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Can neural networks with arbitrary delays be finite-timely synchronized?

Xinsong Yang *

Department of Mathematics, Chongqing Normal University, Chongqing 401331, China

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ABSTRACT

Finite-time synchronization means the optimality in convergence time, thus many contributions have been made to it in the literature. However, to the best of our knowledge, most of the existing results on finite-time synchronization do not include time-delay. Considering the fact that time-delays especially infinite-time distributed delays are inevitably existing in neural networks, this paper aims to study global synchronization in finite time of neural networks with both time-varying discrete delay and infinite-time distributed delay (mixed delays). The techniques that we apply in this paper are not only different from the techniques employed in existing papers, but also applicable to differential systems with or without delay. Based on new Lyapunov–Krasovskii functional candidate and the new analysis techniques, sufficient conditions guaranteeing the finite-time synchronization of the addressed neural networks are derived by using a class of simple discontinuous state feedback controller. Conditions for realizing finite-time synchronization of neural networks with finite-time distributed delay and without delay are also given. Moreover, estimation of the upper bound of synchronization-time is also provided for neural networks with finite-time distributed delay and without delay. It is shown that the synchronization-time depends on both the initial values and the time-delays of the drive-response systems. Numerical examples demonstrate the effectiveness of the theoretical results.

1. Introduction

Drive-response synchronization was first proposed in [1]. In [1], a chaotic system, called the driver (or master), generates a signal sent over a channel to a responder (or slave), which uses this signal to synchronize itself with the driver. Since Aihara first introduced chaotic neural networks to simulate the chaotic behavior of biological neurons [2], synchronization of chaotic neural networks has attracted considerable attention due to its successful applications in combinational optimization [3], associative memory [4], secure communication [5], pattern recognition [6], and so on. Up to date, many kinds of synchronization of chaotic neural networks have been considered which include exponential synchronization [7], asymptotic synchronization [8,9], finite-time synchronization [10].

Among the above-mentioned types of synchronization, finite-time synchronization is optimal [11]. We take chaos-synchronization based secure communication as an example. It is well known that the range of time during which the chaotic oscillators are not synchronized corresponds to the range of time during which the encoded message unfortunately cannot be recovered [11]. Therefore, finite-time synchronization technique enables us to recover the transmitted signals in a setting time, while the transmitted signals can only be obtained as time goes to infinity if asymptotic synchronization or exponential synchronization techniques are utilized. Obviously, compared with exponential synchronization and asymptotic synchronization, finite-time synchronization improves the efficiency and the confidentiality greatly when it is applied to secure communication. Till now, many authors have devoted themselves to investigating finite-time synchronization of chaotic systems. For instance, in [12–15], authors addressed finite-time synchronization of multi-agent systems without delay; in [10,11,16], finite-time synchronization of coupled chaotic systems without time-delay was studied. Note that most of the existing results on finite-time synchronization are obtained based on the finite-time stability theory in [17]. Recently, the authors in [18] studied the finite-time synchronization of a class of complex networks with constant delay by using the finite-time stability theory in [17]. However, the theory in [17] is not applicable to delayed systems (see Remark 4 in this paper). Considering the advantage of finite-time control strategy, authors in [19] investigated the finite-time bounded stable problem for a class of delayed systems. But the techniques in [19] cannot be used to study the complete synchronization in finite time of delayed systems.

* Tel./fax: +86 2365362798.
E-mail address: xinsongyang@163.com

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Due to finite speeds of switching of amplifiers and transmission of signals in hardware implementation, discrete time-delays are inevitably existing in neural networks [20,21]. Moreover, distributed delays should also be introduced in neural networks because neural networks have a special nature of the presence of an amount of parallel pathways with a variety of axon sizes and lengths [22]. In the literature, various time-delays have been considered in studying synchronization of neural networks. For instance, exponential synchronization of neural networks with constant delay was considered in [23], asymptotic synchronization for a class of neural networks with constant discrete and finite-time distributed delays was studied in [24], exponential and asymptotic synchronizations of neural networks with discrete and finite-time distributed time-varying delays were investigated in [20,25], respectively, global exponential synchronization for a class of switched neural networks with time-varying delays and infinite-time distributed delays was studied [22]. Unfortunately, to the best of our knowledge, result on finite-time synchronization of neural networks with both discrete and infinite-time distributed delays has not been reported in the literature. Moreover, the analysis techniques used in the above-mentioned references cannot be extended to the finite-time synchronization of neural networks with infinite-time distributed delays. Therefore, it is urgent to develop some new analysis techniques to study finite-time synchronization of neural networks with both discrete and infinite-time distributed delays.

Based on the above discussions, the objective of this paper is to develop a new analysis technique for finite-time synchronization of neural networks with time-varying discrete delay and infinite-time distributed delay. By constructing proper Lyapunov–Krasovskii functional and employing the property of sign function, a class of discontinuous control law is proposed which guarantees the finite-time synchronization of neural networks with delays. We do not use the usual finite-time stability theory in [17]. The controller is very simple and can be easily implemented in practical applications and can finite-timely synchronize neural networks with or without delay. Moreover, synchronization criteria for neural networks with bounded delays are also derived, and the synchronization time can be estimated when there is no delay or the delays are bounded in the neural networks. Numerical simulations demonstrate the effectiveness of the theoretical results.

The rest of this paper is organized as follows. In Section 2, neural networks with mixed delays are presented. Some necessary assumptions, definitions are also given in this section. In Section 3, finite-time synchronization for the presented model is studied. Then, in Section 4, simulation examples are given to show the effectiveness of the theoretical results. Finally, Section 5 gives some conclusions.

