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Square-mean almost periodic solutions for impulsive stochastic shunting inhibitory cellular neural networks with delays ☆

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ABSTRACT

In this paper, we consider a class of impulsive stochastic shunting inhibitory cellular neural networks with delays. By using the fixed points principle and Gronwall–Bellman's inequality technique, we obtain the existence and exponential stability of square-mean almost periodic solutions for this class of networks. Finally, we give an example to illustrate the feasibility of our results.

1. Introduction

In recent years, shunting inhibitory cellular neural networks (SICNNs) have been extensively applied in psychophysics, speech, perception, robotics, adaptive pattern recognition, vision, and image processing. Hence, they have been the object of intensive analysis by numerous authors. Many important results on dynamical behaviors of SICNNs have been established and successfully applied to signal processing, pattern recognition, associative memories, and so on. We refer the reader to [1–9] and the references cited therein.

The time delay is a natural phenomenon that is commonly encountered in various engineering systems containing chemical processes, hydraulic and rolling mill systems due to the finite switching speed of amplifiers in networks. Moreover, the delay is frequently a source of instability and oscillation of system performance. Therefore, it is important to investigate the stability of delayed neural networks (see [10]).

On the other hand, the theory of impulsive differential equations is now being recognized to be not only richer than the corresponding theory of differential equations without impulses, but also represents a more natural framework for mathematical modelling of many real-world phenomena, such as population dynamical models and neural networks. Since many dynamical processes are characterized by the fact that at certain moments of time, they undergo abrupt changes of state. With the development of the theory of impulsive differential equations (see [11,12]), various kinds of neural networks governed by impulsive differential equations have been proposed and studied extensively (see [13–18]). For example, authors of [19] considered the impulsive SICNNs neural networks with almost periodic coefficients:

$$\begin{cases} \dfrac{dx_{ij}(t)}{dt} = -a_{ij}(t)x_{ij}(t) + \sum\limits_{C_{hl} \in N_r(i,j)} C_{ij}^{hl}(t) f(x_{hl}(t)) x_{ij}(t) + L_{ij}(t), & t \neq t_k, \\ \Delta x_{ij}(t_k) = \alpha_{ijk} x_{ij}(t_k) + I_{ijk}(x_{ij}(t_k)) + L_{ijk}, & t = t_k, \ k \in \mathbb{N}, \end{cases}$$

where $i = 1, 2, \ldots, m$, $j = 1, 2, \ldots, n$, $\Delta x_{ij}(t_k) = x_{ij}(t_k^+) - x_{ij}(t_k^-)$ are impulses at moments t_k and $t_0 < t_1 < t_2 < \cdots < t_k < \cdots$ is a strictly increasing sequence such that $\lim_{t \to \infty} t_k = +\infty$. By using the contraction principle and Gronwall–Bellman's inequality, they obtained some sufficient conditions for the existence and exponential stability of almost periodic solutions.

Besides, in real nervous systems, there are many stochastic perturbations that affect the stability of neural networks. The results in [20] suggested that one neural network could be stabilized or destabilized by certain stochastic inputs. It implies that the stability analysis of stochastic neural networks has primary significance in the design and applications of neural networks. With respect to stochastic neural networks, there are many works on the dynamics of different classes of stochastic neural networks (see [21–32,38,33–35]). For example, the stability of several classes of stochastic neural networks was investigated in [25–33,38]; the existence and exponential stability of periodic solutions for two classes of stochastic neural networks with delays were established in [34,35], respectively.

