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Existence and exponential stability of periodic solution for stochastic Hopfield neural networks on time scales ☆

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ABSTRACT

In this paper, by using the contraction mapping theorem and Gronwall's Inequality on time scales, we establish some sufficient conditions on the existence and exponential stability of periodic solutions for a class of stochastic neural networks on time scales. Moreover, we present an example to illustrate the feasibility of our results and to show that the continuous-time neural network and its discrete-time analogue have the same dynamical behaviors.

1. Introduction

Cellular neural networks have been successfully applied in different areas such as signal and image processing, pattern recognition, and optimization. In the study of cellular neural networks, Hopfield neural networks (HNNs) are an important type of neural networks. The existence and stability of equilibrium points, periodic solutions or almost periodic solutions for HNNs have been studied by many scholars (see [1–6] and references cited therein). For example, in [6], based on Lyapunov functionals, authors obtained sufficient conditions on the stability of Hopfield neural networks on time scales. In papers [7–9], by using the continuation theorem of coincidence degree theory and constructing suitable Lyapunov functionals or utilizing the boundedness of the solutions, authors studied the existence and exponential stability of periodic or anti-periodic solutions of higher-order Hopfield neural networks on time scales. In [10], by using Mawhins's continuation of coincidence degree theory and constructing suitable Lyapunov functionals, the author investigated the periodicity and exponential stability for a class of BAM higher-order Hopfield neural networks on time scales. However, none of the above results considered stochastic effects on the dynamical behavior of neural networks.

As pointed out by Haykin [11] that in real nervous systems synaptic transmission is noisy process brought on by random fluctuations from the release of neurotransmitters and other probabilistic causes. The neural networks could be stabilized or destabilized by some stochastic inputs [12]. Therefore, it is significant to consider stochastic effects on the dynamical behavior of neural networks. And the corresponding neural networks with noise disturbances are called stochastic neural networks. There are many works on the stability and on the synchronization of stochastic neural networks. For example, in [12–18], scholars studied the stability of different classes of stochastic neural networks and in [19,20], scholars studied the synchronization of two classes of stochastic neural networks, respectively. Authors of [21,22] studied the existence and exponential stability of periodic solutions for impulsive stochastic neural networks. Authors of [23] obtained some sufficient conditions ensuring the existence and stability of almost periodic solutions for a stochastic neural network.

Note that most results are on the stochastic neural networks which act in continuous-time manner. When it comes to implementation of continuous-time networks for the sake of computer-based simulation, experimentation or computation, it is usual to discretize the continuous-time networks. Hence, in implementation and applications of neural networks, discrete-time neural networks become more important than their continuous-time counterpart and more suitable to model digitally transmitted signals in a dynamical way. For example, in paper [17], authors considered the exponential stability of discrete-time delayed Hopfield neural networks with stochastic perturbations and impulses. But it is troublesome to study

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the dynamical properties for continuous and discrete systems, respectively. So it is significant to study dynamical systems on time scales (see [6,8–10,24–32] and references cited therein), which helps avoid proving results twice, once for differential equations and once for difference equations.

Recently, some authors studied stochastic differential equations on time scales. For example, authors of [33] studied stochastic processes indexed by a time scale; authors of [34] introduced the Kalman filter for linear stochastic systems on time scales; authors of [35] studied the existence and uniqueness of solutions for random dynamic systems on time scales; authors of [36] introduced the construction of the stochastic integral and the concept of stochastic dynamic equations on time scales. For other results on stochastic dynamic equations on time scales, readers may refer to [37,38].

