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An intelligent method of swarm neural networks for equalities-constrained nonconvex optimization

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ABSTRACT

To deal with equalities-constrained nonconvex optimization problem, an intelligence method of swarm neural networks (SNN) is introduced in this paper. The proposed method handles the problem into two parts, which combines local searching ability of one-layer recurrent neural network (RNN) and global searching ability of shuffled frog leaping algorithm (SFLA). First, a RNN model based on general nonconvex optimization is presented. Then the convergence property of RNN is analyzed and proven. Moreover, based on SFLA framework, neural networks are treated as frogs which must be divided into several memeplexes and evolve by their own differential equations to search a local exact solution. Next, through shuffling the best solution of each memeplex, we can obtain the global best point. Finally, numerical examples with simulation results are given to illustrate the effectiveness and good characteristics of the proposed method solving nonconvex optimization problem.

1. Introduction

Equalities-constrained optimization has many applications in engineering and scientific problems [1–4], such as signal processing, subcarrier allocation in wireless communication, signal transmission, channel or power allocation. Among them, due to the real-time and hardware implementations [5], RNN is one of the effective means to obtain the optimal solution. Hopfield neural networks were firstly proposed for linear programming by Tank and Hopfield [6]. And then, Kennedy and Chua [7] presented a RNN for nonlinear optimization which employed both gradient method and penalty function method to approximate optimal solutions. Because of the pioneering work [8–12], more and more researchers are inspired to develop neural networks for solving optimization problem. In [13], a RNN is presented which performed quadratic optimization subject to bound constraints on each of the optimization variables. Wang [14] utilized a discrete neural network which was capable of determining the shortest paths of a given directed network. Liu [15] studied a one-layer recurrent neural network with a discontinuous activation function which was proposed for linear programming. Xia [16,17] widely investigated several classes of RNN to solve convex optimization subject to nonlinear inequality constraints. Then Hu and Zhang [18] applied RNN to solve *k*-Winners-Take-All Problem. Especially, He [19,20] constructed a new neural network model for solving bilevel programming problem. As projection operator is introduced in recurrent neural network [22–24,33–35], it brings more simpler structure of the recurrent neural network. Ref. [25] solves generally constrained generalized linear variational inequalities using the general projection neural networks. Ref. [26] proposes a projection neural network solving nonlinear convex programming problem without requiring the Lipschitz continuity condition of objective function.

In this paper, it is concerned with nonconvex optimization problem. We exploit penalty method and projection technology to design a novel recurrent neural network model. Because of massive local optimal solutions, the new RNN model is short of ability to guarantee the global best point [27]. Motivated by shortage of RNN mentioned above, we creatively investigate using numerous RNNs in SFLA's framework which is named SNN. SNN can solve the complex nonconvex problem which [17,18,20,21,26] cannot be handled by the existing methods. SFLA belongs to the Memetic algorithm (MA) family. It is a meta-heuristic optimization method inspired from the memetic evolution of a group of frogs when seeking for food. In this algorithm, the evolution of memes is driven by the exchange of information among interactive individuals. SFLA combines the advantages of the genetic-based MA and the social behavior-based Particle Swarm Optimization (PSO). It has been tested on several optimization problems and found to be effective in searching the global solutions [28–30]; owing to associate with RNNs, SLFA's local searching ability will have a huge improvement.

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The main contribution of the paper is: (i) to establish a new recurrent neural network to solve the optimization model with equality constraints and then its convergence has been proven; (ii) to improve the performance of SFLA algorithm through combining with RNNS and (iii) to put forward the idea to solve non-convex optimization problem using swarm neural networks.

The rest of this paper is organized as follows. In Section 2, the one-layer recurrent neural network is described. In Section 3, analysis of the recurrent neural network's convergence is given. In Section 4, swarm neural network optimization is elaborated in detail. In Section 5, three illustrative examples are discussed. Conclusions are found in Section 6.

2. Problem formulation and model description

This section provides the necessary mathematical background which is used to describe nonconvex program problem and then constructs a recurrent neural network model to solve it. Consider an equalities-constrained optimization problem given by

$$\begin{aligned} &\text{minimize} \quad f(x) \\ &\text{s.t.} \quad h_i(x)=0 \\ &x\in\Omega, \quad i=1,\dots,m. \end{aligned} \tag{1}$$

where $x=(x_1,x_2\dots x_n)^T\in\Re^n$ is the decision variable, $f:\Re^n\to\Re$ is a minimized objective function, $h_i:\Re^n\to\Re, i=1,\dots,m$, are equality constraints. $\Omega=\{x\in\Re^n|u\le x\le v\}$ is a box set in $\Re^n$ which is a nonempty and closed convex set.

Definition 1. Let $I=\{i|\,h_i(x)=0\}, x^*\in\Re^n$, be the global optimal solution if existing x^* satisfies $f(x^*)\le f(x)$, for all $x\in\Omega\cap I$. In this paper, we assume that $\Omega\cap I$ is nonempty.

