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Distributed delay control of multi-agent systems with nonlinear dynamics: Stochastic disturbance

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ABSTRACT

For multi-agent systems, significantly adding to the complexity of dynamical behaviors are intrinsic nonlinearity and stochastic noises due to environmental uncertainties. This paper deals with the mean square bounded consensus problem of multi-agent systems with intrinsic nonlinear dynamics, where each agent is affected by stochastic noises. Considering the impact of the former behaviors of nonlinear agents, we put forward a novel type of integral distributed delay protocol which is formed as a weighted sum of historical information exchange over all time interval $[t-\tau, t]$. Compared with the previous work, the protocol expression of this paper is more general and includes many traditional protocols as its special cases. Based on a Lyapunov-based approach, together with results from matrix theory and algebraic graph theory, sufficient conditions are derived for the mean square bounded consensus. Simulation examples with nonlinear even chaotic agents illustrate the theoretical results.

1. Introduction

During the past decades, consensus problems for networks of dynamic agents have been extensively studied by researchers from distinct points of view, due to a broad application of multi-agent systems including wireless sensor networks, satellite formation [1], the direction of fish or birds [2,3], distributed sensor filtering value [4], evolutionary game models [5–7] and so on. Indeed, some topics highly relevant to consensus are synchronization of complex networks [8–10].

In multi-agent systems, much progress has been recently achieved in investigating consensus in networks of identical linear systems, where consensus can be reached if the linear system satisfies some assumptions and the network is connected [11]. Note that typical multi-agent systems did not take full advantage of the powerful nonlinear dynamics. Recently some authors have considered the consensus problem with nonlinear agent dynamics [12–15].

It is well known that the nonlinear intrinsic dynamics are often subjected to uncertainties, such as failures and repairs of the components, changes in the interconnections of subsystems, and disturbances [16] caused by environmental circumstances, which make it difficult or impossible for a networked agent to obtain timely and accurate information of its neighbors. In Minyi Huang's work [17], stochastic consensus over noisy networks with Markovian and arbitrary switches is considered. The recent work of Jun Liu [18] studies stochastic consensus seeking with communication delays. The another important aspect of multi-agent is systems phase transitions. When a parameter changes, the nonlinear dynamical behavior will change too. The change is characterized by a certain type of transition point, either continuous or discontinuous. Two recent studies [19,20] have beautifully demonstrated that a comprehensive control would need to address this in detail since the vicinity of the phase transition point could vitally affect control.

Since sometime the multi-agent systems cannot achieve consensus by itself, one critical issue arising from multi-agent systems is to develop the control strategies based only on local relative information that can guarantee the whole system to evolve into a coordinated behavior. To achieve the aim, suitable neighbor-based rules are usually adopted to interconnect the networked agents, such as impulsive control, adaptive control, distributed control [21–29]. One important challenge is the influence of time delay in the inter-agent information flows. In real systems, time delay always exists due to finite communication speed. Olfati et al. [30] study the average consensus of first-order multi-agent systems with constant and uniform communication time delays under fixed topology. Furthermore, Bliman et al. [31] consider uniform and nonuniform time-varying time-delays in control. Wen et al. [32] investigate a delayed-input approach for consensus of multi-agent systems with nonlinear dynamics, which the protocol uses a point delayed information from the range $[t_k, t_{k+1}]$.

Motivated by above discussion, we provide a novel type of neighbor-based integral distributed delay protocol of nonlinear

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continuous time with stochastic disturbance. The novelty of the approach is that the protocol makes use of a decaying weighted sum of historical information exchange over a time interval $[t-\tau, t]$. The weighted functions depend on practical situation and may be exponentially decaying weighted functions or piece-wise functions. The motivation is that outdated state information is within any control system and deserves consideration, and the memory is rather cheap. Precisely, the designed protocol can guarantee that each individual state converges in mean square bounded to common random variable, whose expectation is right the average of the states of the whole system. Compared with Refs. [25,32], the integral protocol expression is more general. And the theoretical analysis for sufficient conditions does not require the Laplace transform, by properly selecting Lyapunov functions, we convert the convergence analysis of matrix products into that of scalar sequences, the multi-agent systems achieve asymptotically mean square bounded consensus.

The rest of the paper is organized as follows. In Section 2, some preliminaries and the model description are given. We design the novel integral distributed delay protocol and derive the sufficient conditions for mean square bounded consensus of multi-agent systems in Section 3. In Section 4, some numerical examples are given to illustrate the theoretical results. Conclusions are finally drawn in Section 5.

2. Problem formulation and model description

2.1. Network topology

Let $G=(V,\varepsilon,A)$ be a weighted undirected graph of order N, with $V=\{v_1,v_2,\ldots,v_N\}$ representing the sets of nodes, $\varepsilon \in v\times v$ the set of undirected edges, and $A=[a_{ij}]_{N\times N}$ the underlying weighted adjacency matrix with nonnegative elements. It is assumed that $a_{ii}=0$ for $i=1,2,\ldots,N$. The set of neighbors of a node v_i is denoted by $N_i=\{v_j\in V:(v_j,v_i)\in\varepsilon\}$. Let $x_i\in R^n$ denote the position states of the ith agent. A path between nodes v_i and v_j in G is a sequence of edges (v_i,v_{i1}), $(v_{i1},v_{i2}),\ldots(v_{il},v_j)$. A graph is connected if any two distinct nodes of the graph are connected through a path that follows the direction of the edges of digraph. The graph Laplacian L of the network is defined by $l_{ii}=-\Sigma_{j=1,j\neq i}^{N} a_{ij}$ and $l_{ij}=-a_{ij}$.

