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Adaptive almost sure asymptotically stability for neutral-type neural networks with stochastic perturbation and Markovian switching ☆

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ABSTRACT

The problem of stability via adaptive controller is considered for time-delay neutral-type neural networks with stochastic noise and Markovian switching in this paper. A new criterion of almost sure (a.s.) asymptotic stability for a general neutral-type stochastic differential equation is proposed. Based on this criterion, and by using of the generalized Itô's formula and the M-matrix method, a delay dependent sufficient condition is established to ensure the almost sure asymptotic stability for neutral-type neural networks with stochastic perturbation and Markovian switching. Meanwhile, the update law of the feedback control is determined. A numerical example is provided to verify the usefulness of the criterion proposed in this paper.

1. Introduction

Recently, the stability of neutral-type neural networks, which depend on the derivative of the state and the delay state have attracted a lot of attention (see e.g. [1–8] and the references therein) since the fact that some physical systems in the real world can be described by neutral-type models (see [9]).

As we know, the synaptic transmission in real nervous systems can be viewed as a noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes [10]. In general, Gaussian noise has been regarded as the disturbance arising in neural networks (see e.g. [11–19], and the references therein).

Also, it has been shown that many neural networks may experience abrupt changes in their structure and parameters due to the phenomena such as component failures or repairs, changing subsystem interconnections and abrupt environmental disturbances. In this situation, neural networks may be treated as systems which have finite modes, and the modes may switch from one to another at different times, and can be described by finite-state Markov chains. The stability analysis problem for neural networks with Markovian switching has therefore received much research attention (see e.g. [13,14,17,20–22], and the references therein).

Although the importance of adaptive stabilization has been widely recognized, no related results have been established for time-delay neutral-type neural networks with Markovian switching and stochastic perturbation. Motivated by the studies mentioned above, we aim to tackle the problem of almost sure asymptotic stability for time-delayed neural networks with stochastic noise and Markovian switching via adaptive control. A new criterion of almost sure (a. s.) asymptotic stability for a general neutral-type stochastic differential equation is proposed. Based on this criterion, and by using of the generalized Itô's formula and the *M*-matrix method, a delay dependent sufficient condition is established to ensure the almost sure asymptotic stability for neutral-type neural networks with stochastic perturbation and Markovian switching. Meanwhile, the update law of the feedback control is determined. A numerical example is provided to verify the usefulness of the criterion proposed in this paper.

The attributions of this work lie in two aspects. Firstly, a new criterion of almost sure asymptotic stability for a general neutral-type stochastic differential equation is proposed which extends

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the existing results. The second one is that we concern with the M-matrix method to obtain the delay dependent sufficient condition of the almost sure asymptotic stability for neutral-type neural networks with stochastic perturbation and Markovian switching.

2. System and problem description and preliminaries

Consider an n-dimensional time-delay neutral-type neural network with Markovian switching and stochastic noise of the form

$$\begin{aligned} d[x(t)-D(r(t))x(t-\tau)] \\ =[-A(r(t))x(t)+W(r(t))\varphi(x(t)) \\ +W_d(r(t))\varphi(x(t-\tau))+U(r(t))]\,dt \\ +g(t,r(t),x(t),x(t-\tau))\,d\omega(t) \end{aligned} \quad (1)$$

where $x(t)=[x_1(t)\ldots x_n(t)]^T\in\mathbb{R}^n$ is the state vector associated with the n neurons. $\varphi(x(t))=[\varphi_1(x_1(t))\ldots\varphi_n(x_n(t))]^T$ denotes the neuron activation function, τ denotes the constant time-delay.