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Moreover, it is well known that the almost periodicity is more universal than the periodicity and the square-mean almost periodicity is important in probability for investigating stochastic processes [32,36–39]. Such a notion is also of interest for applications arising in mathematical physics and statistics. However, to the best of our knowledge, there exists no result on the existence and uniqueness of mean square almost periodic solutions for impulsive stochastic shunting inhibitory cellular neural networks with delays. Motivated by the above, we consider the following impulsive stochastic SICNNs with delays:

$$\begin{cases} \mathrm{d}x_{ij}(t) &= \Big[-a_{ij}(t)x_{ij}(t)-\sum\limits_{C_{kl}\in N_r(i,j)} C_{ij}^{kl}(t)f(x_{kl}(t-\eta_{ij}(t)))x_{ij}(t) \\ &\quad -\sum\limits_{C_{kl}\in N_q(i,j)} B_{ij}^{kl}(t)\int_0^{+\infty} J_{ij}(u)g(x_{kl}(t-u))\,\mathrm{d}u x_{ij}(t)+L_{ij}(t)\Big]\,\mathrm{d}t \\ &\quad +\sum\limits_{C_{kl}\in N_p(i,j)} D_{ij}^{kl}(t)\sigma_{ij}(t,x_{ij}(t))\,\mathrm{d}w_{ij}(t), \quad t\neq t_k, \\ \Delta x_{ij}(t_k) &= \alpha_{ijk}x_{ij}(t_k)+T_{ijk}(x_{ij}(t_k))+\nu_{ijk}, \quad t=t_k,\ k\in\mathbb{N}, \end{cases} \tag{1.1}$$

where $i=1,2,\ldots,m,\ j=1,2,\ldots,n;$ C_{ij} denotes the cell at the (i,j) position of the lattice, the r-neighborhood $N_r(i,j)$ is given as

$$N_r(i,j)=\{C_{kl}:\max(|k-i|,|l-j|)r,\ 1\le k\le m,\ 1\le l\le n\},$$

$N_q(i,j)$ and $N_p(i,j)$ are similarly specified; x_{ij} is the activity of the cell C_{ij} at time t; L_{ij} is the external input to C_{ij} at time t; $a_{ij}(t)>0$ represents the passive decay rate of the cell activity at time t; $C_{ij}^{kl}\ge 0$, $B_{ij}^{kl}\ge 0$ and $D_{ij}^{kl}\ge 0$ are the connection or coupling strength of postsynaptic activity of the cell transmitted to the cell C_{ij} at time t; the activity functions f,g represent the output or firing rate of the cell C_{kl}; $\eta_{ij}(t)\ge 0$ corresponds to the transmission delay at time t; $J_{ij}(\cdot)$ is the kernel function determining the distributed delay at the cell (i,j); $w(t)=(w_{11}(t),w_{12}(t),\ldots,w_{mn}(t))^T$ is the $m\times n$-dimensional Brownian motion defined on a complete probability space; σ_{ij} is a Borel measurable function and $\sigma=(\sigma_{ij})_{m\times n}$ is a diffusion coefficient matrix, where $i=1,2,\ldots,m,\ j=1,2,\ldots,n;$ $\Delta x_{ij}(t_k)=x_{ij}(t_k^+)-x_{ij}(t_k^-)$ are impulses at moments t_k and $t_0<t_1<t_2<\cdots<t_k<\cdots$ is astrictly increasing sequence such that $\lim_{t\to\infty}t_k=+\infty$, α_{ijk}, $\nu_{ijk}\in\mathbb{R}$.

