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Brief Papers

New mean square exponential stability condition of stochastic fuzzy neural networks ☆

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ABSTRACT

This paper investigates the stability problem for interval type-2 (IT2) stochastic fuzzy neural networks. Firstly, an IT2 stochastic fuzzy neural network is constructed. Secondly, by using stochastic analysis approach and Itô's differential formula, a new sufficient condition ensuring mean square exponential stability is obtained. The condition can be expressed in terms of convex optimization problem. The main contribution of this paper is that the IT2 stochastic fuzzy neural network with parameter uncertainties is first proposed. The parameter uncertainties are bounded, and can be effectively expressed by upper and lower membership functions. Two numerical examples are proposed to show the effectiveness of the proposed scheme.

1. Introduction

Over the past years, neural networks have been widely used in many practical applications such as signal processing, combinatorial optimization and image processing. Many research results have been proposed for neural networks, such as [1–8], and the references therein. In real nervous systems, it is easy to see that stochastic disturbances are nearly inevitable and affect the stability of the neural networks. Hence, it implies that the stability analysis of stochastic neural networks has primary significance in the research of neural networks. Some related research results have been published in [9–13].

Fuzzy logic control scheme has been proposed as an effective method to deal with complex nonlinear systems [14–16]. In recent years, the fuzzy logic control method has been widely used in many practical applications such as chemical processes, automotive systems and robotics systems [17–26]. Takagi–Sugeno (T–S) fuzzy system [27] is a well-known fuzzy system in model-based fuzzy control. The results on stability analysis and stabilization, control problem and filtering problem for T–S fuzzy systems were reported in [28–33]. Recently, the T–S fuzzy control approach has been used in the study of stochastic fuzzy neural network [34–38]. In [35], the authors considered the mean square exponential stability problem of stochastic fuzzy neural networks with time-varying delays. However, it should be mentioned that the above research results are based on type-1 T–S stochastic fuzzy neural networks. By using the type-1 T–S fuzzy model, the stability problem cannot be studied if the nonlinear plants contain parameter uncertainties. Once nonlinear plants contain parameter uncertainties, it will result in the uncertainties of membership functions. To solve this problem, the authors in [39] proposed an IT2 fuzzy logic model, which can be utilized to investigate the nonlinear plants problem. Recently, the authors in [40,41] have studied the problem of the IT2 fuzzy systems by using the upper and lower membership functions which can be used to deal with parameter uncertainties problem. However, it is mentioned that, so far, there are few results on stability analysis of IT2 stochastic fuzzy neural networks.

Motivated by the above discussion, this paper investigates the stability problem for IT2 stochastic fuzzy neural networks subject to parameter uncertainties. The parameter uncertainties, which can be obtained by the lower and upper membership functions, are bounded. By using stochastic analysis approach and Itô differential formula, a novel sufficient condition ensuring mean square exponential stability for stochastic fuzzy neural networks is achieved. Two examples are given to show the feasibility of the proposed scheme. The remaining parts of this paper are organized as follows. Section 2 introduces IT2 stochastic fuzzy neural networks. Section 3 proposes stability condition for IT2 stochastic fuzzy neural networks and

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Section 4 provides two illustrative examples to show the merits of the proposed results. Section 5 concludes this paper.

Notation: The superscript "T" denotes matrix transposition, and "-1" stands for matrix inverse. $\mathbb{R}^n$ is the n-dimensional Euclidean space and the notation $X>0(\geq 0)$ stands for a symmetric and positive definite (semi-definite) matrix. The symbol $\star$ within a symmetric block matrix represents the symmetric terms, and diag$\{\ldots\}$ denotes a block-diagonal matrix. He(A) is defined as $\text{He}(A)=A+A^T$ for simplicity. If not explicitly stated, all matrices are assumed to be compatible dimensions for algebraic operations. $\mathcal{L}_2[0,+\infty)$ stands for the Hilbert space of square integrable functions over $[0,+\infty)$. $\mathcal{E}\{x\}$ means the expectation of the stochastic variable x.

