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Finite-time stability analysis of fractional-order neural networks with delay

Xujun Yang [a], Qiankun Song [a,*], Yurong Liu [b,c], Zhenjiang Zhao [d]

[a] School of Management, Chongqing Jiaotong University, Chongqing 400074, China
[b] Department of Mathematics, Yangzhou University, Yangzhou 225002, China
[c] Communication Systems and Networks (CSN) Research Group, Faculty of Engineering, King Abdulaziz University, Jeddah 21589, Saudi Arabia
[d] Department of Mathematics, Huzhou Teachers College, Huzhou 313000, China

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ABSTRACT

Stability analysis of fractional-order neural networks with delay is addressed in this paper. By using the contracting mapping principle, method of iteration and inequality techniques, a sufficient condition is established to ensure the existence, uniqueness and finite-time stability of the equilibrium point of the proposed networks. Finally, based on the Predictor-Corrector Approach, two numerical examples are presented to illustrate the validity and feasibility of the obtained result.

1. Introduction

The nonlinear systems are pervasive in the real world [1–7], and during the last three decades, neural networks, one of the most important nonlinear systems, have been studied intensively and have been successfully applied to optimization problems [8]. In order to settle problems in the fields of optimization, pattern recognition, and signal processing, etc., neural networks have to be designed such that there is only one equilibrium point and this equilibrium point is stable, thereby avoiding the risk of having spurious equilibria and being trapped at a local minima [9–11]. Thus, it is of significance to study the stability of dynamical neural networks.

In hardware implementation of neural networks, time delays, whether constant or variable, are ubiquitous due to the finite switching speed of amplifiers and will occur in the signal transmission among neurons, which may lead to some complex dynamic behaviors such as oscillation, divergence, chaos and instability or other poor performances of the neural networks [12]. Hence, the stability analysis for delayed neural networks is of both theoretical and practical importance, and over the past dozen years, large quantities of sufficient conditions on the existence, uniqueness and stability of equilibrium point for delayed neural networks were reported under some assumptions, for example, see [13–18] and references therein. Nevertheless, results on the stability analysis of such neural networks are prevailingly based on infinite time interval.

More than sixty years ago, the concept of finite-time stability was introduced by Kamenkov for the first time [19], which was system property concerning the quantitative behavior of the state variables over an assigned finite time interval. Given a bound on the initial condition, a system is said to be finite-time stable if its state norm does not exceed a certain threshold during the specified time interval [20]. It was demonstrated that finite-time stable systems might have not only faster convergence but also better robustness and disturbance rejection properties [21]. Thus, a considerable number of results on finite-time stability for autonomous or non-autonomous systems have been achieved, for example, see [22–25] and references therein.

It should be noted that, in the past few decades, the development of the theory and application of fractional differential equations have tended to mature gradually [26]. Since fractional-order derivative is nonlocal and has weakly singular kernels, it provides an excellent instrument for the description of memory and hereditary properties of various materials and dynamical processes [27]. Thereupon, many phenomena in various fields of engineering, physics and economics, such as viscoelasticity, heat conduction, dielectric polarization, electromagnetic wave, biology and finance can be described by fractional differential [29,30]. Meanwhile, for the first time, Lazarević investigated and discussed the finite-time stability of fractional-order systems with delay [31]. In [32],

* Corresponding author.
E-mail address: qiankunsong@163.com (Q. Song).

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Lazarević and Spasić further studied the finite-time stability of fractional-order dynamic systems by utilizing generalized Gronwall inequality.

In recent years, the fractional calculus has been incorporated into artificial neural networks due to its infinite memory property, and great efforts have been made to study the complex dynamics of fractional-order neural networks to distinguish the classical integer-order models, for example, see [33–43] and references therein. Among them, some authors focused on fractional-order neural networks without delay. In [33], on the basis of classic well-known Hopfield net, the authors presented the fractional-order Hopfield neural networks, and discussed the stability of such networks through energy-like function analysis. In [34], the authors investigated a series of dynamic behaviors involved with fractional-order neural networks, such as stability, multi-stability, bifurcations and chaos. By means of theoretical analysis and numerical simulation, the authors declared that the fractional-order neural networks might demonstrate rich dynamics, and their dynamic behaviors became more and more complex as the fractional order increased, eventually resulting in chaotic behavior. In [35] and [36], by utilizing Lyapunov method and linear control theory, several effective criteria concerning stability and synchronization of fractional-order neural networks and memristor-based fractional-order neural networks were derived, respectively. In [37–39], the authors take the constant delay into account and several delay-dependent or delay-independent sufficient criteria ensuring the existence, uniqueness and uniform stability of the addressed models were derived by means of inequality techniques and the theory of fractional calculus, respectively. In [40], the authors dealt with the fractional-order neural networks with impulsive effects and time-varying delays, and established several sufficient conditions guaranteeing the global Mittag–Leffler stability of the equilibrium point of the neural networks.

