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Adaptive hybrid projective synchronization of two coupled fractional-order complex networks with different sizes

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ABSTRACT

This paper investigates a new hybrid projective synchronization scheme between two coupled fractional-order complex networks with different sizes. The hybrid projective synchronization studied in this paper includes complete synchronization of the states of the nodes in each network and projective synchronization of the states of a pair of nodes from both networks. Based on the stability theorem of fractional-order differential system and adaptive control technique, some sufficient conditions for guaranteeing the existence of the hybrid projective synchronization are derived. Two examples are given to show the effectiveness of the proposed methods.

1. Introduction

In the past few decades, many efforts have been devoted to complex networks due to the wide and potential applications in various fields, such as, World Wide Web, communication networks, social organizations, food webs, power grid networks, genetic regulatory network, and so on [1–8]. Specifically, synchronization, as an important and interesting collective behavior of complex networks in our real life, has drawn increasing attention from different fields such as information science, biological system, image processing, secure communication, etc. [9–15]. Up to now, there are many widely-studied synchronization schemes, which defines the correlated in-time behaviors among the nodes in a dynamical network, such as complete synchronization [16], phase synchronization [17], generalized synchronization [18], projective synchronization [19,20], lag synchronization [21], cluster synchronization [22], synchronization in multi-agent systems [23,24]. The synchronization observed in anetwork is usually called "inner synchronization" as it is a collective behavior within this network. In reality, there are other kinds of network synchronization, for example, synchronization between two or more complex networks regardless if the inner synchronization, which was termed as "outer synchronization", always does exist in our lives. Li et al. [25] investigated the outer synchronization between two unidirectionally coupled networks and a criterion for the synchronization between two networks with identical topological structures. Tang et al. [26] studied the outer synchronization between two networks with different topological structures by adaptive control method. Wu and Lu [27] researched the outer synchronization of uncertain complex delayed networks with adaptive coupling.

The fractional order derivative has its inception in an exchange letters between L'Hospital and Leibniz in 1965. The question was what would happen if the order of a derivative is not the integer. Compared with the classical integer-order models, fractional-order models provide an excellent instrument for the description of memory and hereditary properties of various materials and processes. It would be far better if many practical problems are described by fractional-order dynamical systems rather than integer-order ones. In fact, real-world processes generally or most likely are fractional-order systems, for example, dielectric polarization, electrode–electrolyte polarization, electromagnetic waves, viscoelastic systems, quantitative finance and diffusion waves [28–31]. The advantageous use of this mathematical tool is recognized in the modeling of these dynamical systems and the results demonstrate the importance of fractional calculus and motivate the development of new applications. In recent years, research about the theory and application of fractional calculus has attracted much interest. Specifically, synchronization of fractional-order chaotic systems starts to attract increasing attention due to its potential applications in secure communication and control processing.

Although there are many results about synchronization of complex networks, most efforts have been devoted to complex networks whose nodes are constructed by integer-order ordinary differential equations. As we know, the fractional order calculus has many merits, but due to the limited theories for analyzing the dynamical systems, the related research is still a challenging topic and the existing works is still few. For example, Chai et al. [32] investigated synchronization of general fractional-order complex dynamical

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networks by adaptive pinning method. Yang and Jiang [33] studied the adaptive synchronization in the drive-response fractional-order dynamical networks with uncertain parameters. Si et al. [34] studied the identification problem of fractional-order complex network with unknown system parameters and network topologies. Wu and Lu [35] investigated outer synchronization between two different fractional-order general complex networks.

More recently, coexistence of the hybrid synchronization in fractional-order chaotic systems was investigated intensively [36,37]. To the best of our knowledge, most of these works were concerned with the hybrid synchronization only in two coupled chaotic systems, the hybrid synchronization of two coupled fractional-order networks of different sizes has not been explicitly considered and studied.

Motivated by the above discussions, in this paper, we will study the hybrid projective synchronization of two coupled general fractional-order complex networks with different sizes. In order to realize the generalized projective synchronization, adaptive control method is utilized, which has been widely use in control scheme [38–42]. Based on the stability theorem of fractional-order differential system, several sufficient conditions for guaranteeing the existence of the generalized projective synchronization are obtained.

This paper is organized as follows: in Section 2, some fractional-order definitions, lemmas are given. In Section 3, fractional-order complex networks model and hybrid synchronization definitions are given. In Section 4, the criteria for the hybrid synchronization of the fractional-order complex networks are obtained. The examples are given in Section 5, followed by conclusions in Section 6.

Throughout this paper, the following notations are used. $\|\cdot\|$ is the Euclidean norm of a vector; A^T means the transpose of the matrix A; $I_n \in \mathbb{R}^{n\times n}$ denotes the identity matrix with dimension n; $\otimes$ representation Kronecker product of two matrices and A^s denotes $(A+A^T)/2$.

2. Model description and preliminaries

2.1. Fraction calculus

The fractional-order integer-differential operator is the generalization of integer-order integer-differential operator, which could be denoted by a fundamental operator as follows

$$ {}_aD_t^q = \begin{cases} \frac{d^q}{dt^q}, & R(q)>0, \\ 1, & R(q)=0, \\ \int_a^t (d\tau)^{-q}, & R(q)<0, \end{cases} \tag{1}$$

where q is the fractional-order calculus operator which can be a complex number, a and t are the limits of the operation. There are some definitions for fractional derivatives and the commonly used definitions are Grunwald–Letnikou (GL), Riemann–Liouville (RL), and Caputo (C). In the rest of this paper, the notation ${}_aD_t^q$ is chosen as the Caputo fractional derivation operator.

