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Finite-time state estimation for delayed Hopfield neural networks with Markovian jump

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ABSTRACT

In this paper, the finite-time state estimation problem of delayed Hopfield neural networks with Markovian jump is investigated. The activation functions are assumed to satisfy the section condition. A discontinuous estimator is designed through available output measurements such that the estimation error converges to the origin in finite time. The conditions that the desired estimator parameters need to satisfy are derived by using the Lyapunov stability theory and inequality technique. These conditions are provided in terms of the linear matrix inequalities. Finally, the effectiveness of the proposed method is illustrated by means of a numerical example.

1. Introduction

In the past few years, neural networks have been applied in various areas such as signal processing, parallel computation and optimization solvers [1–4]. In these applications, it is necessary to study the dynamical behaviors of the neural networks such as stability, robustness and oscillation. Specially, time delays are often encountered in dynamical systems, which sometimes may cause these dynamical systems instability and oscillations. Thus, the analysis of delayed neural networks has attracted much research attention and there exist many related results such as [5–8].

It should be pointed out that, for the neural networks composed of large scale neurons, the neuron states usually are not completely available in applications. Thus, we need to estimate the neuron states according to available output measurement and use them to design controller to achieve a given control target, which results in some researchers paying great attention to the state estimation problem of neural networks. For example, for the continuous neural networks, two kinds of guaranteed performance state estimators of static neural networks with time-varying delays are concerned in [9]. The authors investigated the H_∞ state estimation problem for static neural networks with time-varying delays, and both delay-dependent and delay-independent criteria are presented such that the error system is globally asymptotically stable with a guaranteed H_∞ performance in [10]. For the discrete cases, the authors investigated the state estimation problem for a class of discrete-time delayed neural networks with fractional uncertainties and sensor saturations in [11]. The state estimation problem for a class of discrete-time stochastic neural networks with random delays is studied by employing a Lyapunov–Krasovskii functional in [12]. Related works can be found in [13–19].

In practice, one usually utilizes the dynamical systems with Markovian jump to model a class of dynamical systems consisting of a finite number of dynamical modes with random changes in structure or parameters, such as manufacturing systems and communication systems. Similarly, some neural networks have finite discrete modes, and the switching law among these modes satisfies Markovian property. In order to model such neural networks, the neural networks with Markovian jump parameters are introduced. There have been many works to investigate the state estimation problem of this kind of neural networks. For example, Takagi–Sugeno fuzzy model is used to investigate the state estimation of uncertain Markovian jump Hopfield neural networks with mixed interval time-varying delays in [20]. The authors investigated the state estimation problem of Markovian jump Hopfield neural networks with discrete and distributed delays by avoiding the model transformations and cross-terms bounding techniques in [21].

It is worth noting that most of the existing papers investigate the asymptotic convergence of the estimation error, which means that the states of estimated system converge to the states of the system in the sense of the infinite horizon. From the viewpoint of practical applications, it is more meaningful if we can make the state estimation error converge to the origin in a finite time. Moreover, there are many finite-time control methods, such as finite-time stability [22–24] and finite-time control [25–27], to

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realize synchronization or stability in recent years. However, there exist few works to consider the finite-time state estimation for delayed neural networks.

Motivated by the above discussions, in this paper, we study the finite-time state estimation for delayed Hopfield neural networks with Markovian jump. By constructing a Lyapunov–Krasovskii functional and employing inequality techniques, some conditions that the estimator parameters need to satisfy are derived. The contributions of our paper are as follows. (i) The finite-time state estimation of delayed Hopfield neural networks with Markovian jump is considered. (ii) The convergence time can be adjusted by tuning the estimator parameters. (iii) The results are presented in terms of the linear matrix inequalities.

The rest of this paper is organized as follows. In Section 2, the delayed Hopfield neural networks and its estimator are introduced, and some preliminaries are given. In Section 3, some conditions that the estimator parameters should satisfy are derived. In Section 4, a numerical example is provided to illustrate the effectiveness of the proposed method. Finally, this paper is ended with conclusions in Section 5.

