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Output feedback robust stabilization of switched fuzzy systems with time-delay and actuator saturation ☆

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ABSTRACT

This paper investigates the output feedback robust stabilization problem of a class of switched fuzzy systems with immeasurable states and actuator saturation. A switched state observer is designed to obtain the estimations of the unmeasured states. By using parallel compensation design (PDC) scheme, a robust fuzzy output feedback control law and the signal switching law are constructed, respectively. The sufficient conditions of ensuring the switched fuzzy control system asymptotic stabilization are proposed and formulated in the form of linear matrix inequalities (LMIs). It is proved that proposed control scheme can guarantee that whole closed-loop system is asymptotically stable in the sense of the Lyapunov function. One numerical example and a practical example are given to illustrate the effectiveness of the proposed control method.

1. Introduction

In the past decades, the Takagi–Sugeno (T–S) fuzzy model has witnessed a great progress since it is an effective tool in approximating most complex nonlinear systems [1]. Based on the T–S fuzzy models (fuzzy systems), many significant control design methods and stability conditions have been obtained, for example, see [2–16]. Basic stability conditions [3–6] in terms of LMIs were obtained by using the Lyapunov functions theory. Subsequently, several relaxed stability conditions were proposed based on PDC design concept [7–9]. Note that the above mentioned results are developed by a single Lyapunov function method. Thus the obtained stability conditions are conservative. To obtain less conservative stability conditions, Ohtake et al. [10,11] and Li et al. [12,13] proposed switching fuzzy controller designs for a class of for a class of fuzzy systems, and the obtained stability conditions are based on the switching Lyapunov function; Wu et al. [14] proposed the problems of stability analysis for a class of discrete-time T–S fuzzy systems with time-varying state delay based on a novel fuzzy Lyapunov–Krasovskii function; Yang and Dong [15], Wang et al. [16], Dong and Yang [17,18] and Lam [19] investigated a switching fuzzy controller design for a class of fuzzy systems via a switching fuzzy model and a piecewise Lyapunov function, and new stability conditions were developed. However, the aforementioned controller design and stability analysis theories are only for the simple fuzzy systems, instead of switched fuzzy systems.

Recently, the control synthesis of switched systems has been extensively investigated [20–28]. Switched systems are composed of a family of continuous-time or discrete-time subsystems and a logical rule that orchestrates switching between the subsystems. Switching signals are crucial in analysis and design of switched systems, and they can be classified into arbitrary switching and dwell time or average dwell time switching. As for the former, the study mainly based on a common Lyapunov function for all subsystems [20–23]. As for the latter, the average dwell time logic is proposed, and stability problem and switching law design are addressed for switched systems with average dwell time [24–28]. More recently, several control design and stability methods have been explored for switched fuzzy systems, and some useful results have been obtained, such as in [29–34]. However, all the above-mentioned results are based on state feedback control methodology. The assumption of the state variables being available for measurement must be imposed on the controlled systems, which limits the applicability of these control schemes in practical engineering. In addition, the aforementioned control methods do not consider the effect of the actuator saturation on the control performance. In fact, the actuator saturation exists in many practical control systems, and its existence often degrades the performance of a control system and sometimes even leads to the instability of the control systems [35,36]. To the best of our knowledge, to date, there are few results on fuzzy output feedback control design for switched fuzzy systems with immeasurable states and actuator saturation, which motivates us for this study.

This paper investigates the output feedback robust stabilization problem of a class of switched fuzzy systems with immeasurable

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states and actuator saturation. A switched state observer is first designed and the unmeasured states are obtained via it. By using parallel compensation design (PDC) scheme, a robust fuzzy output feedback controller and the signal switching law are constructed, respectively. The sufficient conditions of ensuring the asymptotic stabilization of switched fuzzy control system are proposed and formulated in the form of linear matrix inequalities (LMIs). It is proved that proposed control scheme can guarantee that whole closed-loop system is asymptotically stable in presence of the immeasurable states and actuator saturation.

