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New results on exponential synchronization of memristor-based chaotic neural networks

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ABSTRACT

This paper investigates the exponential synchronization of a general class of memristor-based recurrent neural networks with variable delay. Then, by using stability theory of Lyapunov functionals and linear matrix inequalities, the simple feedback controller is designed to achieve synchronization between the master neural network and slave neural network and the exponential convergence rate is given by the algebraic equation. The new sufficient condition for the synchronization controller is given in term of linear matrix inequalities (LMI). Two examples are given to verify our results.

1. Introduction

According to the equation deduced, Chua [1] in 1971 postulated the existence of the fourth fundamental circuit element called memristor (but the three standard components outside of the resistor, capacitor and inductor). 37 years later, in 2008, the United States advanced research member of the HP Laboratories published a paper [2] in Nature to prove the existence of the memristor. Scientists was full of expectation to the new electronic components, and tried to find out the ways to build the memristor-based neural networks to emulate the human brain [3]. Because their promising potential applications in areas such as image processing, pattern recognition, optimization and other areas, memristor-based neural networks (MNN) have attracted increasing interest in scientific community [16–19,21,22]. Pershin and Di Ventra said that they had built a memristor emulator which realizes all required synaptic properties and demonstrated experimentally the formation of associative memory in a simple neural network by two memristor to emulate synapses. Most importantly, the experimental demonstration opens up new possibilities in the understanding of memristor-based neural network to reproduce complex learning, adaptive and spontaneous behavior [4]. The behavior of MNN became more and more noticeable as new technology process nodes were introduced in integrated circuit design [5], where the memristor may be used as a nonvolatile memory switch [6]. However, uncertainty (switch) may appear in MNN due to parameter fluctuation during their implementation, which may result in complexity performance of MNN [1–11,16–19,21,22,33]. As a special complex networks, MNN have also been found to exhibit some complex and unpredictable behaviors including chaotic attractors [7,8]. Some works dealing with chaos synchronization phenomena in MNN have also appeared [7–9,11]. In [7], a periodically intermittent control law was proposed which guarantees exponential synchronization between the master MNN and the slave MNN. In [11], authors have discussed the synchronization problem of a class of MNN and delay-dependent control law was derived to achieve the exponential synchronization. Especially, to my best knowledge, few references have not been addressed on the exponential convergence rate for the exponential synchronization of MNN with memoryless control. This has motivated the study of the exponential synchronization between the MNN by using LMI technique and interval matrix theory [13–15,20,23–32], and the estimation of the exponential synchronization rate is investigated in this paper.

For simplicity, all matrices have appropriate dimensions in the sequels. in this paper, we will mainly deal with the problem of exponential synchronization for a class of memristor-based recurrent neural networks as follows:

$$\begin{aligned}\dot{x}_i(t) = {} & -x_i(t) + \sum_{j=1}^{n} a_{ij}(x_j(t)) f_j(x_j(t)) \\ & + \sum_{j=1}^{n} b_{ij}(x_j(t-\tau_j(t))) g_j(x_j(t-\tau_j(t))) + I_i, \\ & t \geq 0, \quad i = 1, 2, \ldots, n, \end{aligned} \tag{1}$$

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where

$$a_{ij}(x_j(t)) = \begin{cases} \hat{a}_{ij}, & |x_j(t)| \le T_j, \\ \check{a}_{ij}, & |x_j(t)| > T_j, \end{cases}$$

$$b_{ij}(x_j(t-\tau_j(t))) = \begin{cases} \hat{b}_{ij}, & |x_j(t-\tau_j(t))| \le T_j, \\ \check{b}_{ij}, & |x_j(t-\tau_j(t))| > T_j, \end{cases}$$

in which switching jumps $T_i > 0$, $\hat{a}_{ij}, \check{a}_{ij}, i,j = 1,2,\ldots,n$, are all constant numbers, $f_j(\cdot), g_j(\cdot): R \to R, j = 1,2,\ldots,n$ denotes the neuron activation functions, $x_i(t)$ is the voltage of the capacitor, $\tau_j(t)$ corresponds to the transmission delays and satisfies $0 \le \tau_j(t) \le h$, $\tau_j'(t) \le \mu < 1$, and I_i denotes external bounded input.

