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A finite-time convergent neural dynamics for online solution of time-varying linear complex matrix equation

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ABSTRACT

This paper proposes and investigates a finite-time convergent neural dynamics (FTCND) for online solution of time-varying linear complex matrix equation in complex domain. Different from the conventional gradient-based neural dynamical method, the proposed method utilizes adequate time-derivative information of time-varying complex matrix coefficients. It is theoretically proved that our FTCND model can converge to the theoretical solution of time-varying linear complex matrix equation within finite time. In addition, the upper bound of the convergence time is derived analytically via Lyapunov theory. For comparative purposes, the conventional gradient-based neural dynamics (GND) is developed and exploited for solving such a time-varying complex problem. Computer-simulation results verify the effectiveness and superiorness of the FTCND model for solving time-varying linear complex matrix equation in complex domain, as compared with the GND model.

1. Introduction

The online solution of linear matrix equation is one of the basic problems encountered in a variety of science and engineering fields [1] and applied in linear system theorem [2,3], disturbance decoupling [4], and linear least squares regression [5], etc. Therefore, numerous of numerical algorithms were developed for the iterative solution of linear matrix equations using different techniques such as Taylor's series, homotopy, quadrature formulas, interpolation and decomposition [1–5]. Specifically, extending the Jacobi and Gauss–Seidel iterative method for linear system $Ax=b$, Ding et al. [1,6,7] obtained numerical solutions of linear matrix equation as well as the generalized Sylvester matrix equation, and then further investigated the transformations and relationships between some special matrices [8]. In [9], extending the idea of conjugate gradient method, Song et al. constructed an iterative algorithm to solve coupled Sylvester-transpose matrix equations. Prakash and Mittra [10] introduced a based characteristic-basis-function approach for the solution of matrix equations arising in the method of moments formulation of electromagnetic scattering problems. However, the Jacobi and Gauss–Seidel algorithm has a time complexity $O(n^3)$ to complete the iteration calculation of linear matrix equation [11]. Evidently, such serial-processing iterative algorithms may not be efficient enough in large-scale datum calculation and related real-time applications. For example, when such serial-processing iterative algorithms are applied to online solution of time-varying linear matrix equations, they should be performed within every sampling period. If the sampling rate is too high to allow the algorithms to complete the calculation in a single sampling period, then the algorithms will fail.

In recent decades, neural dynamical methods have drawn extensive interest and investigation of researchers and have been widely applied in scientific computation and engineering applications, such as quadratic program problems [12], system identification [13], automatic speech processing [14] and planar manipulator [15]. As compared to the numerical iterative method, the neural dynamical method has some potential advantages in real-time processing applications (e.g., the hardware-implementation ability and parallel distributed nature). Thus, neural dynamical method is now regarded as a powerful alternative for online solution of various challenging computation problems. In addition, due to the in-depth research on neural dynamical methods, many neural dynamical models have been proposed, developed and analyzed for solving linear matrix equation in the real domain [16,17]. As a typical neural dynamical method, the gradient-based neural dynamics (GND) has been also designed, proposed and investigated for solving linear matrix equations in the real domain [17]. This typical method usually uses the Frobenius norm of the error matrix as the performance criterion and can converge to the theoretical solution with time in the time-invariant case. However, the Frobenius norm of the error matrix cannot converge to zero during the process of solving time-varying linear matrix equation even after infinitely long time, because the velocity compensation of time-varying

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coefficients is lacked [16,17]. In view of this point, a special class of neural dynamics [called Zhang neural dynamics (ZND)] has been proposed to handle the lagging-error problem of the conventional gradient-based neural dynamics when solving time-varying problems [18,19]. Compared with the GND method, a prominent advantage of the ZND method lies in that the lagging error converges to zero exponentially as time t goes on.

It is worth pointing out that most of the above works (including the numerical iterative method and the neural dynamical method) usually aimed at solving linear matrix equation in the real domain, and few literatures have been devoted to the investigation of linear complex matrix equation in the complex domain, not to mention time-varying linear complex matrix equation problem. In addition, it is noted that, in some situations, online solution of linear complex matrix equation may also occur, when the problem contains online frequency domain identification processes [20,21], or when the input signals incorporate both magnitude and phase information [22]. Furthermore, after deep investigation and analysis of Zhang neural dynamics, we found that the ZND model cannot converge to the theoretical solution of time-varying linear matrix equation within finite time, which may limit its large-scale applications in real-time processing. Based on the above considerations, a finite-time convergent neural dynamics (FTCND) is proposed and investigated for online solution of time-varying linear complex matrix equation in complex domain. Different from conventional gradient-based neural dynamics and recently proposed Zhang neural dynamics, our proposed FTCND model can converge to the theoretical solution of time-varying linear complex matrix equation within finite time instead of infinite time. Besides, the upper bound of the convergence time is derived analytically via Lyapunov theory. To the best of authors' knowledge, this is the first time to provide such a finite-time convergent neural dynamics for solving time-varying linear complex matrix equation and its finite-time neural-state solution in the complex domain. Finally, the conventional gradient-based neural dynamical method is exploited and developed for such a complex problem for comparative purposes.

Before ending this section, the main contributions of this paper are summarized and listed as follows.