Definition 1. The Caputo fractional derivative is define as follows

$$ {}_a^cD_t^q x(t) = \begin{cases} \frac{1}{\Gamma(n-q)}\int_a^t (t-\tau)^{n-q-1}x^{(n)}(\tau)d\tau, & n-1<q<n, \\ \frac{d^n}{dt^n}x(t), & q=n, \end{cases} \tag{2}$$

where $\Gamma(\cdot)$ is the Gamma function which is defined by $\Gamma(z)=\int_0^\infty e^{-z}t^{z-1}dt$.

It should be noted that the advantage of the Caputo approach is that the initial conditions for fractional differential equations with Caputo derivatives take on the same form as those for integer-order ones, which have well understood physical meaning.

The following lemmas are needed to derive our main results.

Lemma 1. *[34]. Consider the following fractional order system*

$$ D^qX = f(X), \tag{3}$$

where $X \in \mathbb{R}^n$, $0<q\le 1$. For the nonlinear fractional order systems (3), if there exists a real symmetric positive definite matrix P such that the equation $J = X^TPD^qX \le 0$ always holds for any states $X=(x_1(t),x_2(t),\dots,x_n(t))^T$, then system (3) is asymptotically locally stable.

Lemma 2. *[43]. For any vectors $x, y \in \mathbb{R}^n$ the following matrix inequality holds: $2x^Ty \le x^Tx + y^Ty$.*

2.2. Network model

In this paper, fractional-order complex network model consisting N nodes are described as the following:

$$ D^qx_i(t) = f(x_i(t)) + c\sum_{j=1}^{N} a_{ij}\Gamma x_j(t), \qquad i=1,2,\dots,N_1, \tag{4}$$

where $0<q<1$ is the fractional order, $x_i(t)=(x_{i1}(t),x_{i2}(t),\dots,x_{in}(t))^T \in \mathbb{R}^n$ is the state variable of the ith node, $f:\mathbb{R}^+\times\mathbb{R}^n$ is a smooth nonlinear continuous map, $A=(a_{ij})\in\mathbb{R}^{N\times N}$ denotes he coupling configuration matrix; c is the coupling strength, and $\Gamma=\mathrm{diag}(\zeta_1,\zeta_2,\dots,\zeta_n)\in\mathbb{R}^{n\times n}$ is the inner coupling matrix. The matrix A is defined as follows: if there exist a connection between node i and node $j(i\ne j)$, then $a_{ij}>0$; otherwise, $a_{ij}=0$ and the diagonal elements of matrix A are define as

$$ a_{ii} = -\sum_{j=1, j\ne i}^{N} a_{ij}, \qquad i=1,2,\dots,N_1, \tag{5}$$

We refer to model (4) as the drive complex networks. Correspondingly the response complex network with the control inputs $u_i(t)\in\mathbb{R}^n (i=1,2,\dots,N)$ can be rewritten as

$$ D^qy_i(t) = f(y_i(t)) + c\sum_{j=1}^{N} b_{ij}\Gamma y_j(t) + u_i, \qquad i=1,2,\dots,N_2, \tag{6}$$

where $y_i(t)=(y_{i1},y_{i2},\dots,y_{in})^T\in\mathbb{R}^n$ is the response state vector of the ith node, f, c and Γ have the same meaning as those in Eq. (4), u_i are the controllers to be designed.

3. Mathematical preliminaries

Before starting the main results of this paper, we firstly defined of the complex networks hybrid synchronization.

Definition 2. [44]. The driving networks (4) and the response networks (6) are said to realize the hybrid projective synchronization if

$$ \begin{cases} \lim_{t\to\infty}\|y_i(t,Y_0)-\alpha x_i(t,X_0)\| = 0, & (i=1,2,\dots,N), \\ \lim_{t\to\infty}\|x_i(t,X_0)-x_j(t,X_0)\| = 0, & (i,j=1,2,\dots,N_1), \\ \lim_{t\to\infty}\|y_i(t,Y_0)-y_j(t,Y_0)\| = 0, & (i,j=1,2,\dots,N_2), \end{cases} \tag{7}$$

where $N=\min\{N_1,N_2\}$, and α is the ratio coefficient.

Assumption 1. [45]. Function class *QUAD* (P,Δ): assume that $P=diag(p_1,p_2,\dots,p_n)$ is a positive definite diagonal matrix and $\Delta=\mathrm{diag}(\Delta_1,\Delta_2,\dots,\Delta_n)$ is a diagonal matrix. $f(x):\mathbb{R}^n\to\mathbb{R}^n$ is continuous. We say $f\in\mathrm{QUAD}(P,\Delta)$ if and only if the following inequality

$$ (x-y)^TP(f(x)-f(y)-\Delta(x-y)) \le -\beta(x-y)^T(x-y) \tag{8}$$

holds for some $\beta>0$, all x, $y\in\mathbb{R}^n$ and $t>0$.