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Adaptive impulsive synchronization of fractional order chaotic system with uncertain and unknown parameters

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ABSTRACT

Impulsive synchronization for fractional order chaotic system is a difficult problem that is required to solve. In this paper, a new method to realize the adaptive impulsive synchronization of two general fractional order chaotic systems with uncertain and unknown parameters is investigated. A new function is to be applied to the stability theory of fractional order directly. Then the impulsive synchronization of two fractional order chaotic systems with unknown parameters is discussed. The criteria for synchronization of the systems are established. Finally, some numerical examples are delivered to illustrate the effectiveness of our results.

1. Introduction

Fractional calculus is as old as conventional calculus, but is not as prevalent as conventional calculus. In the last three centuries, this subject was studied only in mathematics. Nevertheless, there are many known systems where the modeling with fractional operators have turned out to be useful. Many systems in biology, physics, engineering and information science fields exhibit impulsive dynamical behavior due to sudden jumps at certain instants in the evolutionary processes. These problems can be modeled by impulsive differential systems. Therefore, the study of impulsive fractional order system is of great importance.

Chaos control, stability analysis and synchronization is a topic in the field of nonlinear systems. Many methods were proposed to integer order chaotic systems, such as the OGY method [1], adaptive method [2], active method [3], backing stepping technique [4], sliding mode method [5], impulsive method [6–8], and so on. Among these methods, impulsive method has interested a great number of researchers because of various systems exhibiting impulsive dynamical behavior. The theory of impulsive control for integer order chaotic system is usually perfect. Moreover, the impulsive hybrid control method of integer order chaotic system has also been puting forward [6–8].

However, fractional order calculus is distinct from integer order calculus. So, a new theory of stability analysis and control for the fractional order system must be set up or extended from integer order chaotic systems. So far, a few methods mentioned above having be brought to fractional order chaotic systems. And most studies about impulsive fractional order systems use the comparison theory of integer order systems [9–19]. Then Hu proposed a universal significance function instead of the Lyapunov function to study fractional order systems [9]. Liu [10] studied impulsive synchronization of fractional order chaotic systems. Ma [13] investigated impulsive synchronization and modified impulsive synchronization of fractional order systems by constructing a suitable response system method. The synchronization scheme for the fractional order systems is transformed into the stability analysis of the integer order error systems. Wang [14] studied the impulsive synchronization of a class of fractional order hyper-chaotic systems. Adaptive impulsive synchronization for a class of fractional order chaotic systems were further studied [15,16]. Stamova [17] investigated the stability of impulsive fractional order systems by constructing a new comparison principle.

On the other hand, uncertainties happen frequently in various systems due to modeling errors, measurement inaccuracy, linear approximation, and so on. At times, even the parameters are unknown. Uncertainties exist in operation research, management science, decision science, information science, system science, computer science and industrial engineering, and many other fields. So uncertainty is applied to solve many problems, such as the vehicle scheduling problem, the key problem, the shortest circuit path, reliability issues, storage, sorting, the location problem, the assignment problem, update, fuzzy clustering, data analysis, information processing, network optimization, project review, project management, e-commerce, investment risk analysis, logistics and supply chain

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management, the decision support system, economic policy, etc. Many researchers have studied the problem of uncertainty [18]. And people found that chaotic systems with uncertain parameters exist extensively. Some research has been being made to solve to solve this problem [19–21]. In addition, the fractional order chaotic system with uncertain parameters was also been studied [22]. However, so far very few related results of impulsive fractional order system with uncertain or unknown parameters have been reported.

In this paper, we shall investigate impulsive synchronization of fractional order chaotic system with uncertain and unknown parameters utilizing the adaptive method. The organization of this paper is presented below. In the next section, some necessary definitions and lemmas are given. In Section 3, a new method is used to realize the adaptive impulsive synchronization of two fractional orders chaotic systems whose parameters are uncertain. In Section 4, the unknown parameters system is studied to realize the adaptive synchronization. In Section 5, the numerical simulations are taken to verify their effectiveness.

2. Preparation

Definition 1. (see [9]). The Caputo fractional derivative of order q of a continuous function $f: R^+ \to R$ is defined as follows:

$$D_t^q f(t) = \begin{cases} \frac{1}{\Gamma(m-q)} \int_0^t \frac{f^{(m)}(\tau)}{(t-\tau)^{q-m+1}} d\tau, & m-1<q<m \\ \frac{d^m}{dt^m} f(t), & q=m \end{cases} \tag{1}$$

where Γ is the gamma function which is $\Gamma(z) = \int_0^\infty e^{-t} t^{z-1}\, dt$ and $\Gamma(z+1) = z\Gamma(z)$.

From the definition of fractional order differential, it is clearly that

$$D_t^q(kx(t)) = kD_t^q x(t) \tag{2}$$

where k is a constant.

When the order satisfies $0<q\le 1$, the solution to the Caputo fractional order derivatives is equivalent to

$$x(t) = x_0 + \frac{1}{\Gamma(q)} \int_0^t (t-\tau)^{q-1} D_\tau^q x(\tau)\, d\tau \tag{3}$$

and for the general fractional order linear differential equation

$$D_t^q x(t) = Ax(t) \tag{4}$$

the general solution of the equation is

$$x(t) = x(0) E_q(At^q) \tag{5}$$

where

$$E_q(z) = \sum_{k=0}^{+\infty} \frac{z^k}{\Gamma(qk+1)},$$

is the Mittag-Leffter function.

