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Fuzzy regional pole placement based on fuzzy Lyapunov functions

Jianjun Bai [a,*], Renquan Lu [a], Xia Liu [b], Anke Xue [a], Zhonghua Shi [c]

[a] Key Lab for IOT and Information Fusion Technology of Zhejiang, Institute of Information Science and Control, Hangzhou Dianzi University, Hangzhou 310018, PR China
[b] Research Institute of Liaoyang Petrochemical Company, PetroChina, PR China
[c] China Petroleum Longhui Automation Engineering Co. Ltd., PR China

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ABSTRACT

This paper studies the regional pole placement problem for a class of T–S fuzzy systems. Firstly, a definition for the fuzzy systems to be $\mathcal{D}$ stable is given from the viewpoint of Lyapunov functions. Then a new sufficient condition is proposed to guarantee all the poles of the fuzzy systems located within a prescribed LMI region by using the fuzzy Lyapunov functions method and introducing some free matrices. And then the controller design approach is given by solving a set of LMIs. Finally, numerical examples are given to illustrate the effectiveness of the proposed approach.

1. Introduction

Nonlinear systems have drawn considerable research attentions in the past several decades [1]. As an efficient method to deal with nonlinear systems, T–S model based fuzzy systems have also attracted more and more researchers, see [2–13] and the references therein. It has been shown that any smooth nonlinear control systems can be approximated exactly by the T–S fuzzy models with a set of linear subsystems [14], which provides a natural, simple and effective design approach to complement other nonlinear control techniques [15–19].

A great number of results on the analysis and design of T–S fuzzy systems have been obtained since it was proposed by Takagi and Sugeno in [20]. A common Lyapunov function was firstly employed in [21] to prove the stability of the fuzzy systems. However, for some cases, i.e. the number of the subsystems is very large, common Lyapunov functions may not exist even if the system is stable. Hence, much efforts have been done to get less conservative stability conditions. In [22], a piecewise Lyapunov function was proposed to solve the stabilization problem, where each subsystem could be designed independently and the individual solutions were combined to get a solution for the overall design problem. The main drawback of the piecewise Lyapunov method is that the result is formulated in form of BMIs [23], which is hard to solve. To overcome this circumstance, a multiple Lyapunov function method was used in [24,25] and a non-PDC (parallel distribute compensation) controller design scheme was proposed to get the LMIs condition. In [26], descriptor system approach was used to further reduced the stability conservativeness. However, as the fuzzy Lyapunov function matrix depending on the premise variable, the upper or lower bounds of the membership functions should be know previously to compute the controller, which increases the conservativeness. In [27], a new fuzzy Lyapunov function was proposed and a stability condition independent of the derivative of membership functions was obtained. Based on [27], less conservative results were obtained in [28] by using the free matrix technique[29–31].

On the other hand, the transient properties of the linear systems could be guaranteed by placing the poles of the system to some specified regions, which is also called the $\mathcal{D}$ stability problem, see [32–34] and the references therein. However, for the nonlinear system, few results have bee addressed about the $\mathcal{D}$ stability problem. By using the common Lyapunov functions method, the H_∞ controller design problem of the fuzzy systems with pole location constraints was considered in [35,36] and experiments showed that satisfactory transient performance can be obtained by confining pole locations of the closed loop system. In [37], the output feedback controller was designed for the LMI regional pole placement problem of fuzzy systems, which was also based on a common Lyapunov function. Piecewise Lyapunov function was used in [38] to study the pole placement problem. In [39], the pole placement problem of fuzzy descriptor system

* Corresponding author.
E-mail address: baijianjun@hdu.edu.cn (J. Bai).

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was studied and the circle region was considered. So far, for the LMI regional pole placement problem of fuzzy systems, there is no result available based on fuzzy Lyapunov functions, which motivates the study of this paper.