2. Problem formulation

As discussed in [34], we consider the following IT2 stochastic fuzzy neural network.

Plant Rule i : **IF** $g_1(x(t))$ *is* η_1^i *AND* $\cdots$ *AND* $g_p(x(t))$ *is* η_p^i, **THEN** :

$$dx(t)=[-A_ix(t)+B_if(x(t))]\,dt+[C_ix(t)+D_if(x(t))]\,dw(t), \tag{1}$$

where $g_a(x(t))$ denotes the premise variable and η_a^i is an IT2 fuzzy set, $i=1,2,\ldots,r, a=1,2,\ldots,p$. p is a positive integer. $x(t)=[x_1(t), x_2(t),\ldots,x_n(t)]^T$ stands for the neural state, $f(x(t))=[f_1(x_1(t)), f_2(x_2(t)),\ldots,f_n(x_n(t))]^T$ denotes the neuron activation function, and $w(t)$ is a one-dimensional Brownian motion satisfying $\mathcal{E}\{dw(t)\}=0$ and $\mathcal{E}\{dw^2(t)\}=dt$. $A_i=\text{diag}\{a_{1i,}a_{2i},\ldots,a_{ni}\}$ is a positive diagonal matrix, $B_i\in\mathbb{R}^{n\times n}$ and $D_i\in\mathbb{R}^{n\times n}$ are the connection weight matrices, $C_i\in\mathbb{R}^{n\times n}$ is the known constant matrix. The following interval sets denote firing strength of the ith rule:

$$W_i(x(t))=\left[\underline{\delta}_i(x(t)),\quad \overline{\delta}_i(x(t))\right],\quad i=1,2,\ldots,r,$$

where $\underline{\delta}_i(x(t))=\prod_{a=1}^{p}\underline{\vartheta}_{\eta_a^i}(g_a(x(t)))\geq 0$ and $\overline{\delta}_i(x(t))=\prod_{a=1}^{p}\overline{\vartheta}_{\eta_a^i}(g_a(x(t)))\geq 0$ denote the lower and upper grades of membership, respectively. $\underline{\vartheta}_{\eta_a^i}(g_a(x(t)))\geq 0$ and $\overline{\vartheta}_{\eta_a^i}(g_a(x(t)))\geq 0$ stand for the lower membership function and upper membership function, respectively. Therefore, it can be found that $\overline{\vartheta}_{\eta_a^i}(g_a(x(t)))\geq\underline{\vartheta}_{\eta_a^i}(g_a(x(t)))$ and $\overline{\delta}_i(x(t))\geq\underline{\delta}_i(x(t))$ for all i. Then, the IT2 stochastic fuzzy neural network is obtained as

$$dx(t)=\sum_{i=1}^{r}\tilde{\delta}_i(x(t))\{[-A_ix(t)+B_if(x(t))]\,dt+[C_ix(t)+D_if(x(t))]\,dw(t)\}, \tag{2}$$

where

$$\tilde{\delta}_i(x(t))=\underline{\alpha}_i(x(t))\underline{\delta}_i(x(t))+\overline{a}_i(x(t))\overline{\delta}_i(x(t))\geq 0,$$

$$0\leq\tilde{\delta}_i(x(t))\leq 1,\qquad \sum_{i=1}^{r}\tilde{\delta}_i(x(t))=1,\forall i,$$

in which $0\leq\underline{\alpha}_i(x(t))\leq 1$ and $0\leq\overline{a}_i(x(t))\leq 1$ are nonlinear functions and possess the trait of $\underline{\alpha}_i(x(t))+\overline{a}_i(x(t))=1$ for all i. $\tilde{\delta}_i(x(t))$ stands for the normalized membership functions. In order to have a simple description, $\tilde{\delta}_i(x(t))$ is denoted as $\tilde{\delta}_i$.

Remark 1. The difference between other existing ones and IT2 stochastic fuzzy neural networks is that the IT2 stochastic fuzzy neural network contains parameter uncertainties. The parameter uncertainties can result in the uncertainties of the membership functions. In this paper, the membership functions with parameter uncertainties can be obtained by the upper and lower membership functions and relevant weighting functions.

