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Multistability of discrete-time delayed Cohen–Grossberg neural networks with second-order synaptic connectivity

Wu Yang [a,b], Yan-Wu Wang [a,b,*], Zhi-Gang Zeng [a,b], Ding-Fu Zheng [a,b]

[a] School of Automation, Huazhong University of Science and Technology, Wuhan, Hubei 430074, PR China
[b] Key Laboratory of Image Processing and Intelligent Control (Huazhong University of Science and Technology), Ministry of Education, PR China

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ABSTRACT

This paper addresses the multistability problem of discrete-time delayed Cohen–Grossberg neural networks (DDCGNNs) with second-order synaptic connectivity. For the neural networks with nondecreasing saturated activation functions possessing 2 corner points, based on the partition space method and reduction ad absurdum, several sufficient conditions are derived to ensure that n-neuron second-order DDCGNNs can have 2^n locally exponentially stable equilibrium points. Then, the analyses are extended to nondecreasing saturated activation functions with $2r$ corner points and some sufficient conditions are given to guarantee that the n-neuron DDCGNNs can have $(r+1)^n$ locally exponentially stable equilibrium points. Moreover, some conditions are obtained to ensure the existence of locally exponentially stable equilibrium point in a predesigned region. Finally, three examples are carried out to show the effectiveness of the proposed criteria.

1. Introduction

Cohen–Grossberg neural networks (CGNNs), including Hopfield neural networks and cellular neural networks as their special case, were firstly proposed by Cohen and Grossberg in 1983 [1]. Due to their potential applications in associative memory, image processing, pattern recognition, and some other areas, the studies of this kind of neural networks and their applications have attracted a tremendous amount of research interests [1–13].

As is well known, the stability of equilibrium point is prerequisite for a successful application of dynamic system. In the past decades, monostability of CGNNs have been extensively investigated by many researchers [14–16]. However, in the applications of associative memory storage and image processing, it is desired that CGNNs have as many equilibrium points as possible [5,17–20]. In detail, by using the Cauchy convergence principle and the partition space method, [5,17] discussed the multistability and the multiperiodicity of CGNNs with time delays. The multiperiodicity of a class of high-order CGNNs was discussed in [18]. Wang et al. [19,20] investigated the stability in Lagrange sense for nonautonomous CGNNs with mixed delays and for autonomous CGNNs with mixed delays, respectively.

Note that, only first-order synaptic connectivity is taken into consideration in these studies on CGNNs with the exception of [18]. As revealed in [6,21–25], second-order or high-order neural networks have been shown with greater storage capacity, stronger approximation property, faster convergence rate, and higher fault tolerance than traditional first-order neural networks. Besides, all the above-mentioned models act in a continuous-time fashion, few papers discuss the multistability of discrete-time CGNNs except for the monostability of discrete-time CGNNs [26–30]. In addition, although the discrete-time analogues reflect the dynamics of their continuous-time counterparts to some extent, the discretization cannot maintain the dynamics of the continuous-time counterpart even for a small sampling period [31]. Therefore, it is quite necessary to do some investigations on the dynamics of discrete-time CGNNs.

Motivated by the above discussion, the multistability of second-order discrete-time delayed Cohen–Grossberg neural networks (SODDCGNNs) are studied in this paper for the first time. Some sufficient conditions are presented to ensure that n-neuron SODDCGNNs with nondecreasing saturated activation functions with 2 corner points can have 2^n locally exponentially stable equilibrium points. When the amplification functions in the CGNNs equal to 1 and the second-order synaptic connectivities are absent, the obtained conditions are less conservative than the

* Corresponding author at: School of Automation, Huazhong University of Science and Technology, Wuhan, Hubei 430074, PR China.
E-mail address: wangyw@mail.hust.edu.cn (Y.-W. Wang).