Next, for the sake of elaborating the following work, we also consider the variational inequality problem $VI(U,\Omega_0)$ which is related to nonlinear program:

$$U(\check{x})^T(x-\check{x})\ge 0, \quad \forall x\in\Omega \tag{2}$$

where $U(x)$ is always the gradient of objective function on convex set Ω, $\check{x}$ is a KKT point which is equivalent to the following conditions [36, p. 2, Example 1]:

(1) if $u<\check{x}<v$, then $f(\check{x})=0$;
(2) if $\check{x}=u$, then $f(\check{x})\ge 0$;
(3) if $\check{x}=v$, then $f(\check{x})\le 0$;

Lemma 1 (*Xia et al. [23]*). *$\check{x}$ is a solution to $VI(U,\Omega)$ if and only if $\check{x}$ satisfies*

$$P_\Omega(\check{x}-\alpha U(\check{x}))=\check{x}. \tag{3}$$

In Eq. (3) α is any positive constant, and $P_\Omega:\Re^n\to\Omega$ is a projection operator which enforces vector ξ in Ω and is defined by

$$P_\Omega(\xi)=\arg\min_{\eta\in\Omega}\|\xi-\eta\|. \tag{4}$$

Under previous work, the way to compute projection of a point onto a box set is defined as follows:

$$P_\Omega(\xi_i)=\begin{cases} v_i, & \xi_i\le v_i \\ \xi_i, & v_i\le\xi_i\le u_i. \\ u_i, & \xi i\ge u_i \end{cases} \tag{5}$$

Moreover, Ω can also be used to express a ball constrain, i.e.

$$\Omega=\{\xi:\|\xi-c\|\le\rho\}$$

with $c\in\Re^n$ and $\rho>0$. Then, the form can be changed as follows:

$$P_\Omega(\xi)=\begin{cases} \xi, & \|\xi-c\|\le\rho \\ c+\dfrac{\rho(\xi-c)}{\|\xi-c\|}, & \|\xi-c\|>\rho. \end{cases} \tag{6}$$

Remark 1. Generally, a problem which is regarded as nonconvex program must satisfy one of the following conditions. Firstly, either the objective function or its constraints should be nonconvex. Secondly, both objective function and its constraints are nonconvex. Unlike convex program, nonconvex program always can obtain many local optimal solutions, but not all of them are the global best.

Recently, projection operator is an effective and simple method for dealing with the constraints, which has been used in neural networks for solving some kinds of constrained optimization problems. Different from some most recent neural network models based on penalty method only, we introduce the following system modeled by a nonautonomous differential equation with projection operator to solve (1)

$$\dot{x}(t)=-x(t)+P_\Omega\{x(t)-\alpha U[x(t)]\}. \tag{7}$$

If $f(x)$ and $h(x)$ are continuous and differentiable, we could replace $U[x(t)]$ with $\nabla f(x)+2c\nabla h(x)h(x)$, where $\nabla f(x)$ is the gradient of $f(x)$, $\nabla h(x)$ is the gradient of $h(x)$ and $\nabla h(x)=(\nabla h_1(x),\dots,\nabla h_m(x))$, and $c\in\Re^n$, is the coefficient of $\nabla h(x)$. Let $\alpha=1$, so Eq. (7) can be changed to

$$\dot{x}(t)=-x(t)+P_\Omega\{x(t)-\nabla f[x(t)]-2c\nabla h[x(t)]h[x(t)]\}. \tag{8}$$

Obviously, the model (8) is a RNN which only has one-layer structure and each decision variable corresponds to one neuron. The relationship between model's equilibria and the KKT points will be shown in the next section. Meanwhile, the convergence of the model will be demonstrated.

3. Theoretical analysis

Lemma 2 (*Xia and Wang [31]*). *If there exists a point (x^*,y^*,z^*) such that for all $x\in\Re^n$, and all $(z,y)\in\Re^{r+m}$ with $z\ge 0$*

$$L(x^*,z,y)\le L(x^*,z^*,y^*)\le L(x,z^*,y^*). \tag{9}$$

Then x^* is an optimal solution to nonlinear program, where $L(x,z,y)=f(x)+z^Tg(x)+y^Th(x)$ is referred to as a Lagrange function.

Where there are only equality constraints, it follows that (x^*,y^*) satisfies the Karush–Kuhn–Tucker conditions below

$$\nabla f(x)+\nabla h(x)y=0, \quad h(x)=0. \tag{10}$$

For convex program, the Karush–Kuhn–Tucker condition (10) is the sufficient and necessary condition to optimal solution. The previous work has given the proof that if problem (1) is convex, dynamic system (8) can globally converge to the exact optimal solution to problem (1). But for nonconvex program, (10) is just a necessary condition. However, when objective $f(x)$ is nonconvex on Ω, global property cannot be guaranteed.

For the equality constraints $h_i=0$, $i=1,\dots m$, the penalty method is introduced into the network for solving (1). The penalty function used in this paper can be given in any expression, which satisfies the following condition:

- $P(x)=0$ for $x\in Q=\{x|\,h_i(x)=0, i=1,\dots,m\}$, and $P(x)>0$ for x is not in Q.
- $P(x)$ is convex on Ω.

So the problem (1) can be transformed into a unconstrained form:

$$\bar{f}=f(x)+c\sum_{i=1}^{m}\left[h_i(x)\right]^2, \tag{11}$$

where we let $P(x)=c\sum_{i=1}^{m}\left[h_i(x)\right]^2$.