Neurocomputing 156 (2015) 297–305

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Short Communication

Observer-based consensus tracking for second-order leader-following nonlinear multi-agent systems with adaptive coupling parameter design

Xiaole Xu [a,b], Shengyong Chen [b], Lixin Gao [c,*]

[a] Wenzhou Vocational College of Science & Technology, Zhejiang 325006, China
[b] College of Information Engineering, Zhejiang University of Technology, Zhejiang 310023, China
[c] Institute of Intelligent Systems and Decision, Wenzhou University, Zhejiang 325027, China

ARTICLE INFO

Article history:
Received 30 May 2014
Received in revised form
4 August 2014
Accepted 17 December 2014
Communicated by H. Jiang
Available online 31 December 2014

Keywords:
Multi-agent system
Distributed control
Consensus
Tracking control
Observer

ABSTRACT

This paper considers the leader-following tracking problem of second-order nonlinear multi-agent systems with a reference leader. It is assumed that all following agents can only access the relative position information with its neighbors, the position and velocity information of the leader is only accessed by a subset of the following agents, and the leader's bounded reference input cannot be available by any following agents. To track the active leader, a distributed discontinuous observer-based consensus protocol is proposed for each following agent, whose observer is used to estimate the agent's unmeasured velocity. Under connected undirected interaction topology, the proposed protocol can be used to solve the second-order nonlinear multi-agent consensus problem. While the reference input of the leader is known, the proposed protocol can solve the consensus problem under a class of directed switching interaction topologies. Furthermore, to make the designed approach in fully distributed fashion, the distributed adaptive laws are provided to design the protocol coupling parameters. Finally, a numerical example is presented to illustrate our obtained result.

1. Introduction

Recent years have witnessed a huge and rapidly growing literature on multi-agent systems due to its numerous potential applications. It is well-known that the consensus problem is one fundamental problem for coordinated control of multi-agent systems. Many topics, such as agreement [1], flocking [2], formation control [3], swarm stability [4], consensus filtering [5] and so on, are related with consensus problem. Usually, we pay attention to the distributed approaches based only on the local relative information to solve the consensus problem.

In the past decade, numerous interesting results have been obtained for the consensus problem. The first-order consensus problem has been discussed sufficiently via the stochastic matrix and Lyapunov methods [6–9]. But, in many practical systems especially the mechanical systems, the involved dynamical systems are often modeled by second-order systems. Ren and Atkins [10] showed that unlike the first-order consensus, the existence of a directed spanning tree is a necessary rather than a sufficient condition to reach second-order consensus. Thus, it is nontrivial to extend first-order consensus to second-order one, which is more complicated and challenging than the first-order case [11]. The second-order consensus problem has been probed significantly by many references [10,12–17]. The consensus problem of multi-agent system with general linear dynamics has been investigated by [18,19]. The leader-following configuration is often used to design the multi-agent systems. Many interesting and useful results, such as [2,3,6,11,12] and so on, have been obtained for the leader-following tracking problem. However, in many circumstances, nonzero control input might be implemented on the leader in order to achieve certain objective. Most existed results are based on the assumption that the input of the leader is zero or can be available by all following agents. Of course, it is impractical to assume that all the following agents can access the leader's reference input while the scale of the network is very large.

Since nonlinear systems are ubiquitous in practice, research on the distributed control of multiple nonlinear systems has emerged and developed rapidly. In [20], the authors studied the leaderless consensus problem for second-order multi-agent systems with intrinsic nonlinear dynamics under directed interaction topology. Song et al. [11] studied the second-order leader-following consensus problem of nonlinear multi-agent systems via pinning control. The distributed adaptive gain design approach for second-order multi-agent consensus with nonlinear dynamics was provided in [21]. In [22], the discontinuous protocols based on the relative state

* Corresponding author.
E-mail address: gao-lixin@163.com (L. Gao).

http://dx.doi.org/10.1016/j.neucom.2014.12.037
0925-2312/

information are proposed to solve the leader-following multi-agent consensus problem with Lipschitz-type dynamics. The second-order consensus of multi-agent systems with heterogeneous nonlinear dynamics and time-varying delays via adaptive gain strategies was investigated by [23]. The second-order consensus problem of multi-agent systems with nonlinear dynamics via impulsive control protocol was discussed by [24].