Remark 1. The authors in [7–9] have given a clear exposition about the relation between memristances and the coefficients of MNN (1), so researchers can consult [7–9] to get more explanation.

The organization of this paper is as follows. Some preliminaries are introduced in Section 2. The main results are given in Section 3. And then, numerical simulations are given to demonstrate the effectiveness of the proposed approach in Section 4. Finally, this paper ends by a conclusion in Section 5.

2. Preliminaries

Throughout this paper, unless otherwise specified, solutions of all the systems considered in the following are intended in Filippovs sense (see [12]). $A \gg 0$ denotes nonnegative definite matrix, and $A > 0$ denotes positive definite symmetric matrix. Let A^T, A^{-1}, $\lambda_{min}(A)$, $\lambda_{max}(A)$, denotes, respectively the transpose of, the inverse of, the minimum eigenvalue of, and the maximum eigenvalue of a square matrix A. $[\cdot,\cdot]$ represents the interval. In Banach space of all continuous functions $C([-h,0],R^n), i = 1,2,\ldots,n$, we define $\|v\| = [\sum_{i=1}^n v_i^2]^{1/2}$, for $\forall v_i(s) \in C([-h,0],R)$. And $\|Q\|$ denotes the operator norm of matrix Q, i.e., $\|Q\| = [\lambda_{max} Q^T Q]^{1/2}$. Let $\overline{a}_{ij} = \max\{\hat{a}_{ij}, \check{a}_{ij}\}$, $\underline{a}_{ij} = \min\{\hat{a}_{ij}, \check{a}_{ij}\}$, for $i,j = 1,2,\ldots,n$. For any $a, b \in R$, $a \bigvee b = \max\{a,b\}$ and $a \bigwedge b = \min\{a,b\}$. For matrix $M = (m_{ij})_{n\times n}, N = (n_{ij})_{n\times n}, M \gg N (M \ll N$ means that $m_{ij} \ge n_{ij}$ $(m_{ij} \le n_{ij})$, for $i,j = 1,2,\ldots,n$. And by the interval matrix $[M,N]$, it follows that $M \ll N$. For $\forall \mathcal{L} = (l_{ij}) \in [M,N]$, it means $M \ll \mathcal{L} \ll N$, i.e., $m_{ij} \le l_{ij} \le n_{ij}$ for $i,j = 1,2,\ldots,n$.

In addition, the initial conditions of system (1) are given by $x_i(s) = \psi_i(s) \in C([-h,0],R)$.

Consider system (1) the master and the corresponding slave system is as

$$\dot{y}_i(t) = -y_i(t) + \sum_{j=1}^n a_{ij}(y_j(t)) f_j(y_j(t)) + \sum_{j=1}^n b_{ij}(y_j(t-\tau_j(t))) g_j(y_j(t-\tau_j(t))) + I_i + u_i(t),$$
$$t \ge 0, \quad i = 1,2,\ldots,n, \tag{2}$$

with initial conditions are of the form $y_i(s) = \phi_i(s) \in C([-h,0],R)$, and where $u_i(t)$ $(i = 1,2,\ldots,n)$ are the appropriate control input that will be designed to obtain synchronization between the master system (1) and the corresponding slave system (2).

In this paper, the following Assumptions (A1) or (B1) and (A2) are needed:

(A1): The functions f_i, g_i, $i \in 1,2,\ldots,n$ are bounded and satisfy the Lipschitz condition with the Lipschitz constants $\rho_i, l_i > 0$, i.e.,

$$|f_i(x) - f_i(y)| \le \rho_i |x-y| \quad \text{for all } x, y \in R.$$

$$|g_i(x) - g_i(y)| \le l_i |x-y| \quad \text{for all } x, y \in R.$$

(B1): The functions f_i, g_i, $i \in 1,2,\ldots,n$ are bounded and satisfy the Lipschitz condition with the Lipschitz constants $\rho_i, l_i > 0$, i.e.,

$$0 \le \frac{f_i(x) - f_i(y)}{x-y} \le \rho_i \quad \text{for all } x, y \in R, x \ne y.$$

$$0 \le \frac{g_i(x) - g_i(y)}{x-y} \le l_i \quad \text{for all } x, y \in R, x \ne y.$$