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results by reducing from the multiperiodicity result in [32] to the multistability case. When the amplification functions in the CGNNs equal to 1 and CGNNs have first-order delayed synaptic connectivities, the obtained conditions are also less conservative than the results by reducing from the multiperiodicity result in [33] to the multistability case. The multistability results are further extended to the SODDCGNNs with nondecreasing saturated activation functions with $2r$ corner points and it is shown that there are $(r+1)^n$ locally exponentially stable equilibrium points in this case. Moreover, some sufficient conditions are obtained to guarantee the existence of a unique stable equilibrium point in a designated region. Some examples are also given to illustrate the effectiveness of the proposed results and to show the less conservatism of the results comparing with the multistability case of [32,33].

The remaining part of this paper is organized as follows. In Section 2, the model and some preliminaries are given. In Section 3, main results of this paper are presented. In Section 4, three illustrative examples are provided with simulation results. Finally, some conclusions are made in Section 5.

Notation: The notations $\mathbb{Z}$ and $\mathbb{Z}^+$ denote the set of all integers and positive integers, respectively. $[a,b]=\{a,a+1,\dots,b-1,b\}$, where a, $b\in\mathbb{Z}$, $a\le b$.

2. System description and preliminaries

Consider a class of SODDCGNNs by the system of difference equations:

$$\begin{aligned}x_i(k+1)=x_i(k)-a_i(x_i(k))\Bigg[&b_ix_i(k)-\sum_{j=1}^{n}c_{ij}f_j(x_j(k))\\&-\sum_{j=1}^{n}d_{ij}f_j(x_j(k-\tau_j(k)))-I_i\\&-\sum_{j=1}^{n}\sum_{l=1}^{n}e_{ijl}f_j(x_j(k-\delta_j(k)))f_l(x_l(k-\delta_l(k)))\Bigg],\end{aligned}$$

$$i=1,2,\dots,n,k=0,1,\dots, \tag{1}$$

where $n\ge 2$ is the number of neurons, $x_i(k)$ denotes the state variable of the ith neuron at time k, $a_i(\cdot)$ represents amplification function, and $b_i>0$. c_{ij}, d_{ij} and e_{ijl} are the first-order and second-order connection weights, respectively. I_i is the constant input. $f_j(\cdot)$ is the neuron activation function. $\tau_j(k)$ and $\delta_j(k)$ are time delays of jth neuron at time k and satisfy $0\le\tau_j(k)\le\tau_j$ and $0\le\delta_j(k)\le\delta_j$, respectively, where τ_j and δ_j are nonnegative integers. Initial condition of (1) is assumed to be $x(s)=\phi(s)$, $s\in[-\tau,k_0]$, $\tau=\max_{0\le j\le n}\{\tau_j,\delta_j\}$, where $\phi(s)=(\phi_1(s),\phi_2(s),\dots,\phi_n(s))^T$ and $\phi_i(s)\in C([-\tau,k_0],D)$ is a continuous function, $C([-\tau,k_0],D)$ is the space of all continuous functions mapping $[-\tau,k_0]$ into D with the norm defined by $\|\phi\|=\max_{1\le i\le n}\{\sup_{-\tau\le s\le k_0}|\phi_i|\}$.

Remark 1. Note that when $a_i(\cdot)=1$, $d_{ij}=0$, i, $j=1,\dots,n$, and I_i replaced by $I_i(k)$, then neural networks (1) reduce to the following second-order discrete-time delayed neural networks (SODDNNs), which are investigated in [33]:

$$\begin{aligned}x_i(k+1)=x_i(k)-b_ix_i(k)+\sum_{j=1}^{n}c_{ij}f_j(x_j(k))+I_i(k)\\+\sum_{j=1}^{n}\sum_{l=1}^{n}e_{ijl}f_j(x_j(k-\delta_j(k)))f_l(x_l(k-\delta_l(k))).\end{aligned}\tag{2}$$

Moreover, when $a_i(\cdot)=1, e_{ijl}=0$, $i, j, l=1,\dots,n$, I_i replaced by $I_i(k)$, that is, there is no second-order synaptic connectivity in (1), then neural networks (1) reduce to the following first-order discrete-time delayed neural networks (FODDNNs), which are investigated in [32]:

$$x_i(k+1)=x_i(k)-b_ix_i(k)+\sum_{j=1}^{n}c_{ij}f_j(x_j(k))+\sum_{j=1}^{n}d_{ij}f_j(x_j(k-\tau_j(k)))+I_i(k). \tag{3}$$

Therefore, SODDCGNNs (1) studied in this paper possess the characteristics of being general in structure.

