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Brief Papers

Synchronization control of stochastic memristor-based neural networks with mixed delays

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ABSTRACT

In this paper, the synchronization control of stochastic memristor-based neural networks with mixed delays is studied. Based on the drive-response concept, the stochastic differential inclusions theory and Lyapunov functional method some new criteria are established to guarantee the exponential synchronization in the *p*th moment of stochastic memristor-based neural networks. The obtained sufficient conditions can be checked easily and improve the results in earlier publications. Finally, a numerical example is given to illustrate the effectiveness of the new scheme.

1. Introduction

In 1971, based on physical symmetry arguments, Chua [1] conceived and predicted that besides the resistor, capacitor and inductor, there should be a fourth fundamental two-terminal circuit element called a memristor, defined by a nonlinear relationship between charge and flux linkage. In 2008, members of the Hewlett-Packard Laboratories [2] realized the memristor in devise form. The memristor is a two-terminal passive device whose value depends on the magnitude and polarity of the voltage applied to it and the length of the time that the voltage has been applied. In other words, the memristor has variable resistance and exhibits the memory characteristic. For these properties, it is shown that the memristor device has many promising applications such as device modeling, signal processing, one of which is to emulate synaptic behavior [3–8]. As we know, the artificial neural networks can be realized by nonlinear circuits. In the circuits, the connection weights are implemented by fixed value resistors, which are supposed to represent the strength of synaptic connections between neurons in brain. The strength of synapses changes and accords with Hebbian learning rule while the resistance is invariable [9,10]. Therefore, in order to simulate the artificial neural network of human brain better, the resistor is replaced by the memristor, which leads to a new model of neural networks (memristor-based neural networks).

During the past decade, there has been increasing interest in potential applications of chaos synchronization of dynamics systems in many areas such as secure communication, image processing, and harmonic oscillation generation [11–13]. In addition, we note that a plethora of complex nonlinear behaviors including chaos appear even in a simple network of memristor, so a detailed analytical study of synchronization problem of the basic oscillator is necessary. Recently, some achievements about synchronization control of memristor-based neural networks have been obtained. For instance, Wu et al. [14] discussed the synchronization control of a general class of memristor-based recurrent neural networks with time delays. A delay-independent feedback controller is derived based on the drive-response concept, linear matrix inequalities and Lyapunov functional method. Furthermore, Wang and Shen [16] improved the results by employing the Newton–Leibniz formulation and novel inequality technique, and Chandrasekar et al. [17] extended the notion of synchronization of the memristor-based recurrent neural networks with two delay components based on second-order reciprocally convex approach. According to the fuzzy theory, Wen et al. [18] analyzed the global exponential synchronization of memristor-based recurrent neural networks. For more related research, refer to [15,19,20].

On the other hand, in real nervous systems, synaptic transmission is a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. Thus, noise is unavoidable in actual applications of artificial neural

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networks. Meanwhile, stochastic perturbations may make important effects on the dynamic behaviors of delayed system, and a neural network could be stabilized or destabilized by certain stochastic inputs [21,22]. Therefore, it is significant and of prime importance to consider stochastic effects to the synchronization of neural networks with delays. Some results concerning stochastic neural networks have already been presented. Especially, Sun et al. [23] dealt with the exponential synchronization problem for a class of stochastic perturbed chaotic delayed neural networks. By virtue of stochastic analysis, Halanay inequality, time-delay feedback control techniques, several sufficient conditions are proposed to guarantee the exponential synchronization of two identical delayed neural networks with stochastic perturbation. Yu and Cao [24] introduced a novel control method to ensure the global asymptotical stability in mean square for error system based on the Lyapunov functional method and linear matrix inequality technique. Furthermore, Li et al. [25] studied the exponential synchronization of stochastic perturbed chaotic neural networks with mixed delays based on output coupling with delay feedback and linear matrix inequality approach. Liu et al. [26] examined the pth moment exponential synchronization of a class of stochastic perturbed chaotic neural networks with time-varying delays and unbounded distributed delays by establishing two new integro-differential inequalities. Recently, a stochastic memristor-based neural networks is proposed in [27] and the global exponential stability in the mean square for this system is considered. However, to the best of our knowledge, the research on global exponential synchronization of stochastic memristor-based neural networks is still an open problem.