However, to the best of our knowledge, there is no paper published on the existence and stability of periodic solutions for stochastic neural networks on time scales. Motivated by the above discussion, in this paper, we consider the following stochastic neural networks on time scales

$$\Delta x_i(t)=\left[-a_i(t)x_i(t)+\sum_{j=1}^{n}b_{ij}(t)f_j(x_j(t))+\sum_{j=1}^{n}c_{ij}(t)g_j(x_j(t-\tau_{ij}(t)))+I_i(t)\right]\Delta t$$
$$+\sum_{j=1}^{n}\delta_{ij}(x_j(t))\,\Delta w_j(t),\quad t\in\mathbb{T},\ i=1,2,\ldots,n, \tag{1.1}$$

where $\mathbb{T}$ is a periodic time scale, n is the number of neurons in layers, $x_i(t)$ denotes the activation of the ith neuron at time t; a_i represents the rate with which the ith neuron will reset its potential to the resting state in isolation when they are disconnected from the network and the external inputs at time t; f_j, g_j are the input–output functions (the activation functions); $\tau_{ij}:\mathbb{T}\to[0,+\infty)\cap\mathbb{T}$ and satisfy $t-\tau_{ij}(t)\in\mathbb{T}$; b_{ij},c_{ij} are elements of feedback templates at time t; I_i denotes the input of the ith neuron at time t, $i=1,2,\ldots,n$; $w(t)=(w_1(t),\ldots,w_n(t))^T$ $(t\in\mathbb{T})$ is the n-dimensional Brownian motion defined on complete probability space $(\Omega,\mathcal{F},\mathbb{P})$; here, we denote by $\mathcal{F}$ the associated σ-algebra generated by $\{w(t)\}$ with the probability measure $\mathbb{P}$; δ_{ij} are Borel measurable functions, $A=(\delta_{ij})_{n\times n}$ is the diffusion coefficient matrix.

Remark 1.1. If $\mathbb{T}=\mathbb{R}$, then (1.1) reduces to

$$\mathrm{d}x_i(t)=\left[-a_i(t)x_i(t)+\sum_{j=1}^{n}b_{ij}(t)f_j(x_j(t))+\sum_{j=1}^{n}c_{ij}(t)g_j(x_j(t-\tau_{ij}(t)))+I_i(t)\right]\mathrm{d}t$$
$$+\sum_{j=1}^{n}\delta_{ij}(x_j(t))\,\mathrm{d}w_j(t),\quad t\in\mathbb{R},\ i=1,2,\ldots,n.$$

In [22], by establishing new integral inequalities and using the properties of spectral radius of nonnegative matrices, authors obtained some sufficient conditions for the existence and global p-exponential stability of periodic solutions for the above system with impulses effects. If $\mathbb{T}=\mathbb{Z}$, then (1.1) reduces to

$$x_i(t+1)-x_i(t)=-a_i(t)x_i(t)+\sum_{j=1}^{n}b_{ij}(t)f_j(x_j(t))$$
$$+\sum_{j=1}^{n}c_{ij}(t)g_j(x_j(t-\tau_{ij}(t)))+I_i(t)$$
$$+\sum_{j=1}^{n}\delta_{ij}(x_j(t))w_j(t),\quad t\in\mathbb{Z},\ i=1,2,\ldots,n.$$

In [39], based on Lyapunov stability theory and stochastic approaches, authors derived criteria ensuring the robust exponential stability in the mean square for the above system.

Our main purpose of this paper is by using the contraction mapping theorem to establish some sufficient conditions on the existence of periodic solutions for (1.1). Besides, by using Gronwall's Inequality on time scales, we also explore the exponential stability of periodic solutions of (1.1). Our results are new even in both cases of $\mathbb{T}=\mathbb{R}$ or $\mathbb{Z}$.

