $n\times m$ real matrices. For a real symmetric matrix X and Y, the notation $X\geq Y$ (respectively, $X>Y$) means that the matrix $X-Y$ is semi-positive definite (respectively, positive definite). I is the identity matrix with appropriate dimension. O denotes a matrix with all the elements are zero. For symmetric block matrices or long matrix expressions, the symbol "$*$" is used to represent a term that is induced by symmetry. The superscript T stands for the transpose of a matrix or a vector, diag$(\cdots)$ denotes a block-diagonal matrix. The notation $A\otimes B$ stands for the Kronecker product of matrices A and B. Let $\tau>0$ and $C([-\tau,0],\mathbb{R}^n)$ denote the family of continuous function φ, from $[-\tau,0]$ to $\mathbb{R}^n$ with the norm $|\varphi|=\sup_{-\tau\leq\theta\leq 0}\|\varphi(\theta)\|$, where $\|x\|$ is the Euclidean norm in $\mathbb{R}^n$. Moreover, $(\Omega,\mathcal{F},\{\mathcal{F}_t\},P)$ is a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., the filtration contains all P-null sets and is right continuous). Denote by $L_{F_0}([-\tau,0],\mathbb{R}^n)$ the family of all F_0-measurable $C([-\tau,0],\mathbb{R}^n)$-valued random variable which satisfy $\sup_{-\tau\leq\theta\leq 0}E\{\|\varphi(\theta)\|^2\}\leq\infty$. $E\{\cdot\}$ represents the mathematical expectation, that is $\|x\|^2=x^Tx$. $\lambda_{\max}(\cdot)$ means the largest eigenvalue of a matrix. Matrices, if not explicitly stated, are assumed to have compatible dimensions.

2. System formulation and preliminaries

Neural networks are often distributed by environmental noises that affect the stability of the equilibrium point and by the varying structure parameters that satisfy the Markov process. In this paper, we introduce a more general model of complex networks composing of identical neural networks with Markovian jumping stochastically hybrid couplings and both mixed time-delays as follows:

$$E_{r_t}\dot{x}_i(t)=-D_{r_t}x_i(t)+A_{r_t}f(x_i(t))+B_{r_t}g(x_i(t-d_{r_t}))+C_{r_t}\int_{t-\tau_{r_t}}^{t}h(x_i(s))\,ds+c\sum_{j=1}^{N}G_{ij}\Gamma_{r_t}x_j(t-d_{r_t}) \tag{1}$$

where $x(t)=(x_1(t),x_2(t),\ldots,x_n(t))^T\in R^n$ is the state vector associated with n neurons; E_{r_t} is the singular matrix satisfying rank$(E_{r_t})=r\leq n$; $D_{r_t}=\mathrm{diag}\{d_{1,r_t},d_{2,r_t},\ldots,d_{n,r_t}\}>0$ is a positive matrix; A_{r_t}, $B_{r_t},C_{r_t}\in R^{n\times n}$ are respectively, the connection weight matrix, the discretely delayed connection weight matrix and the distributively delayed connection weight matrix; the matrices A_{0,r_t},B_{0,r_t}, $C_{0,r_t},D_{0,r_t},D_{1,r_t}$ are known real constant matrices with appropriate dimensions; the bounded functions d_{r_t} and τ_{r_t} represent unknown the discrete time delay and the distributed delay of system with $0\leq d_{r_t}\leq d_m$, $0\leq\tau_{r_t}\leq\tau_m$; $c>0$ represents coupling strength; $\Gamma=\mathrm{diag}\{\gamma_1,\gamma_2,\ldots,\gamma_n\}$ is a inner-coupling matrix; $G=(G_{ij})_{N\times N}$ denotes the coupling configuration matrix, if there is a connection between node i and j, then $G_{ij}=G_{ji}=1$ $(i\neq j)$, otherwise $G_{ij}=G_{ji}=0$, the diagonal elements of matrix G are defined by $\sum_{j=1}^{N}G_{ij}=0$ $(i=1,2,\ldots,N)$; $\{r_t=r(t),t\geq 0\}$ is a homogeneous, finite-state Markovian process with right continuous trajectories and taking values in finite set $\mathcal{U}=\{1,2,\ldots,m\}$ with given probability space $(\Omega,\mathcal{F},\{\mathcal{F}_t\},P)$ and the initial mode r_0; let $\Pi=(\pi_{ij})$ $(i,j\in\mathcal{U})$, which denotes the transition rate matrix with transition probability:

$$\Pr(r_{t+\Delta t}=j|r_t=i)=\begin{cases}\pi_{ij}\Delta t+o(\Delta t), & i\neq j\\ 1+\pi_{ii}\Delta t+o(\Delta t), & i=j\end{cases} \tag{2}$$

where $\Delta t>0$, $\lim_{\Delta t\to 0}(o(\Delta t)/\Delta t)=0$, and π_{ij} is the transition rate mode i to mode j satisfying $\pi_{ij}\geq 0$ for $i\neq j$ with $\pi_{ii}=-\sum_{j=1,j\neq i}^{m}\pi_{ij}, i,j\in\mathcal{U}$; $\varphi_0(t)$ is a real-valued initial vector function that is continuous on the interval $[-\tau,0]$, and

$$f(x(t))=(f_1(x_1(t)),f_2(x_2(t)),\ldots,f_n(x_n(t)))^T$$
$$g(x(t))=(g_1(x_1(t)),g_2(x_2(t)),\ldots,g_n(x_n(t)))^T$$
$$h(x(t))=(h_1(x_1(t)),h_2(x_2(t)),\ldots,h_n(x_n(t)))^T$$

denote the continuous nonlinear vector functions.

Throughout this paper, we make the following assumptions, definitions and lemmas:

Assumption 1 (*Liu et al. [19]*). Nonlinear functions $f(\cdot)$, $g(\cdot)$ and $h(\cdot)$ are bounded functions satisfying $f(0)=g(0)=h(0)=0$, and for $i\in\{1,2,\ldots,n\}$ there exist constant l_i^-, l_i^+, σ_i^-, σ_i^+, v_i^- and v_i^+ such that

$$l_i^-\leq\frac{f_i(s_1)-f_i(s_2)}{s_1-s_2}\leq l_i^+$$

$$\sigma_i^-\leq\frac{g_i(s_1)-g_i(s_2)}{s_1-s_2}\leq\sigma_i^+$$

$$v_i^-\leq\frac{h_i(s_1)-h_i(s_2)}{s_1-s_2}\leq v_i^+$$

for all $s_1,s_2\in\mathbb{R}$, $s_1\neq s_2$, where l_i^-, l_i^+, σ_i^-, σ_i^+, v_i^- and v_i^+ are some fixed constants.

Remark 1. Assumption 1 was first introduced in [15]. The constants l_i^-, l_i^+, σ_i^-, σ_i^+, v_i^- and v_i^+ are allowed to be positive, negative or zero. Hence, the resulting nonlinearities functions maybe non-monotonic and more general that the usual sigmoid functions and Lipschitz-type conditions. By adopting such presentation, it would be possible to reduce the conservatism of the main results caused by quantifying the nonlinear functions via an LMI technique.

Lemma 1 (*Koo et al. [6]*). *Given any real matrix $M>0$, any scalars a and b with $a<b$, and a vector function $x(t):[a,b]\to\mathbb{R}^n$ such that the integrals concerned as well defined, then*

$$\left[\int_a^b x(s)\,ds\right]^T M\left[\int_a^b x(s)\,ds\right]\leq(b-a)\int_a^b x^T(s)Mx(s)\,ds \tag{3}$$

Lemma 2 (*Langville and Stewart [28]*). *By the definition of Kronecker product, the following properties hold:*