2. Preliminaries

A neural network with time-varying discrete delay and infinite-time distributed delay is described as follows:

$$\begin{cases} \dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_{ij}(t))) \\ \qquad + \sum_{j=1}^{n} d_{ij} \int_{-\infty}^{t} K_{ij}(t-s) f_j(x_j(s))\, ds + J_i(t), \\ x(s) = \phi(s) \in C([-\infty, 0], \mathbb{R}^n), \end{cases} \tag{1}$$

where $i = 1, 2, \ldots, n$, n corresponds to the number of neurons; $x(t) = (x_1(t), x_2(t), \ldots, y_n(t))^T \in \mathbb{R}^n$ is the state vector of the network at time t, $f(x(t)) = (f_1(x_1(t)), \ldots, f_n(x_n(t)))^T$ is the neuron activation function at time t; $C = \mathrm{diag}(c_1, c_2, \ldots, c_n)$ with $c_i > 0$, $A = (a_{ij})_{n\times n}$, $B = (b_{ij})_{n\times n}$ and $D = (d_{ij})_{n\times n}$ are the connection weight matrices, $J(t) = (J_1(t), J_2(t), \ldots, J_n(t))^T \in \mathbb{R}^n$ is the external input vector. The bounded function $\tau_{ij}(t)$ represents time-varying discrete delay of the jth unit from the ith unit. $K_{ij}(t)$ is the non-negative bounded scalar function defined on $[0, +\infty)$ describing the delay kernel of the infinite-time distributed delay along the axon of the jth unit from the ith unit. $\phi(s) = (\phi_1(s), \phi_2(s), \ldots, \phi_n(s))^T$ is the initial value, $C([-\infty, 0], \mathbb{R}^n)$ denotes the Banach space of all continuous functions from $[-\infty, 0]$ to $\mathbb{R}^n$ with the norm $\|\phi\| = \sup_{-\infty \le s \le 0}\{\sum_{i=1}^{n} |\phi_i(s)|\}$. $x(t)$ can be any desired state: an equilibrium point, a nontrivial periodic orbit, or even a chaotic orbit.

Remark 1. Model (1) is very general. It includes the models in references as a special case. Specially, when the delay kernels satisfy the following condition:

$$K_{ij}(t) = \begin{cases} 0, & t > \theta_{ij}, \\ 1, & 0 \le t \le \theta_{ij}, \end{cases} \tag{2}$$

where $\theta_{ij} > 0$ $(i = 1, 2, \ldots, n)$ are constants, the model (1) becomes the following neural network with finite-time distributed delays:

$$\dot{x}_i(t) = -c_i x_i(t) + \sum_{j=1}^{n} a_{ij} f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(x_j(t-\tau_{ij}(t))) + \sum_{j=1}^{n} d_{ij} \int_{t-\theta_{ij}}^{t} f_j(x_j(s))\, ds + J_i(t). \tag{3}$$

It should be mentioned that all the analysis methods developed in most of the existing papers for stability or chaos synchronization cannot be extended to study the finite-time synchronization of the models (1) and (3).

Based on the concept of drive-response synchronization, which was initially proposed by Pecora and Carrol in [1], we take (1) as the drive system. The corresponding response system is constructed as follows:

$$\begin{cases} \dot{u}_i(t) = -c_i u_i(t) + \sum_{j=1}^{n} a_{ij} f_j(u_j(t)) + \sum_{j=1}^{n} b_{ij} f_j(u_j(t-\tau_{ij}(t))) \\ \qquad + \sum_{j=1}^{n} d_{ij} \int_{-\infty}^{t} K_{ij}(t-s) f_j(u_j(s))\, ds + J_i(t) + U_i(t), \\ u(s) = \varphi(s) \in C([-\infty, 0], \mathbb{R}^n), \end{cases} \tag{4}$$

where $u(t) = (u_1(t), u_2(t), \ldots, u_n(t))^T \in \mathbb{R}^n$ is the state vector of the response system at time t and $U_i(t)$ is the control input to be designed.

In order to investigate the problem of finite-time synchronization between (1) and (4), we define the synchronization error signal $e_i(t) = u_i(t) - x_i(t)$. Subtracting (1) from (4) yields the following error system:

$$\begin{cases} \dot{e}_i(t) = -c_i e_i(t) + \sum_{j=1}^{n} a_{ij} g_j(e_j(t)) + \sum_{j=1}^{n} b_{ij} g_j(e_j(t-\tau_{ij}(t))) \\ \qquad + \sum_{j=1}^{n} d_{ij} \int_{-\infty}^{t} K_{ij}(t-s) g_j(e_j(s))\, ds + U_i(t), \\ e(s) = \psi(s) \in C([-\infty, 0], \mathbb{R}^n), \end{cases} \tag{5}$$

where $g_j(e_j(\cdot)) = f_j(u_j(\cdot)) - f_j(x_j(\cdot))$, $e(t) = (e_1(t), e_2(t), \ldots, e_n(t))^T$, $\psi(s) = \phi(s) - \varphi(s)$.

The following assumptions for the delays and the activation functions are needed in this paper.

(H_1) There are positive constants $\overline{\tau}_{ij}$ and $\mu_{ij} < 1$ such that $0 < \tau_{ij}(t) \le \overline{\tau}_{ij}$, $\dot{\tau}_{ij}(t) \le \mu_{ij}$, $i, j = 1, 2, \ldots, n$.

(H_2) For any $u, v \in \mathbb{R}$, $u \ne v$, there exist constants l_i $(i = 1, 2, \ldots, n)$ such that $|f_i(u) - f_i(v)| \le l_i |u - v|$.

(H_3) There are positive constants $\overline{k}_{ij}$ such that $\int_0^{+\infty} K_{ij}(u)\, du = \overline{k}_{ij}$, $i, j = 1, 2, \ldots, n$.

Before starting the main results, we introduce the definition of finite-time synchronization.