For more details, one can see Refs. [2,18,32].

2.2. Model description

The commonly studied first-order continuous-time multi-agent systems [8] are described by $\dot{x}_i(t)=u_i(t), i=1,\ldots,N$, where $x_i\in R^n$ is the position state of the ith agent, and $L=[l_{ij}]_{N\times N}$ is the Laplacian matrix of the communication topology $G(A)$. $u_i(t)\in R^n$ is a control input to be designed. Usually, $u_i(t)=-\sum_{j=1}^{N} l_{ij}x_j(t)$. However, the velocity of each agent is generally not a constant but a time-varying variable. Many works [12,13,32] study multi-agent systems with nonlinear dynamics

$$\dot{x}_i(t)=f(x_i(t),t)+u_i(t), \quad i=1,\ldots,N$$

where $f(x_i(t),t)\in R^n$ describes the intrinsic nonlinear dynamics of the ith agent. The consensus states are that $\lim_{t\to\infty}\|x_i(t)-x_j(t)\|=0$ for all $i,j=1,2,\ldots,N$. Furthermore, in realistic applications, uncertainties in subsystems or environmental disturbances always exist. To cope with this, we consider the following multi-agent dynamical systems:

$$\dot{x}_i(t)=f(x_i(t),t)+\sigma(t,x_i(t))n(t)+u_i(t), \quad i=1,\ldots,N \tag{1}$$

where $\sigma(\cdot,\cdot)$ is the noise intensity function vector. $n(t)$ is a scalar zero mean Gaussian white noise process. Recall that the time derivative of a Wiener process (Brownian motions) is a white noise process, we have $dw(t)=n(t)\,dt$. Only the information of v_i itself and its neighbors are available in forming the state feedback for the node v_i. The objective here is to design a distributed control protocol $u_i(t)$ to achieve the stochastic bounded consensus.

Definition 1. The system (1) under the protocol $u_i(t)$ is said to reach the mean square bounded consensus, if there exists $x^*(t)$ such that

$$\lim_{t\to\infty} E(x_i(t)-x^*(t))^2 \le C<\infty; \ \forall \ i\in I.$$

where C is a bounded positive constant independent of t.

The mean square bounded consensus definition is adopted [33,35] for multi-agent systems. In this paper, we will first deal with the stochastic consensus problem for the system (1) with nonlinear agents. By constructing new Lyapunov Krasovskii functionals, sufficient conditions for consensus problems are derived.

3. Main results

In this section, the mean square bounded consensus problem for system (1) is investigated. The integral distributed delay neighbor-based protocol is designed as

$$u_i(t)=\sum_{v_j\in N_i} a_{ij}\left[\int_{t-\tau}^{t} K(t-s)x_j(s)\,ds-\int_{t-\tau}^{t} K(t-s)x_i(s))\,ds\right],$$
$$i=1,\ldots,N \tag{2}$$

where $K(t)=diag(k_1(t),\ldots,k_n(t))$, and the weighted functions $k_j(t)$ is a real-valued nonnegative continuous function defined on $[0,\infty)$, such as an exponentially decaying weighted function $k_i(t)=\exp(-t)$. Moreover, there exist two constants $\nu>0$, $c>0$, and matrix $\overline{K}(\nu)=diag(\overline{k}_1(\nu),\ldots,\overline{k}_n(\nu))>0$ such that

$$\int_0^{\tau} k_j(\theta)\,d\theta\le c, \qquad \int_0^{\tau} k_j(\theta)e^{\nu\theta}\,d\theta=\overline{k}_j(\nu)<\infty \tag{3}$$

Substituting (2) into (1) gives

$$\dot{x}_i(t)=f(x_i(t),t)+\sum_{v_j\in N_i} a_{ij}\left[\int_{t-\tau}^{t} K(t-s)(x_j(s)-x_i(s))\,ds\right]+\sigma(t,x_i(t))n(t),$$
$$i=1,\ldots,N \tag{4}$$

Remark 1. Distributed delays over a certain duration of time have been introduced in [27,34]. Distributed delays are due primarily to the presence of an amount of parallel pathways of a variety of agents sizes and lengths. We propose the integral distributed delay protocol (2) to achieve mean square bounded consensus in multi-agent systems. As can be seen in the sequel, the inclusion of such a distributed delay expression will bring additional difficulty in the analysis and a novel Lyapunov Krasovskii function will need to be established.

Remark 2. If the weighted functions are chose as exponentially decaying functions $k_i(t)=\exp(-t)$, it can describe many typical phenomena such as signal transmission among neurons, the distant past has less influence compared to the recent behavior of the state.

Remark 3. Delay consensus protocol for continuous type of multi-agents system has been investigated by several authors [18,32]. Here we propose a novel delay consensus criterion for nonlinear dynamics in environmental uncertainties via the previous and current information, which uses a decaying weighted sum of historical information exchange over a time interval $[t-\tau, t]$. The goal of strategy is to make full use of the delay information, which thus allow for a better interpretation and measure complexity arising in nonlinear systems. In fact, this consensus protocol is a