Motivated by the above discussion, we focus our attention on the synchronization problem of stochastic memristor-based neural networks with mixed delays. The main contributions of this paper are to design feedback controllers and give some sufficient conditions to ensure the exponential synchronization in the pth moment of the neural networks system based on the drive-response concept, the stochastic differential inclusions theory and Lyapunov functional method. In this paper, the mixed delays are considered which made the Lyapunov functional become more complicated compared with the ones in [14–20]. Moreover, some inequality techniques are introduced. Our results could be considered as a continuation of the ones in [27]. The structure of this paper is outlined as follows. Some preliminaries are introduced in Section 2. The main results are given in Section 3. And numerical simulations are given to demonstrate the effectiveness of the proposed approach in Section 4. In the last section, a conclusion is drawn.

2. Model description and preliminaries

In this section, referring to some relevant works in [14–20], which deal with the detailed construction of some general classes of memristor-based recurrent neural networks from the aspects of circuit analysis and memristor physical properties, and taking random disturbances into account, we propose a class of stochastic memristor-based neural networks model with discrete and distributed delays described by the following stochastic differential equations:

$$dx_i(t)=\Bigg[-d_ix_i+\sum_{j=1}^{n}a_{ij}(x_i(t))f_j(x_j(t))+\sum_{j=1}^{n}b_{ij}(x_i(t))f_j(x_j(t-\tau(t)))+\sum_{j=1}^{n}c_{ij}(x_i(t))\int_{t-\rho(t)}^{t}f_j(x_j(s))\,ds\Bigg]dt+\beta_i(t,x_i(t),x_i(t-\tau(t))\,d\omega_i(t),\quad i=1,2,\dots,n,\tag{1}$$

where $x(t)=(x_1(t),\dots,x_n(t))^T$ denote the voltage applied on capacitor C_i. The self-feedback connection matrix is $D=\text{diag}\{d_1,d_2,\dots,d_n\}$ with $d_i>0,\ i=1,2,\dots,n$. $f(x(t))=(f_1(x_1(t)),f_2(x_2(t)),\dots,f_n(x_n(t)))^T$ represents the neuron feedback functions. $(\omega_1(t),\omega_2(t),\dots,\omega_n(t))^T$ is n dimensional Brown motion. $\tau(t),\rho(t)$ are two time-varying delays satisfying $0\le\tau(t)\le\tau_1,\ \ 0\le\rho(t)\le\tau_2$, $|\tau'(t)|\le\sigma<1$. $a_{ij}(x_i(t)),b_{ij}(x_i(t))$ and $c_{ij}(x_i(t))$ are memristive synaptic connection weights, and

$$a_{ij}(x_j(t))=\frac{M_{ij}}{C_i}\times\text{sign}_{ij},\quad b_{ij}(x_j(t))=\frac{\tilde{M}_{ij}}{C_i}\times\text{sign}_{ij},$$
$$c_{ij}(x_j(t))=\frac{\overline{M}_{ij}}{C_i}\times\text{sign}_{ij},\quad \text{sign}_{ij}=\begin{cases}1, & i\neq j\\ -1, & i=j\end{cases}$$

in which $M_{ij},\tilde{M}_{ij},\overline{M}_{ij}$ denote the memductances of memristors $R_{ij},\tilde{R}_{ij},\overline{R}_{ij}$, respectively. In addition, R_{ij} represents the memristor between the feedback function $f_i(x_i(t))$ and $x_i(t)$, $\tilde{R}_{ij}$ represents the memristor between the feedback function $f_i(x_i(t-\tau(t))$ and $x_i(t)$, and $\tilde{R}_{ij}$ represents the memristor between the feedback function $\int_{t-\rho(t)}^{t}f_i(x_i(s))\,ds$ and $x_i(t)$. As is well known, capacitor C_i is invariant while memductances of memristors $M_{ij},\tilde{M}_{ij},\overline{M}_{ij}$ respond to change in pinched hysteresis loops [14,17–19]. Consequently, $a_{ij}(x_i(t))$, $b_{ij}(x_i(t))$ and $c_{ij}(x_i(t))$ will change. Based on the feature of memristor and current–voltage characteristic, we let