This paper studies the regional pole placement problem for a class of T–S fuzzy systems. A new sufficient condition is firstly proposed to guarantee all the poles of the fuzzy systems located within a prescribed LMI region by using the fuzzy Lyapunov functions method and introducing some free matrices. And then the controller design approach is given by solving a set of LMIs. Finally, numerical examples are given to illustrate the effectiveness of the proposed approach. The main contributions of this paper lie in the following two aspect: (i) the fuzzy Lyapunov function method is firstly used to solve the LMI reginal pole placement problem; (ii) some free matrices are introduced to further reduce the conservativeness.

The rest of this paper is organized as follows: Section 2 presents the problem statement and some preliminaries; The main results are given in Section 3; examples are given in Section 4 to show the effectiveness of the proposed approach and Section 5 is the conclusion.

Notation: Throughout this paper, $\mathbb{R}^n$ denotes the n dimensional real Euclidean space, $\mathbb{C}$ denotes the complex plane, I_k is the $k\times k$ identity matrix, the superscripts 'T' and '-1' stand for the matrix transpose and inverse respectively, $\bar{s}$ denotes the conjugate of s, '$*$' denotes the symmetric element in a matrix, $\mathbb{C}^-$ denotes the left-hand side of complex plane. $W>0$ ($W\geq 0$) means that W is real, symmetric and positive definite (positive semidefinite), $\otimes$ denotes the Kronecker product. $L+M+(\bullet)$ denotes $L+M+M^T$. If not explicitly stated, the matrices are assumed to have compatible dimensions.

2. Problem statement and preliminaries

Consider the continuous time T–S fuzzy system described by the following fuzzy rules [27]:

$$\begin{aligned}\textit{Model rule } i: \quad & \text{If } x_1(t) \text{ is } M_1^{\alpha_{i1}} \text{ and } \ldots x_j(t) \text{ is } M_j^{\alpha_{ij}} \text{and} \ldots x_n(t) \text{ is } M_n^{\alpha_{in}}\\ & \text{Then } \dot{x}(t)=A_i x(t)+B_i u(t)\end{aligned} \tag{1}$$

where i denotes the ith fuzzy rule, $x(t)\in\mathbb{R}^n$ is the state vector, $u(t)\in\mathbb{R}^m$ is the control input, $A_i\in\mathbb{R}^{n\times n}$ and $B_i\in\mathbb{R}^{n\times m}$ are the local system matrices. the state variable $x_j(t)(j=1,2,\ldots,n)$ is the premise variable. $M_j^{(*)}$ is the jth fuzzy set. Let r_j be the number of x_j-based fuzzy sets, then the total number of fuzzy rules is $r=\prod_{j=1}^{n} r_j$ and $\alpha_{ij}(1\leq\alpha_{ij}\leq r_j)$ denotes which x_j-based fuzzy set is used in the ith fuzzy rule.

Remark 1. This model is proposed in [27]. To better understand this model, consider the following fuzzy system:

$R1$: If $x_1(t)$ is M_1^1 and $x_2(t)$ is M_2^1, then $\dot{x}(t)=A_1x(t)+B_1u(t)$
$R2$: If $x_1(t)$ is M_1^2 and $x_2(t)$ is M_2^1, then $\dot{x}(t)=A_2x(t)+B_2u(t)$
$R3$: If $x_1(t)$ is M_1^1 and $x_2(t)$ is M_2^2, then $\dot{x}(t)=A_3x(t)+B_3u(t)$
$R4$: If $x_1(t)$ is M_1^2 and $x_2(t)$ is M_2^2, then $\dot{x}(t)=A_4x(t)+B_4u(t)$

Then $\alpha_{11}=1,\ \alpha_{12}=1,\ \alpha_{21}=2,\ \alpha_{22}=1, \alpha_{31}=1,\ \alpha_{32}=2,\ \alpha_{41}=2,\ \alpha_{42}=2$.