In this paper, the following notations are used. R^n and $R^{n\times m}$, respectively, denote the n-dimensional Euclidean space and the set of all $n\times m$ dimensional real matrices. For a vector $v=(v_1,v_2,\ldots,v_n)^T$, $||v||=\sqrt{\sum_{i=1}^{n}v_i^2}$ denotes its norm, $\text{sign}(v)=(\text{sign}(v_1),\text{sign}(v_2),\ldots,\text{sign}(v_n))^T$, where $\text{sign}(\cdot)$ is the sign function. A^T denotes the transpose of matrix A and "$*$" means the symmetric parts of the main diagonal of a matrix. $\lambda_{max}\{A\}$ and $\lambda_{min}\{A\}$ denote the maximum and minimum eigenvalue of matrix A, respectively. The notation $X\ge Y$ (respectively, $X>Y$) means that X–Y is a symmetric semi-definite matrix (respectively, positive definite matrix), where X, Y are symmetric matrices. I_n is the $n\times n$ identity matrix. Throughout this paper, all matrices are assumed to have appropriate dimensions.

2. Problem statements

Consider the following delayed Hopfield neural networks with Markovian jump:

$$\begin{cases}\dot{x}(t)=-A(r_t)x(t)+B(r_t)f(x(t))+B_\tau(r_t)f(x(t-\tau(t))),\\ y(t)=C(r_t)x(t),\\ x(t)=\phi(t),\quad t\in[-\tau,0],\end{cases}\tag{1}$$

where $x(t)=(x_1(t),x_2(t),\ldots,x_n(t))^T$ is the state vector associated with n neurons. r_t is a continuous-time Markov process with taking values in a finite state set $\overline{N}=\{1,2,\ldots,q\}$, and its transition probability matrix $M=[m_{ij}]_{q\times q}$ is given by

$$\Pr\{r_{t+\Delta}=j|r_t=i\}=\begin{cases}m_{ij}\Delta+o(\Delta), & i\ne j,\\ 1+m_{ii}\Delta+o(\Delta), & i=j,\end{cases}\tag{2}$$

where

$$\lim_{\Delta\to0^+}\frac{o(\Delta)}{\Delta}=0,\tag{3}$$

and $m_{ij}>0, i\ne j, i,j\in\overline{N}$, is the transition rate from mode i at time t to mode j at time $t+\Delta$, which satisfies $m_{ii}=-\sum_{j=1,j\ne i}^{q}m_{ij}$. $A(r_t)\in R^{n\times n}$ is a diagonal matrix whose entries are positive constants. $B(r_t)\in R^{n\times n}$ and $B_\tau(r_t)\in R^{n\times n}$ are the connection weight matrices and the delayed connection weight matrices which switch with the Markov process r_t, respectively. $f(x(t))=(f_1(x_1(t)),f_2(x_2(t)),\ldots,f_n(x_n(t)))^T$ represents a continuous activation function and satisfies $f(0)=0$. Time-varying delay $\tau(t)$ is a continuous differential function and satisfies $0\le\tau(t)<\tau$ and $\dot{\tau}(t)\le\nu<1$, where τ and ν are given positive constants. $y(t)\in R^m$ describes the measurement output of neural network (1), $C(r_t)\in R^{m\times n}$ is a known constant matrix. The initial condition $\phi(t)\in R^n$ is a continuous vector function.