$$a_{ij}(x_i(t))=\begin{cases}\hat{a}_{ij}, & |x_i(t)|\le T_i\\ \check{a}_{ij}, & |x_i(t)|>T_i;\end{cases}\quad b_{ij}(x_i(t))=\begin{cases}\hat{b}_{ij}, & |x_i(t)|\le T_i\\ \check{b}_{ij}, & |x_i(t)|>T_i;\end{cases}$$
$$c_{ij}(x_i(t))=\begin{cases}\hat{c}_{ij}, & |x_i(t)|\le T_i\\ \check{c}_{ij}, & |x_i(t)|>T_i\end{cases}$$

for $i,j=1,2,\dots,n$, where $\hat{a}_{ij},\check{a}_{ij},\ \hat{b}_{ij},\ \check{b}_{ij},\hat{c}_{ij},\ \check{c}_{ij}$ are known constants with respect to memristance. Moreover, the initial condition of system (1) is assumed to be $x(s)=\varphi(s),\varphi(s)\in C([-\tau,0],R^n),\tau=\max\{\tau_1,\tau_2\}$.

Remark 1. According to the analysis above, $a_{ij}(x(t))$, $b_{ij}(x(t))$, $c_{ij}(x(t))$ in this network are changed according to the state of the system as memristance are switched. Therefore, memristor-based neural networks is considered as a stochastic system with state-dependent switching. When $a_{ij}(x(t)),\ b_{ij}(x(t)),\ c_{ij}(x(t))$ are constants, (1) becomes the general stochastic recurrent neural networks.

Throughout this paper, solutions of all the system considered in the following are intended in Filippov's sense, where $[\cdot,\cdot]$ represents the interval. R^n and $R^{n\times m}$ are the n-dimensional Euclidean space and the set of all $m\times n$ real matrices respectively, and $x\in\{x_1,x_2,\dots,x_n\}^T\in R^n$ denotes a column vector defined by $\|x\|=(\sum_{i=1}^{n}|x_i|^p)^{1/p},\ p\ge1$. $C([-\tau,0],R^n)$ is a Banach space of all continuous functions. $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\ge0},P)$ is a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t\ge0}$ satisfying the usual condition. Let $L^p_{\mathcal{F}_0}([-\tau,0];R^n)$ be the family of all $\mathcal{F}_0$–measurable $C([-\tau,0],R^n)$–valued random variables $\xi=\{\xi(s):-\tau\le s\le0\}$ such that $\sup_{-\tau\le s\le0}E|\xi(s)|^p<\infty$. Set $\overline{a}_{ij}=\max\{\hat{a}_{ij},\check{a}_{ij}\},\underline{a}_{ij}=\min\{\hat{a}_{ij},\check{a}_{ij}\},\overline{b}_{ij}=\max\{\hat{b}_{ij},\check{b}_{ij}\},\underline{b}_{ij}=\min\{\hat{b}_{ij},\check{b}_{ij}\}$, $\overline{c}_{ij}=\max\{\hat{c}_{ij},\check{c}_{ij}\}$, $\underline{c}_{ij}=\min\{\hat{c}_{ij},\check{c}_{ij}\}$, $\overline{A}=(\overline{a}_{ij})_{n\times n}$, $\underline{A}=(\underline{a}_{ij})_{n\times n}$, $\overline{B}=(\overline{b}_{ij})_{n\times n}$, $\underline{B}=(\underline{b}_{ij})_{n\times n}$, $\overline{C}=(\overline{c}_{ij})_{n\times n}$, $\underline{C}=(\underline{c}_{ij})_{n\times n}$ for $i,j=1,2,\dots,n$. $\text{co}[u,v]$ denotes closure of the convex hull generated by real numbers u and v or real matrices u and v. $\text{diag}\{\cdots\}$ stands for a block-diagonal matrix. The superscript "T" denotes the transposition and the notation $X\le Y$ (similarly, $X>Y$), where X and Y are symmetric matrices, means that $X-Y$ is positive semi-definite (similarly, positive definite).

By applying the theories of set-valued maps and stochastic differential inclusions above, the memristor-based network (1) can