To the best of our knowledge, the global exponential stability and uniform stability are mainly concerned with the asymptotical behavior and seldom concerned with specified bounds on the states. However, in many practical applications, people pay more attention on the finite-time behavior of the networks than the asymptotic behavior, thus, it is necessary to maintain the states within some bounds during a specific time-interval, whereupon, some interesting results on finite-time stability of fractional-order neural networks were presented. In [41], the authors investigated the finite-time stability of fractional-order neural networks by Laplace transform, the generalized Gronwall inequality and estimates of Mittag–Leffler functions. In [42], a delay-dependent effective criterion regarding the finite-time stability of fractional-order neural networks with distributed delay was demonstrated by using the theory of fractional calculus and the generalized Gronwall–Bellman inequality approach. In addition, the authors illustrated that the stability performance of the addressed networks was dependent on the time delay and the order of fractional derivative over a finite time interval. In [43], by using Bellman–Gronwall inequality, differential mean value theorem and contracting mapping principle, the authors investigated the finite-time stability of the unique equilibrium point for fractional-order Cohen–Grossberg neural networks with time delay.

Motivated by the above discussions, the objective of this paper is to study the finite-time stability of fractional-order neural networks with delay. First, we will introduced a generalized Gronwall-type inequality. Afterwards, by utilizing this generalized inequality, we will demonstrate the stability of the unique equilibrium point of the proposed model. Meanwhile, it should be pointed out that the research method of iteration and recursion has not yet seen in the field of fractional-order neural networks, and the obtained result is delay-dependent and novel.

The rest of the paper is structured as follows. In Section 2, we will present the proposed neural networks model and recall some necessary definitions and lemmas. In Section 3, a sufficient condition ensuring the existence, uniqueness and finite-time stability of the equilibrium point of such model is established. The effectiveness and feasibility of the theoretical result is illustrated by two numerical examples in Section 4. Finally, the paper concludes in Section 5.

2. Model description and preliminaries

In this paper, we consider the following fractional-order neural networks with delay:

$$D^{\alpha}x_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau)) + I_i, \quad i = 1, 2, \dots, n, \tag{1}$$

or in the vector form:

$$D^{\alpha}x(t) = -Cx(t) + Af(x(t)) + Bf(x(t-\tau)) + I, \tag{2}$$

for $t \in \Theta = [0, T]$, where D^{α} denotes the Caputo fractional derivative of order α, $0 < \alpha < 1$; $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T \in \mathbb{R}^n$ corresponds to the state vector at time t; $f(x(t)) = (f_1(x_1(t)), f_2(x_2(t)), \dots, f_n(x_n(t)))^T$ denotes the neuron activation at time t; $C = diag(c_1, c_2, \dots, c_n)$, where c_i $(c_i > 0)$ describes the rate with which the ith neuron will reset its potential to the resting state in isolation when disconnection from the networks and the external inputs; $A = (a_{ij})_{n\times n}$ and $B = (b_{ij})_{n\times n}$ are referred to the connection of the jth neuron to the ith neuron at time t and $t-\tau$, respectively, where τ denotes the nonnegative constant transmission delay; $I = (I_1, I_2, \dots, I_n)^T$ is the constant input vector.

The initial conditions are of the form:

$$x_i(s) = \psi_i(s), \quad s \in [-\tau, 0], \quad i = 1, 2, \dots, n, \tag{3}$$

where $\psi_i(s)$ denotes the real-valued continuous functions defined on $[-\tau, 0]$, with the norm given by $\|\psi\| = \sup_{s\in[-\tau,0]} \sum_{i=1}^{n} |\psi_i(s)|$.

In the following, we will give some useful definitions and lemmas.

Definition 1 (*Podlubny [26]*). The Riemann–Liouville fractional integral of order $\alpha > 0$ for a continuous function $g: \mathbb{R}^+ \to \mathbb{R}$ is defined as

$$I^{\alpha}g(t) = \frac{1}{\Gamma(\alpha)} \int_{t_0}^{t} (t-s)^{\alpha-1} g(s)\, ds,$$

where I^{α} denotes the Riemann–Liouville fractional integral of order α, $\Gamma(\cdot)$ is the Gamma function.

Definition 2 (*Podlubny [26]*). The Caputo fractional derivative of order $\alpha > 0$ for a function $g \in C^{n+1}([t_0, +\infty], \mathbb{R})$ (the set of all $n+1$ order continuous differentiable functions on $[t_0, +\infty]$) is defined as

$$\begin{aligned} D^{\alpha}g(t) &= I^{n-\alpha}D^{n}g(t) \\ &= \frac{1}{\Gamma(n-\alpha)} \int_{t_0}^{t} (t-s)^{n-\alpha-1} g^{(n)}(s)\, ds, \end{aligned}$$

where n is the first integer greater than α, that is, $n-1 < \alpha < n$.

Particularly, when $0 < \alpha < 1$

$$D^{\alpha}g(t) = \frac{1}{\Gamma(1-\alpha)} \int_{t_0}^{t} (t-s)^{-\alpha} g'(s)\, ds.$$

Definition 3 (*Ke and Miao [43]*). The equilibrium point $x^* = (x_1^*, x_2^*, \dots, x_n^*)^T$ of system (1) is said to be finite time stable with respect to $\{t_0, \delta, \varepsilon, \Theta, \tau\}$, $0 < \delta < \varepsilon$, $\delta, \varepsilon \in \mathbb{R}$, $\Theta = [t_0, t_0 + T]$, such that for any solution $x(t) = (x_1(t), x_2(t), \dots, x_n(t))^T$ of system (1) with initial conditions (3), if and only if

$$\|\psi - x^*\| < \delta,$$