$\{r(t),t\ge 0\}$ is a right-continuous Markov chain on the complete probability space $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\ge0},\mathbb{P})$ with a filtration $\{\mathcal{F}_t\}_{t\ge0}$ satisfying the usual conditions (i.e., it is increasing and right continuous while $\mathcal{F}_0$ contains all $\mathbb{P}$-null sets). The Markov chain takes values in a finite state space $\mathbb{S}=\{1,2,\ldots,S\}$ with generator $\Gamma=(\gamma_{ij})_{S\times S}$ given by

$$\mathbb{P}\{r(t+\Delta)=j\,|\,r(t)=i\}=\begin{cases}\gamma_{ij}\Delta+o(\Delta) & \text{if } i\ne j\\ 1+\gamma_{ii}\Delta+o(\Delta) & \text{if } i=j\end{cases}$$

where $\Delta>0$. Here $\gamma_{ij}\ge 0$ is the transition rate from i to j if $i\ne j$ while $\gamma_{ii}=-\sum_{j=1,j\ne i}^{S}\gamma_{ij}$.

$A(r(t))\in\mathbb{R}^{n\times n}$ (A_i, $r(t)=i$, for short) is a diagonal matrix with the all positive elements. $W(r(t))\in\mathbb{R}^{n\times n}$ and $W_d(r(t))\in\mathbb{R}^{n\times n}$ are the connection weight matrix and the time-delay connection weight matrix, respectively. $D(r(t))\in\mathbb{R}^{n\times n}$ is external input matrix, $U(r(t))\in\mathbb{R}^n$ is the control input vector.

g is the noise intensity function satisfying $g:\mathbb{R}\times\mathbb{S}\times R^n\times\mathbb{R}^n\to\mathbb{R}^{n\times m}$.

$\omega(t)=[\omega_1(t),\omega_2(t),\ldots,\omega_m(t)]^T$ be an m-dimensional $\mathcal{F}_t$-adapted Brownian motion. It is assumed that $\omega(t)$ in system (1) are independent.

The initial data is given by $\{x(\theta):-\tau\le\theta\le 0\}=\xi(\theta)\in L^2_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n])$, $r(0)=r_0$, $\xi(0)=0$.

For system (1), we impose the following assumptions.

Assumption 1. Each function $\varphi_j:\mathbb{R}\to\mathbb{R}$ is nondecreasing and there exists a positive constant Φ such that

$$|\varphi_j(u)-\varphi_j(v)|\le\Phi|u-v| \quad \forall u,v\in\mathbb{R},\ \ j=1,2,\ldots,n.$$

Assumption 2. $\forall i\in\mathbb{S}$, there exist two positive constants G_1 and G_2 such that

$$\begin{aligned}&\mathrm{tr}[g^T(t,i,x(t),x(t-\tau))g(t,i,x(t),x(t-\tau))]\\ &\quad\le x^T(t)G_1x(t)+x^T(t-\tau)G_2x(t-\tau)\end{aligned} \quad (2)$$

and $g(t,i,0,0)=0$.

Assumption 3. For the external input matrix $D_i(i\in\mathbb{S})$, there exists positive constant $\kappa_i\in(0,1)$, such that

$$\rho(D_i)=\kappa_i\le\kappa\in(0,1),$$

where $\kappa=\max_{i\in\mathbb{S}}\kappa_i$ and $\rho(D_i)$ is the spectral radius of matrix D_i.

We now begin with the following concept of a.s. asymptotical stability in mean square.

Definition 1. The neutral-type neural networks (1) is said to be almost sure asymptotical stable if for any $\xi(t)\in L^2_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n])$ and $r_0\in\mathbb{S}$,

$$\mathbb{P}(\lim_{t\to\infty}|x(t;i_0,\xi(t))|=0)=1. \quad (3)$$

where $x(t;i_0,\xi(t))$ is the solution of the system (1) for the initial condition $\xi(t)$.

Now, we describe the problem to solve in this paper as follows.

Target description. For the neutral-type neural network (1), by using Lyapunov functional, general Itô's formula, and M-matrix method, this paper will obtain some criteria of almost surely asymptotically stability for the neutral-type time-delay neural networks with stochastic disturbance and Markovian switching.

The following lemmas are useful for obtaining the main result.