(1) This paper focuses on solving time-varying linear complex matrix equations in complex domain rather than conventionally investigated static or time-varying linear matrix equations in real domain.
(2) A finite-time convergent neural dynamical model is proposed and investigated for online solution of time-varying linear complex matrix equation in complex domain. As compared to the conventional gradient-based neural dynamics and the recently proposed Zhang neural dynamics, our proposed model has a superior convergence performance (i.e., finite-time convergence instead of infinite-time convergence) for solving time-varying linear complex matrix equation.
(3) The paper carries out an in-depth theoretical analysis for our proposed FTCND model. It is theoretically proved that our model can converge to the theoretical solution of time-varying linear complex matrix equation within finite time. In addition, the upper bound of the convergence time is derived analytically via Lyapunov theory.

2. Problem formulation

In mathematics, the problem of time-varying linear complex matrix equation can be generally formulated as

$$A(t)Z(t) = B(t) \in \mathbb{C}^{n\times m}, \tag{1}$$

where $Z(t) \in \mathbb{C}^{n\times m}$ is an unknown time-varying complex matrix to be obtained, complex matrices $A(t) \in \mathbb{C}^{n\times n}$ and $B(t) \in \mathbb{C}^{n\times m}$ are smoothly time-varying coefficients of (1). For convenience, let $Z^*(t) \in \mathbb{C}^{n\times m}$ denote the time-varying theoretical solution of (1). Note that, in most of past literatures on linear matrix [1–5], coefficient matrices are often considered to be real and static; i.e., $A \in \mathbb{R}^{n\times n}$ and $B \in \mathbb{R}^{n\times m}$. Thus, their research processes are relatively easy.

It is noted that any complex matrix can be treated as the combination of its real and imaginary parts, i.e., $A(t) = A_{re}(t) + jA_{im}(t)$, $Z(t) = Z_{re}(t) + jZ_{im}(t)$ and $B(t) = B_{re}(t) + jB_{im}(t)$ with $j = \sqrt{-1}$ denoting an imaginary unit. Therefore, time-varying linear complex matrix equation (1) can be rewritten evidently and equivalently as

$$[A_{re}(t) + jA_{im}(t)][Z_{re}(t) + jZ_{im}(t)] = B_{re}(t) + jB_{im}(t), \tag{2}$$

where $A_{re}(t) \in \mathbb{R}^{n\times n}$, $A_{im}(t) \in \mathbb{R}^{n\times n}$, $Z_{re}(t) \in \mathbb{R}^{n\times m}$, $Z_{im}(t) \in \mathbb{R}^{n\times m}$, $B_{re}(t) \in \mathbb{R}^{n\times m}$ and $B_{im}(t) \in \mathbb{R}^{n\times m}$.

Considering that the real (or imaginary) part of the left-side and right-side of Eq. (2) always holds equal, the following time-varying linear matrix equations can be further derived from Eq. (2) as

$$\begin{cases} A_{re}(t)Z_{re}(t) - A_{im}(t)Z_{im}(t) = B_{re}(t) \in \mathbb{R}^{n\times m}, \\ A_{re}(t)Z_{im}(t) + A_{im}(t)Z_{re}(t) = B_{im}(t) \in \mathbb{R}^{n\times m}, \end{cases} \tag{3}$$

which can be equivalently expressed in a compact matrix form as

$$\begin{bmatrix} A_{re}(t) & -A_{im}(t) \\ A_{im}(t) & A_{re}(t) \end{bmatrix} \begin{bmatrix} Z_{re}(t) \\ Z_{im}(t) \end{bmatrix} = \begin{bmatrix} B_{re}(t) \\ B_{im}(t) \end{bmatrix} \in \mathbb{R}^{2n\times m}. \tag{4}$$

From the above problem reformulation procedure, we can draw the conclusion that the solution to time-varying linear complex matrix equation (1) is equivalent to solving the above time-varying linear real matrix equation (4).

In order to ensure the existence of the unique complex and time-varying theoretical solution $Z^*(t) \in \mathbb{C}^{n\times m}$ at any time instant $t \in [0, +\infty)$, and also for simplicity and clarity, we limit the discussion for the situation that the following condition is satisfied throughout this paper:

$$\det\left(\begin{bmatrix} A_{re}(t) & -A_{im}(t) \\ A_{im}(t) & A_{re}(t) \end{bmatrix}\right) \neq 0,$$

where operator $\det(\cdot)$ denotes the determinant of a square matrix.

3. Finite-time convergent neural dynamics

In this section, inspired by the study on finite-time stability of autonomous system [19,23–25], we propose and investigate a finite-time convergent neural dynamics (FTCND) for solving online time-varying linear complex matrix equation (1). To lay a basis for further discussion, the design method of Zhang neural dynamics is first presented. Then, by using a specially constructed nonlinear activation function (called the sign-bi-power activation function), a finite-time convergent neural dynamics is proposed for solving time-varying linear complex matrix equation (1) with the upper bound of the convergence time derived analytically.

3.1. ZND model

To monitor the solving process of time-varying linear complex matrix equation (1) as well as time-varying linear real matrix equation (4), following Zhang el al.' design method [16], we can define the following matrix-valued error function (instead of the scalar-valued nonnegative energy function usually used in gradient-based neural dynamical