2. System description

Consider the following switched fuzzy system, which is composed of l fuzzy subsystems as follows:

R_σ^i: IF $z_1(t)$ is $F_{\sigma 1}^i$, $z_2(t)$ is $F_{\sigma 2}^i$, …,$z_p(t)$ is $F_{\sigma p}^i$, then

$$\begin{cases} \dot{x}(t)=A_{\sigma i}x(t)+A_{d\sigma i}x(t-d)+B_{\sigma i}u_\sigma(t) \\ y(t)=C_{\sigma i}x(t) \end{cases}, \quad i=1,2,\ldots,N_\sigma, \tag{1}$$

where $z(t)=[z_1(t),z_2(t)\ldots z_p(t)]^T$ are the premise variables, and $F_{\sigma j}^i$ are the fuzzy sets; $\sigma\in F=\{1,2,\ldots,l\}$ is a switching signal, which is a piecewise constant function; d is the constant bounded time delay; $A_{\sigma i}$, $A_{d\sigma i}$, $B_{\sigma i}$ and $C_{\sigma i}$ are known real constant matrices with appropriate dimensions. $x(t)\in R^n$ is the state variable vector, $y(t)$ is the output of the system, and $u_\sigma(t)\in R^m$ is the control input with actuator saturation, which is defined as follows [31]:

$$u_\sigma(t)=\mathrm{sat}(\vartheta_\sigma(t))$$
$$\mathrm{sat}(\vartheta_\sigma)=[\mathrm{sat}(\vartheta_{\sigma 1}),\mathrm{sat}(\vartheta_{\sigma 2}),\ldots,\mathrm{sat}(\vartheta_{\sigma m})]^T$$

where

$$\mathrm{sat}(\vartheta_{\sigma i})=sign(\vartheta_{\sigma i})\min\{1,|\vartheta_{\sigma i}|\}.$$

According to Wang et al. [6,7], the switched fuzzy system can be represent as

$$\begin{cases} \dot{x}(t)=\sum_{i=1}^{N_\sigma}h_{\sigma i}(z(t))[A_{\sigma i}x(t)+A_{d\sigma i}x(t-d)+B_{\sigma i}u_\sigma(t)] \\ y(t)=\sum_{i=1}^{N_\sigma}h_{\sigma i}(z(t))C_{\sigma i}x(t) \end{cases} \tag{2}$$

where

$$\mu_{\sigma i}(z(t))=\prod_{j=1}^{p}F_{\sigma j}^i(z_j(t)),\quad h_{\sigma i}(z(t))=\frac{\mu_{\sigma i}(z(t))}{\sum_{i=1}^{N\sigma}\mu_{\sigma i}(z(t))}.$$

Hence, $F_{\sigma j}^i(z_j(t))$ is the grade of the membership of $z_j(t)$ in $F_{\sigma j}^i$, $h_{\sigma i}(z(t))$ satisfies the following conditions:

$$0<h_{\sigma i}(z(t))<1,\qquad \sum_{i=1}^{N_r}h_{\sigma i}(z(t))=1$$

For the matrices $H\in R^{n\times n}$, denote the qth row of H as H_q and define the symmetric polyhedron

$$\ell(H_q):=\{x_\sigma(t)\in R^n:|H_q|\le 1,\quad q=1,2,\ldots,m\}$$

Let ν be the set of $m\times m$ diagonal matrices whose diagonal elements are either 1 or 0. For example, if $m=2$, then

$$v=\left\{\begin{bmatrix}1&0\\0&1\end{bmatrix},\begin{bmatrix}1&0\\0&0\end{bmatrix},\begin{bmatrix}0&0\\0&1\end{bmatrix},\begin{bmatrix}0&0\\0&0\end{bmatrix}\right\}$$

Suppose that each element of ν is labeled as E_l, $l=1,2,\ldots,2^m$. Denote $E_l^-=I-E_l$. Note that E_l^- is also an element of ν if $E_l\in\nu$.

The following lemma, which captures certain properties of dynamical system with actuator saturation, will be used in this paper.