Lemma 1 (*Wang et al. [11]*). *Let $x,y\in\mathbb{R}^n$, then the inequality $x^Ty+y^Tx\le\epsilon x^Tx+\epsilon^{-1}y^Ty$ holds for any $\epsilon>0$.*

Lemma 2 (*See Kolmanovskii et al. [23]*). *Let $p\ge1$, and Assumption 3 holds. Then*

$$\begin{aligned}&-|x(t)-D_ix(t-\tau)|^p\\ &\quad\le-(1-\kappa)^{p-1}|x(t)|^p+\kappa(1-\kappa)^{p-1}|x(t-\tau)|^p.\end{aligned}$$

Lemma 3 (*Yong inequality, see Mao and Yuan [24]*). *Let $a,b\in\mathbb{R}$ and $\phi\in[0,1]$. Then $|a|^{\phi}|b|^{1-\phi}\le\phi|a|+(1-\phi)|b|$.*

Lemma 4 (*See Mao and Yuan [24]*). *If $M=(m_{ij})_{n\times n}\in\mathbb{R}^{n\times n}$ with $m_{ij}<0(i\ne j)$, then the following statements are equivalent:*

(i) *M is a nonsingular M-matrix.*
(ii) *Every real eigenvalue of M is positive.*
(iii) *M is positive stable. That is, M^{-1} exists and $M^{-1}>0$ (i.e. $M^{-1}\ge0$ and at least one element of M^{-1} is positive).*

Lemma 5 (*See Zhou et al. [25]*). *Let Assumptions 1–3 hold. Assume that there are functions $V\in\mathbb{C}^{2,1}(\mathbb{R}_+\times\mathbb{S}\times\mathbb{R}^n;\mathbb{R}_+)$, the family of positive real-valued functions defined on $\mathbb{R}_+\times\mathbb{S}\times\mathbb{R}^n$ which are continuously twice differentiable in $x\in\mathbb{R}^n$ and once differentiable in $t\in\mathbb{R}_+$, $\gamma\in\mathbb{L}^1(\mathbb{R}_+;\mathbb{R}_+)$, and $W^1,W^2,W^3\in\mathbb{C}(\mathbb{R}^n;\mathbb{R}_+)$ such that*

(C1) $$\lim_{|x|\to\infty}[\inf_{(t,i)\in\mathbb{R}_+\times\mathbb{S}}V(t,i,x)]=\infty. \quad (4)$$

(C2) $$\begin{aligned}&\mathcal{L}V(t,i,x,x_\tau)\\ &\quad\le\gamma(t)-W^1(x)+W^2(x_\tau)-W^3(x-D_ix_\tau)\end{aligned} \quad (5)$$

for $(t,i,x,x_\tau)\in\mathbb{R}_+\times\mathbb{S}\times\mathbb{R}^n\times\mathbb{R}^n$, where $\mathcal{L}V$ is the weak infinitesimal operator of the random process $\{(r(t),x(t)):t\ge0\}$, and $x_\tau=x(t-\tau)$.

(C3) $$W^1(0)=W^2(0)=W^3(0)=0, W^1(x)\ge W^2(x)\forall x\ne=0 \quad (6)$$

Then for any initial data $\{x(\theta):-\tau\le\theta\le0\}=\xi\in\mathbb{C}^b_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$ and $r(0)=i_0\in\mathbb{S}$,

(R1) *Eq. (1) has a unique global solution $x(t;i_0,\xi)$.*
(R2) *Assume that $W^3(x)=0$ if and only if $x=0$. The solution $x(t;i_0,\xi)$ obeys that*

$$\lim_{t\to\infty}x(t;i_0,\xi)=0\ a.s. \quad (7)$$

i.e. $x(t;i_0,\xi)$ is almost surely asymptotically stable.

3. Main results

We are now in a position to derive the condition under which the neutral-type time-delay neural networks (1) with stochastic disturbance and Markovian switching is almost surely asymptotically stable. We will divide the discussion into two parts: (1) $p\ge3$ and (2) $p=2$.

Theorem 1. *Let Assumptions 1–3 hold, and $p\ge3$. Assume that $M:=-\mathrm{diag}\{\underbrace{\eta,\eta,\ldots,\eta}_{S}\}-\Gamma$ is a nonsingular M-matrix, where*

$$\eta=-p\varsigma+(1/2)(p-2)(\kappa^2+\overline{\alpha}^2+2\Phi^2+(p-1)(G_1+G_2))$$