In this paper, we intend to design the following full-order state estimator of neural network (1):

$$\begin{cases}\dot{\hat{x}}(t)=-A(r_t)\hat{x}(t)+B(r_t)f(\hat{x}(t))+B_\tau(r_t)f(\hat{x}(t-\tau(t)))+K_1[y(t)-\hat{y}(t)]\\ \quad-\mu(P(r_t))^{-1}C^T(r_t)K_2^TK_2[y(t)-\hat{y}(t)]\\ \quad-\kappa_2\cdot(P(r_t))^{-1}C^T(r_t)\,\text{sign}\{y(t)-\hat{y}(t)\}\|y(t)-\hat{y}(t)\|^\alpha,\hat{y}(t)\\ =C(r_t)\hat{x}(t),\end{cases}\tag{4}$$

where

$$\mu=\frac{\kappa_1[\int_{t-\tau(t)}^{t}(y(s)-\hat{y}(s))^TK_2^TQK_2(y(s)-\hat{y}(s))\,ds]^{(\alpha+1)/2}}{\|K_2[y(t)-\hat{y}(t)]\|^2}$$

when $\|y(t)-\hat{y}(t)\|^2\ne0$, and $\mu=0$ as $\|y(t)-\hat{y}(t)\|^2=0$. $K_1\in R^{n\times m}$, $K_2\in R^{n\times m}$, $P(r_t)\in R^{n\times n}$ and $Q\in R^{n\times n}$ are unknown matrices to be determined, $0<\alpha<1$, κ_1 and κ_2 are given positive constants.

Definition 1. System (4) is said to be the finite-time state estimator of Hopfield neural network (1) if there exists a positive scalar $T>0$ such that

$$\lim_{t\to T^-}(x(t)-\hat{x}(t))=0,$$

and $x(t)\equiv\hat{x}(t)$ for all $t\ge T$.

Let the state estimation error be $e(t)=x(t)-\hat{x}(t)$ and write $A(r_t)=A_i$, other matrices are similarly denoted as them, then one yields the following error system:

$$\begin{aligned}\dot{e}(t)=&-A_ie(t)+B_ig(e(t))+B_{\tau i}g(e(t-\tau(t)))+K_1C_ie(t)\\ &-\mu P_i^{-1}C_i^TK_2^TK_2[y(t)-\hat{y}(t)]\\ &-\kappa_2\cdot P_i^{-1}C_i^T\,\text{sign}\{y(t)-\hat{y}(t)\}\|y(t)-\hat{y}(t)\|^\alpha,\end{aligned}\tag{5}$$

where $g(e(t))=f(x(t))-f(\hat{x}(t))$ and $g(e(t-\tau(t)))=f(x(t-\tau(t)))-f(\hat{x}(t-\tau(t)))$.

Remark 1. There exist a great deal of works such as [9–21] to study the state estimation problem for various kind of neural networks, but the results on finite-time state estimation are few. On the other hand, it is obvious that the state of estimator (4) follows with the state of neural network (1) in finite time only if error system (5) converges to the origin in finite time.

In what follows, we will study how to choose suitable parameters in (4) such that system (5) is stable in finite time for the nonlinear activation function $f(\cdot)$ and time delay. In order to achieve this goal, we firstly give the following assumption and lemmas.

Assumption 1. Assume that there exist two known real matrices $U_1\in R^{n\times n}$ and $U_2\in R^{n\times n}$ such that the nonlinear activation function $f(\cdot)$ satisfies

$$[f(\varsigma_1(t))-f(\varsigma_2(t))-U_1(\varsigma_1(t)-\varsigma_2(t))]^T[f(\varsigma_1(t))-f(\varsigma_2(t))-U_2(\varsigma_1(t)-\varsigma_2(t))]\le0$$

for any $\varsigma_1(t),\ \varsigma_2(t)\in R^n$.

From Assumption 1, it yields that

$$(e^T(t),g^T(e(t)))\Theta(e^T(t),g^T(e(t)))^T\le0,\tag{6}$$

where

$$\Theta=\begin{bmatrix}\frac{U_1^TU_2+U_2^TU_1}{2} & -\frac{U_1^T+U_2^T}{2}\\ * & I_n\end{bmatrix}.$$

It should be noticed that this assumption is more general than the commonly used Lipschitz conditions (see [11,21]).

Remark 2. Since error system (5) is discontinuous, we assume that it has an equilibrium point in the origin in the sense of Filippov [28].