Let $\omega_j^{\alpha_{ij}}(x_j(t))$ denotes the membership function of $M_j^{\alpha_{ij}}$, then the normalized membership function is

$$\phi_j^{\alpha_{ij}}(x_j(t))=\frac{\omega_j^{\alpha_{ij}}(x_j(t))}{\sum_{\alpha_{ij}=1}^{r_j}\omega_j^{\alpha_{ij}}(x_j(t))}$$

which satisfies

$$0\leq\phi_j^{\alpha_{ij}}(x_j(t))\leq 1,\quad \sum_{\alpha_{ij}=1}^{r_j}\phi_j^{\alpha_{ij}}(x_j(t))=1$$

Then the normalized membership function of the ith fuzzy rule becomes

$$h_i(x(t))=\prod_{j=1}^{n}\phi_j^{\alpha_{ij}}(x_j(t))$$

and

$$0\leq h_i(x(t))\leq 1,\quad \sum_{i=1}^{r}h_i(x(t))=1$$

Omitting t in $x(t)$ and $u(t)$ for simplicity, and by using a singleton fuzzifier, product inference and a center-average defuzzifier, the following dynamic global model can be obtained:

$$\dot{x}=\sum_{i=1}^{r}h_i(x)\{A_ix+B_iu\} \tag{2}$$

The unforced system is given as

$$\dot{x}=\mathcal{A}(h)x=\sum_{i=1}^{r}h_i(x)A_ix \tag{3}$$

Before giving out our main result, some definitions about $\mathcal{D}$ stability are given as follows.

Definition 1 (*Chilali and Gahinet [32]*). A subset $\mathcal{D}$ of the complex plane is called an LMI region if there exist a symmetric matrix $R_1\in\mathbb{R}^{d\times d}$ and a matrix $R_2\in\mathbb{R}^{d\times d}$ such that

$$\mathcal{D}=\{s=\sigma+j\omega\in\mathbb{C}: R_1+sR_2+\bar{s}R_2^T<0\} \tag{4}$$

d is called the order of the LMI region.

When $R_1=0,\ R_2=1$, the LMI region corresponds to the left half plane of the complex plane. When $R_1=\begin{bmatrix}-r & q\\ q & -r\end{bmatrix}$ and $R_2=\begin{bmatrix}0 & 0\\ 1 & 0\end{bmatrix}$, the LMI region is a disk with center at $(-q,0)$ and radius r.

Definition 2. Given an LMI region defined by (4), a nonlinear system $\dot{x}=f^T(x)x$ is said to be $\mathcal{D}$ stable if there exists a Lyapunov function $V(x)$ satisfying $(1/2)(\dot{V}(x)/V(x))\in\mathcal{D}$, i.e.

$$R_1\otimes 1+R_2\otimes\frac{1}{2}\frac{\dot{V}(x)}{V(x)}+R_2^T\otimes\frac{1}{2}\frac{\dot{V}(x)}{V(x)}<0$$

Remark 2. This definition is adopted from [37] and the transient performance of the nonlinear system can be guaranteed by choosing the corresponding LMI regions. When the LMI region $\mathcal{D}$ reduces to the left half plane, the $\mathcal{D}$ stability problem reduces to the stability problem.

Our aim in this paper is to design a parallel distribute compensation (PDC) controller such that the fuzzy closed loop system is $\mathcal{D}$ stable. Let K_i be the state feedback gain of the ith local model, the local control laws are as follows:

$$\begin{aligned}\textit{Control rule } i: \quad & \text{If } x_1 \text{ is } M_1^{\alpha_{i1}} \text{and} \ldots x_j \text{ is } M_j^{\alpha_{ij}} \text{and} \ldots x_n \text{ is } M_n^{\alpha_{in}}\\ & \text{Then } u=K_ix\end{aligned} \tag{5}$$

Then the global fuzzy controller can be calculated by

$$u=\sum_{i=1}^{r}h_i(x)K_ix \tag{6}$$

and the closed-loop system is

$$\dot{x}=\mathcal{A}(h)x=\sum_{i=1}^{r}\sum_{j=1}^{r}h_i(x)h_j(x)(A_i+B_iK_j)x. \tag{7}$$