Lemma 1. *[31] Let $F\in R^{m\times n}$ and $H\in R^{m\times n}$ be given. If $x_\sigma(t)\in\ell(F)$, then $\mathrm{sat}(Fx(t))$ can be represented as*

$$\mathrm{sat}(Fx_\sigma(t))=\sum_{k=1}^{2^m}\eta_k(t)(E_kF+E_k^-H)x_\sigma(t)$$

$\eta_k(t)$ for $k=1,2,\ldots 2^m$ are some scalars which satisfy $0\le\eta_k(t)\le 1$ and $\sum_{k=1}^{2^m}\eta_k(t)=1$.

The aim of this study is to design a fuzzy output feedback controller with the switching law such that the closed-loop fuzzy system is asymptotically stable in presence of the actuator saturation.

3. Fuzzy controller design and stability analysis

Since the states in (1) are unavailable for feedback control design, a state observer is first established for estimating the unmeasured states. Suppose the following fuzzy switched observer is proposed to deal with the state estimation of switched fuzzy system (2).

R_σ^i: IF $z_1(t)$ is $F_{\sigma 1}^i$, $z_2(t)$ is $F_{\sigma 2}^i$, …,$z_p(t)$ is $F_{\sigma p}^i$, then

$$\begin{cases} \dot{\hat{x}}(t)=A_{\sigma i}\hat{x}(t)+A_{d\sigma i}\hat{x}(t-d)+B_{\sigma i}u_\sigma(t) \\ \qquad +L_{\sigma i}(y(t)-\hat{y}(t)) \qquad i=1,2,\ldots,N_\sigma \\ \hat{y}(t)=C_{\sigma i}\hat{x}(t) \end{cases} \tag{3}$$

where $L_{\sigma i}$ is the observer gain for the ith observer rule within the σth switched fuzzy subsystem.

Next, we consider the switching signal $\sigma=\sigma(\hat{x}(t))$, which is determined by a segmentation of R^n: $\tilde{\Omega}_1,\tilde{\Omega}_2,\ldots\tilde{\Omega}_l$ $\cup_{i=1}^l\tilde{\Omega}_i=R^n-\{0\}$, and $\tilde{\Omega}_i\cap\tilde{\Omega}_j=\varphi, i\ne j$, the switching law is $\sigma=\sigma(\hat{x}(t))=r$, when $\hat{x}(t)\in\tilde{\Omega}_r$. Define the switching function as

$$v_r(\hat{x}(t))=\begin{cases}1 & \hat{x}(t)\in\tilde{\Omega}_r \\ 0 & \hat{x}(t)\notin\tilde{\Omega}_r\end{cases},\quad r\in F. \tag{4}$$

The overall fuzzy switched observers are inferred as follows:

$$\begin{cases} \dot{\hat{x}}(t)=\sum_{r=1}^{l}\sum_{i=1}^{N_r}v_r(\hat{x}(t))h_{ri}(z(t))[A_{ri}\hat{x}(t)+A_{dri}\hat{x}(t-d) \\ \qquad +B_{ri}u_r(t)+L_{ri}(y(t)-\hat{y}(t))] \qquad , \quad i=1,2,\ldots,N_r \\ \hat{y}(t)=\sum_{r=1}^{l}\sum_{i=1}^{N_r}v_r(\hat{x}(t)h_{ri}(z(t))C_{ri}\hat{x}(t) \end{cases} \tag{5}$$

Based on PDC scheme [7,37–38], we design the following fuzzy output feedback control law as:

$$u_r(t)=-\sum_{i=1}^{N_r}h_{ri}(z(t))sat(K_{ri}\hat{x}(t)) \tag{6}$$

So the closed-loop fuzzy switched system is represented as follows:

Denote the state estimation error vector as

$$e(t)=x(t)-\hat{x}(t) \tag{8}$$

$$\begin{cases} \dot{x}(t)=\sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r}v_r(\hat{x}(t))h_{ri}(z(t))h_{rj}(z(t))[A_{ri}x(t)+A_{dri}x(t-d)-B_{ri}sat(K_{rj}\hat{x}(t))]y(t)=\sum_{r=1}^{l}\sum_{i=1}^{N_r}\sum_{j=1}^{N_r}v_r(\hat{x}(t))h_{ri}(z(t))h_{rj}(z(t))C_{ri}x(t) \end{cases} \tag{7}$$