Denote by $BC^b_{\mathcal{F}_0}(\mathbb{T},\mathbb{R}^n)$ the family of bounded $\mathcal{F}_0$-measurable, $\mathbb{R}^n$-valued random variables $x(t)$, that is, the value of $x(t)$ is an n-dimensional real vector and can be decided from the values of $w(s)$ for $s\le 0$. For convenience, we denote $[a,b]_{\mathbb{T}}=\{t\,|\,t\in[a,b]\cap\mathbb{T}\}$. For an ω-periodic rd-continuous function $f:R\to R$, denote $\overline{f}=\sup_{t\in[0,\omega]_{\mathbb{T}}}|f(t)|$, $\underline{f}=\inf_{t\in[0,\omega]_{\mathbb{T}}}|f(t)|$. The initial condition of (1.1) is

$$x_i(s)=\varphi_i(s), \tag{1.2}$$

where $\varphi_i\in BC^b_{\mathcal{F}_0}([-\tau_0,0]_{\mathbb{T}},\mathbb{R})$, $i=1,2,\ldots,n$, $\tau_0=\max_{1\le i,j\le n}\sup_{t\in[0,\omega]_{\mathbb{T}}}\tau_{ij}(t)$.

Throughout this paper, we assume that the following conditions hold:

(H_1) $a_i(t)>0$ with $-a_i\in\mathcal{R}$, $b_{ij}(t),c_{ij}(t),I_i(t),J_j(t),\tau_{ij}(t)$ are all periodic rd-continuous functions with period ω for $t\in\mathbb{T}$, where $\mathcal{R}$ denotes the set of regressive functions on $\mathbb{T}$, $i=1,2,\ldots,n$;

(H_2) f_j,g_j,δ_{ij} are Lipschitz-continuous with Lipschitz constants $L^f_j>0$, $L^g_i>0$, $l_{ij}>0$, respectively, $i,j=1,2,\ldots,n$.

This paper is organized as follows: In Section 2, we introduce some definitions and state some preliminary results which are needed in later sections. In Section 3, we establish some sufficient conditions for the existence of periodic solutions of (1.1). In Section 4, we prove that the periodic solution obtained in Section 3 is exponentially stable. In Section 5, we give an example to illustrate our results obtained in previous sections.

2. Preliminaries

In this section, we introduce some definitions and state some preliminary results.

At first, we recall some basic definitions and results on time scales.

Definition 2.1 (*Hilger [40]*). Let $\mathbb{T}$ be a nonempty closed subset (time scale) of $\mathbb{R}$. The forward and backward jump operators σ, $\rho:\mathbb{T}\to\mathbb{T}$ and the graininess $\mu:\mathbb{T}\to[0,\infty)$ are defined, respectively, by

$$\sigma(t)=\inf\{s\in\mathbb{T}:s>t\},\quad \rho(t)=\sup\{s\in\mathbb{T}:s<t\}\quad\text{and}\quad \mu(t)=\sigma(t)-t.$$

Definition 2.2 (*Bohner and Peterson [41]*). We say that a time scale $\mathbb{T}$ is periodic if there exists $p>0$ such that if $t\in\mathbb{T}$, then $t\pm p\in\mathbb{T}$. For $\mathbb{T}\ne\mathbb{R}$, the smallest positive p is called the period of the time scale.

Definition 2.3 (*Bohner and Peterson [41]*). Let $\mathbb{T}\ne\mathbb{R}$ be a periodic time scale with period p. We say that the function $f:\mathbb{T}\to\mathbb{R}$ is periodic with period ω if there exists a natural number n such that $\omega=np$, $f(t+\omega)=f(t)$ for all $t\in\mathbb{T}$ and ω is the smallest positive number such that $f(t+\omega)=f(t)$.

Definition 2.4 (*Bohner and Peterson [41]*). A point $t\in\mathbb{T}$ is called left-dense if $t>\inf\mathbb{T}$ and $\rho(t)=t$, left-scattered if $\rho(t)<t$, right-dense if $t<\sup\mathbb{T}$ and $\sigma(t)=t$, and right-scattered if $\sigma(t)>t$. If $\mathbb{T}$ has a left-scattered maximum m, then $\mathbb{T}^k=\mathbb{T}\setminus\{m\}$; otherwise $\mathbb{T}^k=\mathbb{T}$. If $\mathbb{T}$ has a right-scattered minimum m, then $\mathbb{T}^k=\mathbb{T}\setminus\{m\}$; otherwise $\mathbb{T}^k=\mathbb{T}$.