(A2) For $i,j = 1,2,\ldots,n$,

$$a_{ij}(y_j(t)) f_j(y_j(t)) - a_{ij}(x_j(t)) f_j(x_j(t)) \subseteq [\underline{a}_{ij}, \overline{a}_{ij}](f_j(y_j(t)) - f_j(x_j(t))),$$

$$b_{ij}(y_j(t)) g_j(y_j(t)) - b_{ij}(x_j(t)) g_j(x_j(t)) \subseteq [\underline{b}_{ij}, \overline{b}_{ij}](g_j(y_j(t)) - g_j(x_j(t))).$$

By the systems (1) and (2), we can get the synchronization error dynamical system as

$$\dot{e}_i(t) = -e_i(t) + \sum_{j=1}^n [a_{ij}(y_j(t)) f_j(y_j(t)) - a_{ij}(x_j(t)) f_j(x_j(t))] + \sum_{j=1}^n [b_{ij}(y_j(t-\tau_j(t))) g_j(y_j(t-\tau_j(t))) - b_{ij}(x_j(t-\tau_j(t))) g_j(x_j(t-\tau_j(t)))] + u_i(t), t \ge 0, \quad i = 1,2,\ldots,n, \tag{3}$$

where $e_i(t) = y_i(t) - x_i(t)$.

Supposed that the Assumption A2 is satisfied, applying the theories of set-valued maps and differential inclusions, we have the following inclusion system:

$$\dot{e}_i(t) \in -e_i(t) + \sum_{j=1}^n [\underline{a}_{ij}, \overline{a}_{ij}] F_j(e_j(t)) + \sum_{j=1}^n [\underline{b}_{ij}, \overline{b}_{ij}] G_j(e_j(t-\tau_j(t))) + u_i(t),$$
$$t \ge 0, \quad i = 1,2,\ldots,n, \tag{4}$$

or equivalently, for $i,j = 1,2,\ldots,n$, there exist $\theta_{ij} \in [\underline{a}_{ij}, \overline{a}_{ij}]$, $\upsilon_{ij} \in [\underline{b}_{ij}, \overline{b}_{ij}]$, such that

$$\dot{e}_i(t) = -e_i(t) + \sum_{j=1}^n \theta_{ij} F_j(e_j(t)) + \sum_{j=1}^n \upsilon_{ij} G_j(e_j(t-\tau_j(t))) + u_i(t),$$
$$t \ge 0, \quad i = 1,2,\ldots,n, \tag{5}$$

with initial conditions $\varphi_i(t) = \phi_i(t) - \psi_i(t)$, $i = 1,2,\ldots,n$, where $F_j(e_j(t)) = f_j(y_j(t)) - f_j(x_j(t))$, $G_j(e_j(t)) = g_j(y_j(t)) - g_j(x_j(t))$, $i,j = 1,2,\ldots,n$.

Remark 2. The system (3) and the system (4) are not equivalent, but the system (4) contains the system (3). That is to say, if the system (4) is stabilized then the system (3) must be stabilized, and not vice versa.

Under the Assumption (A1) or (B1), applying the existence theorem for Fillippov solution ([12]), we know that the each solution $x_i(t)$ of the system (4) or (5) with the initial condition exists on the interval $[0, +\infty)$.

For convenience, transform the error system (5) into the vector form as

$$\dot{e}(t) = -e(t) + \Theta F(e(t)) + VG(e(t-\tau(t))) + U(t), \tag{6}$$

where $e(t) = (e_1(t), e_2(t), \ldots, e_n(t))^T$, $\Theta = (\theta_{ij})_{n\times n}, V = (\upsilon_{ij})_{n\times n}$, $U(t) = (u_1(t), \ldots, u_n(t))^T$, $F(e(t)) = (F_1(e_1(t)), F_2(e_2(t)), \ldots, F_n(e_n(t)))^T$, $G(e(t)) = (G_1(e_1(t)), G_2(e_2(t)), \ldots, G_n(e_n(t)))^T$.

To synchronize the master system (1) with the corresponding slave system (2), the controller $U(t)$ will be designed to stabilize the zero of the system (6).

In many engineering application, the following simple memoryless controller is considered:

$$U(t) = \Omega e(t), \tag{7}$$

where $\Omega = (w_{ij})_{n\times n}$.

In this section, we shall give the following definition.