Let $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\ge 0},P)$ be a complete probability space with a filtration $\{\mathcal{F}_t\}_{t\ge 0}$ satisfying the usual conditions (i.e., it is right continuous, and $\mathcal{F}_0$ contains all P-null sets), and let $E[\cdot]$ be the expectation operator with respect to the probability measure, $\mathbb{R}^n(\mathbb{R}^n_+)$ be the space of n-dimensional (nonnegative) real column vectors and $\mathbb{R}^{mn}$ be the space of $m\times n$-dimensional real column vectors. Set $PC(\mathbb{R},\mathbb{R}^{mn})=\{\varphi:\mathbb{R}\to\mathbb{R}^{mn}$, φ is continuous everywhere except at the points t_k, $\{t_k\}\in\mathcal{B}$ at which $\varphi(t_k^+)$ and $\varphi(t_k^-)$ exist, where $\varphi(t_k^-)=\varphi(t_k)\}$, where $\mathcal{B}=\{\{t_k\}\,|\,t_k\in\mathbb{R},\ t_k<t_{k+1},\ \lim_{t\to\infty}t_k=+\infty\}$. For $p>0$ and $t\ge 0$, let $PC^p_{\mathcal{F}_t}(\mathbb{R},\mathbb{R}^{mn})$ denote the family of all $\mathcal{F}_t-$measurable $PC(\mathbb{R},\mathbb{R}^{mn})-$valued random variables φ such that $\sup_{t\in\mathbb{R}}E|\varphi(t)|^p<\infty$. In this paper, we let $p=2$ and denote the norm $\|\varphi\|_\infty=\sup_{t\in\mathbb{R}}(E\|\varphi(t)\|^2)^{1/2}$, where $\|\varphi(t)\|=\max_{(i,j)}\{\|\varphi_{ij}(t)\|_2\}$, $\|\varphi_{ij}(t)\|_2=(\int_\Omega|\varphi_{ij}(t)|^2\,\mathrm{d}s)^{1/2}$, $i=1,2,\ldots,m$, $j=1,2,\ldots,n$.

For convenience, we denote

$$\overline{f}=\sup_{t\in\mathbb{R}}|f(t)|,\quad \overline{g}=\sup_{(t,x)\in\mathbb{R}\times PC^p_{\mathcal{F}_t}(\mathbb{R},\mathbb{R}^{mn})}|g(t,x)|,$$

where $f(t)$ is a square-mean almost periodic function and $g(t,x)$ is a square-mean uniformly almost periodic function in the sense of Bohr.

Throughout this paper, we assume that

(H_1) $a_{ij}(t),\eta_{ij}(t),C_{ij}^{kl}(t),B_{ij}^{kl}(t),D_{ij}^{kl}(t),L_{ij}(t)$ are all square-mean almost periodic functions, and there exists a positive constant λ such that $a_{ij}(t)\ge\lambda$, $i=1,2,\ldots,m,\ j=1,2,\ldots,n$.

(H_2) The activation functions f,g are square-mean almost periodic and there exist positive numbers M, N such that $E\|f(x)-f(y)\|^2\le M_fE\|x-y\|^2$, $E\|g(x)-g(y)\|^2\le M_gE\|x-y\|^2$, $x,y\in PC^2_{\mathcal{F}_t}(\mathbb{R},\mathbb{R})$.

(H_3) σ_{ij} is uniformly square-mean almost periodic and there exists positive numbers R_{ij} such that $E\|\sigma_{ij}(t,x)-\sigma_{ij}(t,y)\|^2\le R_{ij}E\|x-y\|^2$, $x,y\in PC^2_{\mathcal{F}_t}(\mathbb{R},\mathbb{R})$, $i=1,2,\ldots,m,\ j=1,2,\ldots,n$.

(H_4) $\{\alpha_{ijk}\}$, $\{\nu_{ijk}\}$ are almost periodic sequences and $-1\le\alpha_{ijk}\le 0$, $H_{ij}=\sup_{k\in\mathbb{Z}}|\nu_{ijk}|$; T_{ijk} are square-mean almost periodic uniformly with respect to x_{ij} satisfying $E\|T_{ijk}(x)-T_{ijk}(y)\|^2\le\tilde{R}_{ij}E\|x-y\|^2$, $x,y\in PC^2_{\mathcal{F}_t}(\mathbb{R},\mathbb{R})$, $Q_{ij}=\sup_{k\in\mathbb{Z}}|T_{ijk}|$, $k\in\mathbb{N}$, $i=1,2,\ldots,m,\ j=1,2,\ldots,n$.

(H_5) The set of sequences $\{t_k^j\}$, $t_k^j=t_{k+j}-t_k$, $k,j\in\mathbb{N}$ are uniformly almost periodic and $\inf_k t_k^1=\xi>0$.