For second-order multi-agent systems, most mentioned references are required the velocity measurements to implement the consensus protocols. Unfortunately, this requirement is not always satisfied in reality because the velocity measurements are inaccurate or it is not contained with constrains in space, cost and weight. Thus, it is meaningful to design the consensus protocols only with the position information. To achieve the control goal, the observers are usually adopted in protocol to estimate unmeasured velocity variables. To track the leader with unmeasured velocity, a distributed protocol was proposed by [25] for each first-order following-agents based on a velocity observer, which was generalized for the second-order following-agents to track the active leader in [26]. The coordinated tracking protocol was presented in [27] with only position measurements. In [28], the authors proposed consensus protocol based on velocity filters for second-order multi-agent systems with nonlinear dynamics. A distributed observer-based protocol was provided for the first-order following-agents to track the general active leader in [29], whose result was extended to the time-delay case in [30]. To track the accelerated motion leader, a distributed observer-based protocol for the second-order follower-agents was proposed by [31]. For leader-following consensus problem with general linear dynamics, an observer-based protocol framework was proposed by [32]. The distributed reduced-order observed-based protocols were proposed in [33], which was generalized to solve leader-following consensus problem under switching topology in [34]. Generally, in most the mentioned papers, the gain matrices used in the consensus protocols are decoupled from the interaction topology, but the nonzero Laplacian eigenvalue with the smallest real part associated with interaction topology plays a key role to design the coupling parameters of the consensus protocols. Unfortunately, the eigenvalue of the Laplacian matrix belongs to the global information in the sense that each agent has to know the entire interaction topology to compute it. Even if the entire interaction topology is known, it is not an easy work to compute the eigenvalue with the large scale interaction topology. Strictly speaking, almost all consensus protocols proposed in the above-mentioned references cannot be implemented in a fully distributed fashion. To overcome this limitation, distributed adaptive approach to design the coupling parameters was investigated by [21,23].

Motivated by the above works, we focus our research on leader-following second-order nonlinear multi-agent system under three constraints: (i) the systems are with intrinsic nonlinear dynamics; (ii) only relative position measurements between neighbors except the leader can be used, and the position and velocity information of the leader is only accessed by a subset of the following agents; (iii) the bounded reference input of the leader cannot be available by any following agents. By constructing a local velocity observer, a discontinuous distributed observer-based control protocol is provided for the following agents to solve the leader-following consensus problem. A simple sufficient consensus condition is establish under connected undirected interaction topology. For the special case that the leader's reference input is known by all following agents, we prove that the proposed protocol can be used to solve the consensus problem under a class of directed switching topologies. To implement protocol in fully distributed fashion, the distributed adaptive consensus protocols are proposed, whose adaptive laws are assigned to design a time-varying coupling weights. Compared to [21], the adaptive consensus protocols proposed in this paper have two advantages. First, the proposed protocol in this paper is only based relative position measurements, but that of [21] needs the relative position and velocity information. Second, there is a independent adaptive law to design the coupling weight associated with the reference input, which is beneficial to adjust the control gain. Compared to [28], our proposed observer is very simple, the input of the leader is assumed to nonzero or can be available by all following agents, and adaptive coupling weight design is provided to implement protocol in fully distributed fashion.

The subsequent sections are organized as follows: Section 2 introduces the related preliminaries and problem formulation. Distributed observer-based protocols are designed for the second-order follower-agents in Section 3. In Section 4, the distributed consensus protocols with adaptive coupling parameter design is discussed. An illustrative example to verify the effectiveness of the theoretical result is provided in Section 5. Conclusions are drawn in Section 6.