We give the following assumption and definition which are used over this paper.

Assumption 1. The neural activation function $f_j(x_j)$ satisfies

$$h_j^-\leq\frac{f_j(x_j)}{x_j}\leq h_j^+,\quad \forall x_j\in\mathbb{R},\quad j=1,2,\ldots,n,$$

it can follows that

$$\frac{f_j(x_j)-h_j^-x_j}{x_j}\geq 0,\quad \frac{h_j^+x_j-f_j(x_j)}{x_j}\geq 0,$$

where h_j^- and h_j^+ are some constants.

Definition 1. The IT2 stochastic fuzzy neural network (2) is said to be mean square exponentially stability if there is a positive constant λ such that

$$\lim_{t\to+\infty}\sup\frac{1}{t}\log\,\mathcal{E}\{\|x(t)\|^2\}\leq-\lambda. \tag{3}$$

The main purpose of this paper is to obtain the stability condition under which the IT2 stochastic fuzzy neural network (2) is mean square exponentially stability.

3. Main results

In this section, the stability condition for the IT2 stochastic fuzzy neural network (2) is first proposed in Theorem 1. Based on the linear matrix inequality (LMI) approach, we can have the following theorem. For presentation convenience, we denote $L_1=\text{diag}\{h_{1,}^-h_{2,}^-\ldots,h_n^-\}$, $L_2=\text{diag}\{h_{1,}^+h_{2,}^+\cdots,h_n^+\}$.

Theorem 1. *The IT2 stochastic fuzzy neural network (2) is mean square exponentially stable, if there exist matrices $P>0$, $G_1=\text{diag}\{\mu_{1,}\mu_{2,}\ldots,\mu_n\}\geq 0$, $G_2=\text{diag}\{\nu_{1,}\nu_{2,}\ldots,\nu_n\}\geq 0$, $K=\text{diag}\{k_{1,}k_{2,}\ldots,k_n\}\geq 0$ with appropriate dimensions, such that the following LMI holds for $i=1,2,\ldots,r$:*

$$\begin{bmatrix}\Theta_{11} & \Theta_{12} & C^T\overline{P}\\ \star & \Theta_{22} & D^T\overline{P}\\ \star & \star & -\overline{P}\end{bmatrix}<0, \tag{4}$$

where

$$\Theta_{11}=\text{He}(-PA_i+L_1G_1A_i-L_2G_2A_i-L_2KL_1),$$
$$\Theta_{12}=PB_i-L_1G_1B_i+L_2G_2B_i-A_iG_1+A_iG_2+L_2K+L_1K,$$
$$\Theta_{22}=\text{He}(G_1B_i-G_2B_i-K),$$
$$\overline{P}=P+G_1L_2-G_1L_1+G_2L_2-G_2L_1.$$

Proof. Denoting $\Phi_1(t)=-A_ix(t)+B_if(x(t))$, $\Phi_2(t)=C_ix(t)+D_if(x(t))$, then the system (2) can be obtained as follows:

$$dx(t)=\sum_{i=1}^{r}\tilde{\delta}_i\{\Phi_1(t)\,dt+\Phi_2(t)\,dw(t)\}. \tag{5}$$

From Assumption 1, for any positive diagonal matrix K, we can have

$$-2\sum_{j=1}^{n}k_j(f_j(x_j(t))-h_j^+x_j(t))(f_j(x_j(t))-h_j^-x_j(t))\geq 0,$$

that is

$$-2(f(x(t))-L_2x(t))^TK(f(x(t))-L_1x(t))\geq 0. \tag{6}$$

Consider the following Lyapunov function:

$$V(t)=x^T(t)Px(t)+2\sum_{j=1}^{n}\mu_j\int_0^{x_j}(f_j(s)-h_j^-s)\,ds$$
$$+2\sum_{j=1}^{n}\nu_j\int_0^{x_j}(h_j^+s-f_j(s))\,ds. \tag{7}$$