Let $t_0\in\mathbb{R}$. Denote by $PC^2_{\mathcal{F}_{t_0}}(t_0)$, the space of all $\mathcal{F}_{t_0}-$measurable functions $\phi:[t_0-\overline{\tau},t_0]\to\Omega\in\mathbb{R}^{n\times m}$ with respect to a sequence $\{\theta_s\}_{s\in\mathbb{N}}\subset(t_0-\overline{\tau},t_0)_{\mathbb{T}}$, where $\overline{\tau}=\max_{\max(i,j)}\{\sup_{t\in\mathbb{T}}|\eta_{ij}(t)|\}$.

Let φ_0 be an element of $PC^2_{\mathcal{F}_{t_0}}(t_0)$. Denote by

$$x(t)=x(t;t_0,\varphi_0)=(x_{11}(t),x_{12}(t),\ldots,x_{mn}(t))^T,$$

the solution of system (1.1) satisfying the initial condition:

$$\begin{cases} x(t;t_0,\varphi_0)=\varphi_0(t), & t\in(t_0-\overline{\tau},t_0)_{\mathbb{T}}, \\ x(t_0^+;t_0,\varphi_0)=\varphi_0(t_0). \end{cases} \tag{1.2}$$

Our main purpose of this paper is to study the existence and exponential stability of square-mean almost periodic solutions to (1.1) by means of the fixed points principle and Gronwall–Bellman's inequality technique.

2. Preliminaries

In this section, we shall recall some basic definitions, lemmas which are used in what follows.

Definition 2.1 (*Lakshmikantham et al.* [11]). The set of sequences $\{\tau_i^j\}$, $\tau_i^j=\tau_{i+j}-\tau_i$, $i,j\in\mathbb{N}$ is said to be uniformly almost periodic if for any $\varepsilon>0$ there exists a relatively dense set in $\mathbb{R}$ of ε-almost periods common for all of the sequences.

Definition 2.2 (*Lakshmikantham et al.* [11]). The piecewise continuous function $\varphi\in PC(\mathbb{R},\mathbb{R})$ is said to be square-mean almost periodic, if the following hold:

(i) the set of sequences $\{\tau_i^j\}$, $\tau_i^j=\tau_{i+j}-\tau_i$, $i,j\in\mathbb{N}$ is uniformly almost periodic;

(ii) for any $\varepsilon>0$, there is a positive number $\delta=\delta(\varepsilon)$ such that if the points t' and t'' belong to the same interval of continuity and $|t'-t''|<\delta$, then $E\|\varphi(t')-\varphi(t'')\|^2<\varepsilon$;

(iii) for any $\varepsilon>0$, there is relative dense set Γ such that if $\tau\in\Gamma$, then $E\|\varphi(t')-\varphi(t'')\|^2<\varepsilon$ for all $t\in\mathbb{R}$ which satisfy the condition $|t-\tau_i|>\varepsilon$, $i\in\mathbb{Z}$.

Lemma 2.1 (*Lakshmikantham et al.* [11]). *Let the set of sequences $\{\tau_i^j\}$, $\tau_i^j=\tau_{i+j}-\tau_i$, $i,j\in\mathbb{N}$ is uniformly almost periodic. Then for each $l>0$ there exists a positive integer N such that on each interval of a length l, there exist no more than N elements of the sequence $\{\tau_i\}$ and*

$$i(s,t)\le N(t-s)+N,$$

where $i(s,t)$ is the number of points τ_i in the interval (s,t).

In order to study system (1.1), we consider the linear system

$$\begin{cases} x'_{ij}(t)=-a_{ij}(t)x_{ij}(t), & t\neq t_k, \\ \Delta x_{ij}(t_k)=\alpha_{ijk}x_{ij}(t_k), & t=t_k,\ k\in\mathbb{N},\ i=1,2,\ldots,m,\ j=1,2,\ldots,n. \end{cases} \tag{2.1}$$