Lemma 1. *(see [9]). For the fractional order system $d^q x(t)/dt^q = f(x(t))$, $x=(x_1, x_2, \ldots, x_n)^T$, and $f(x(t))$ satisfies the Lipschitz condition, when the order $0<q\le 1$, if there exists a positive definite matrix P such that*

$$J = x^T(t) P \frac{d^q x(t)}{dt^q} \le 0 \tag{6}$$

Then the system is asymptotic stability.

Proof. Constructing a positive definite function

$$V(t) = \tfrac{1}{2} x^T(t) P x(t) \tag{7}$$

then

$$DV = x^T(t) P dx(t)$$

$$\begin{aligned} &= x^T(t) P \frac{1}{\Gamma(q)} \lim_{\delta t \to 0} \int_{t-\delta t}^t (t-\tau)^{q-1} f(x(\tau)) d\tau \\ &= \frac{1}{\Gamma(q)} \lim_{\delta t \to 0} \int_{t-\delta t}^t (t-\tau)^{q-1} x^T(t) P f(x(\tau)) d\tau \end{aligned} \tag{8}$$

because

$$J = x^T(t) P \frac{d^q x(t)}{dt^q} = x^T(t) P f(x(t)) \le 0$$

and function $f(x(t))$ satisfies the Lipschitz condition, when $\delta t \to 0$ and $\tau \in (t-\delta t, t]$, $x(t) P f(x(\tau)) \le 0$. Then

$$\begin{aligned} V'(t) &= \lim_{\delta t \to 0} \frac{\delta V}{\delta t} \\ &= \lim_{\delta t \to 0} \frac{1}{\Gamma(q)} \lim_{\delta t \to 0} \int_{t-\delta t}^t (t-\tau)^{q-1} x^T(t) P f(x(\tau)) d\tau (\delta t)^{-1} \\ &\le 0 \end{aligned} \tag{9}$$

based on the Lyapunov stability theory

$$\lim_{t\to\infty} V(t) = \lim_{t\to\infty} \frac{1}{2} x^T(t) P x(t) = 0 \tag{10}$$

then $\lim_{t\to\infty} x(t) = 0$.

3. Uncertain parameters system

First, we define the drive (master) systems as follows:

$$\begin{cases} D_t^q x(t) = Ax(t) + f(x(t)), & t \ne t_k \\ \Delta x(t_k) = x(t_k^+) - x(t_k) = H_k x(t_k), & k = 1,2,3,\ldots \end{cases} \tag{11}$$

where $x(t) = (x_1(t), x_2(t), \cdots, x_n(t))^T$ is the state variable, $A \in R^{n\times n}$, $H_k \in R^{n\times n}$, $f(x(t)) = (f_1(x(t)), f_2(x(t)), \cdots, f_n(x(t)))^T$ is the nonlinear function. H_k is the impulsive matrix. t_k, $k=1,2,\ldots$ are the time series.

Next, we define the response (slave) systems as follows:

$$\begin{cases} D_t^q y(t) = By(t) + g(y(t)) + \Delta G(y(t)) + d(t) + u(t), & t \ne t_k \\ \Delta y(t_k) = y(t_k^+) - y(t_k) = H_k y(t_k), & k = 1,2,3,\ldots \end{cases} \tag{12}$$

where $y(t) = (y_1(t), y_2(t), \cdots, y_n(t))^T$ is the state variable. $B \in R^{n\times n}$, and $g(y(t)) = (g_1(y(t)), g_2(y(t)), \cdots, g_n(y(t)))^T$ is the nonlinear function. $u(t)$ is the controller which need to designed. $\Delta G(y(t)) = (\Delta G_1(y(t)), \Delta G_2(y(t)), \cdots, \Delta G(y(t)))^T$ represents the parameters uncertainties. $d(t) = (d_1(t), d_2(t), \cdots, d_n(t))^T$ is the external noise perturbations.

We assume that the nonlinear part satisfies the Lipschitz condition, which means there exist two constants L_f and L_g such that

$$\|f(y) - f(x)\| \le L_f \|y - x\| \tag{13}$$

$$\|g(y) - g(x)\| \le L_g \|y - x\| \tag{14}$$

Let us define the synchronization error as

$$e(t) = y(t) - x(t)$$

where $e(t) = (e_1(t), e_2(t), \cdots, e_n(t))^T$.

Then the error system can be described by

$$\begin{cases} D_t^q e(t) = Be(t) + g(y(t)) - f(x(t)) + (B-A)x(t) + \Delta G(y(t)) + d(t) + u(t), & t \ne t_k \\ \Delta e(t_k) = e(t_k^+) - e(t_k) = H_k(e(t_k)), & k = 1,2,3,\ldots \end{cases} \tag{15}$$

Since chaos is bounded, the parameters uncertainties and external noise perturbations are usually small, hence they are bounded too. So there exists a constant γ such that $\|\Delta G(y(t)) + d(t)\| \le \gamma$.

Because the nonlinear parts of (11) and (12) are satisfied with the Lipschitz condition, so the mathematical expectation of the unknown Lipschitz constant is designed for the controller. Besides,