2. Preliminaries and problem formulation

2.1. Notations and graph theory

Standard notations are used in this paper. R is the real number set. I is an identity matrix with compatible dimension and $\mathbf{1}$ represents the vector with all entries being one. For matrix A, A^T denotes its transpose. A matrix is said to be positive stable if all its eigenvalues are located in open right half plane. $A>0$ ($\geq 0, <0$, or ≤ 0) means that A is positive definite (positive semi-definite, negative or negative semi-definite). $\mathrm{diag}\{g_1, g_2, \ldots, g_N\}$ represents a diagonal matrix with diagonal elements g_i. For matrix A whose all eigenvalues are real, $\lambda_{max}(A)$ and $\lambda_{min}(A)$ represent its maximum and minimum eigenvalues, respectively. $\|T\|_1$, $\|T\|_2$ and $\|T\|_\infty$ denote the 1-norm, 2-norm and ∞-norm of a matrix T, respectively. The vector signum function $\mathrm{sgn}(x)$ is denoted by $\mathrm{sgn}(x) = [\mathrm{sgn}(x_1), \mathrm{sgn}(x_2), \ldots, \mathrm{sgn}(x_n)]^T$. $\otimes$ denotes the Kronecker product, which satisfies (1) $(A \otimes B)(C \otimes D) = (AC) \otimes (BD)$; (2) if $A \geq 0$ and $B \geq 0$, then $A \otimes B \geq 0$.

For the multi-agent system, the interaction relationships among N agents can be conveniently described by a simple graph $\mathcal{G}$. Let $\mathcal{G} = (\mathcal{V}, \mathcal{E}, A)$ be a weighted directed graph with N nodes. The set of nodes is $\mathcal{V} = \{v_1, v_2, \ldots, v_N\}$ and $\mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$ is the edges set. v_j is called a neighbor of v_i if $(v_i, v_j) \in \mathcal{E}$, and the neighbor set of vertex v_i is denoted as $\mathcal{N}_i = \{j | (v_i, v_j) \in \mathcal{E}\}$. $A = [a_{ij}]_{N \times N}$ represents weighted adjacency matrix associated with graph $\mathcal{G}$, where $a_{ij} > 0$ if $(v_i, v_j) \in \mathcal{E}$ and $a_{ij} = 0$ otherwise. Correspondingly, the Laplacian matrix L is defined as $l_{ii} = \sum_{j=1, j\neq i}^{N} a_{ij}$ and $l_{ij} = -a_{ij}$. A weighted graph is called undirected graph if for all $(v_i, v_j) \in \mathcal{E}$, we have $(v_j, v_i) \in \mathcal{E}$ and $a_{ij} = a_{ji}$.

For the considered multi-agent system with a reference leader, we concern another graph $\overline{\mathcal{G}}$ associated with the system consisting of N agents and one leader (labeled as 0). $\overline{\mathcal{G}}$ contains a subgraph $\mathcal{G}$ and v_0 with the directed edges from some agents to the leader, where $\mathcal{G}$ is used to describe the interaction topology of N following agents. $a_{ij} > 0$ represents that agent i is connected to agent j. The connection weight constant d_i is taken as positive constant if agent i is connected to the leader and otherwise is taken as zero. Let L be the Laplacian matrix of the interaction graph $\mathcal{G}$, and D is an $N \times N$ diagonal matrix whose i-th diagonal element is d_i. For convenience, let $H = L + D$. Matrix H has following property [12].

Lemma 1. *If $\overline{\mathcal{G}}$ contains a directed spanning tree with root v_0, then the related matrix H is positive stable.*

Furthermore, if the interaction graph $\mathcal{G}$ associated with all following agents is undirected, then matrix H is symmetric and positive definite. For simplicity, the interaction graph $\overline{\mathcal{G}}$ is said to be connected and undirected if $\overline{\mathcal{G}}$ contains a directed spanning tree with root v_0 and its subgraph $\mathcal{G}$ associated with all following agents is undirected.

The following well-known Schur Complement Lemma will be used to establish our result.