It is noted that the characteristics of activation functions have key effect on the existence and the stability of equilibrium points for neural networks. In some existing studies, the standard activation function was usually assumed to be [34,35]:

$$f_i(s)=\frac{|s+1|-|s-1|}{2}. \tag{4}$$

In order to increase the flexibility when designing activation functions, the standard activation function gradually gave way to the following activation functions [36,37]:

$$f_i(s)=\begin{cases}u_i & \text{if } -\infty<s<p_i,\\ \dfrac{v_i-u_i}{q_i-p_i}(s-p_i)+u_i & \text{if } p_i\le s\le q_i,\\ v_i & \text{if } q_i<s<+\infty,\end{cases}\tag{5}$$

where u_i, v_i, p_i, q_i are constants and satisfy $u_i<v_i$, $p_i<q_i$.

In this paper, we consider a general class of activation functions as follows, which is employed in [36,37]:

$$f_i(s)=\begin{cases}u_i^1 & \text{if } -\infty<s<p_i^1,\\ \dfrac{u_i^2-u_i^1}{q_i^1-p_i^1}(s-p_i^1)+u_i^1 & \text{if } p_i^1\le s\le q_i^1,\\ u_i^2 & \text{if } q_i^1<s<p_i^2,\\ \dots & \dots\\ \dfrac{u_i^{r+1}-u_i^r}{q_i^r-p_i^r}(s-p_i^r)+u_i^r & \text{if } p_i^r\le s\le q_i^r,\\ u_i^{r+1} & \text{if } q_i^r<s<+\infty,\end{cases}\tag{6}$$

where $r\ge 1$, u_i^l, p_i^l, q_i^l are constants and satisfy $u_i^1<u_i^2<\dots<u_i^{r+1}$, $-\infty<p_i^1<q_i^1<p_i^2<q_i^2<\dots<p_i^r<q_i^r<+\infty$. It is well known that the number of the corner points of activation functions is the key factor in increasing the number of equilibrium points for neural networks with piecewise linear activation functions. Note that, activation functions (6) have $2r$ corner points and specifically, when $r=1$, then activation functions (6) reduce to (5).

In addition, the following definition and assumption are useful for the main results.

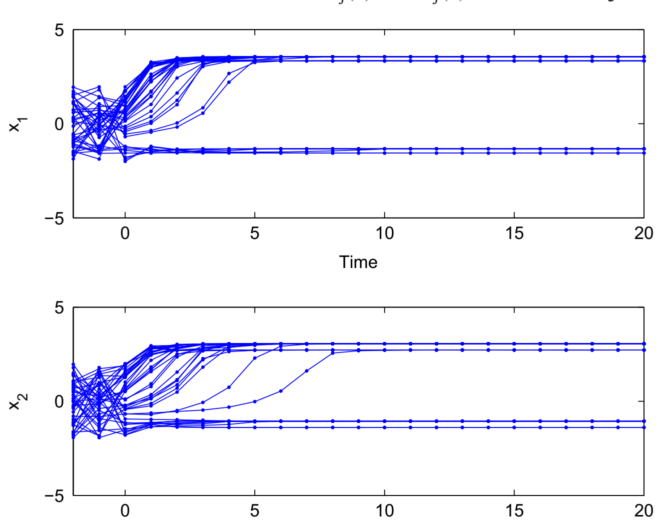

Fig. 1. Trajectories of state variables of the system in Example 1, Case 1.