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# Parameter estimation of the exponentially damped sinusoids signal using a specific neural network

Xiuchun Xiao [a,b], Jian-Huang Lai [b,d,*], Chang-Dong Wang [c,d]

[a] College of Information, Guangdong Ocean University, Zhanjiang 524025, PR China
[b] School of Information Science and Technology, Sun Yat-sen University, Guangzhou 510006, PR China
[c] School of Mobile Information Engineering, Sun Yat-sen University, Zhuhai 519082, PR China
[d] SYSU-CMU Shunde International Joint Research Institute (JRI), Shunde 528300, PR China

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ABSTRACT

The problem of estimating the parameters of exponentially damped sinusoids (EDSs) signal has received very much attention in many fields. In this paper, a specific neural network termed EDSNN for parameter estimation of the EDSs has been proposed. Aiming at effectively evaluating the parameters of the EDSs signal, we construct a specific topology of EDSNN strictly following the mathematic formulation of EDSs signal. Then, what should be further done is how to train EDSNN using the data-set sampled from the EDSs signal. For this purpose, a modified Levenberg–Marquardt algorithm is derived for iteratively solving the weights of EDSNN by optimizing the pre-defined objective function. Profiting from good performance in fault tolerance of neural network, the proposed algorithm possesses a good performance in resistance to noise. Several computer simulations have been conducted to apply this method to some EDSs signal models. The results substantiate that the proposed EDSNN can synchronously obtain a higher precision for the damped factors, frequencies, also amplitudes and initial phases of all the EDSs than the state-of-the-art algorithm for noise free or noise case.



## 1. Introduction

Many practical signals such as speech and audio signal, power system transient signal and radar/sonar signal can be regarded as signals of the sum of exponentially damped sinusoids (EDSs) signal. In fact, estimating the parameters of EDSs signal is now becoming a very important task in many practical applications such as speech analysis [1,2], power system transient detection [3–5], radar/sonar signal analysis [6], and nuclear magnetic resonance image processing [7].

In the recent years, the problem of estimating the parameters of exponentially damped sinusoids (EDSs) signal has received very much attention. A number of techniques have been proposed to tackle this problem in the past. These techniques can be mainly classified as nonparametric [8–12] and parametric ones [1,13–15]. The nonparametric techniques are commonly computationally efficient and have less sensitivity to algorithm specific parameters. Unfortunately, these techniques often have inherent limitations such as suffering from frequency resolution or leakage effects in an unsynchronized sampling [13,16]. On the contrary, compared with nonparametric methods, most parametric methods are model-based methods and can commonly achieve a relatively high accuracy [15]. But they assume that the generated models satisfy a real multi-component signal.

The most widely used nonparametric technique is the fast Fourier transform (FFT) derived from the discrete Fourier transform (DFT) [17]. However, the picket phenomenon and spectrum leakage phenomenon occur when applying FFT to an unsynchronized sampling sequence analysis [6,13]. Consequently, in order to improve the accuracy and reduce its sensitivity to de-synchronization, specific synchronization hardware should be adopted [16–18]. Another general nonparametric method is wavelet analysis [19]. It is an analysis tool with good feature in both time domain and frequency domain. Recently, many researches have been conducted to develop wavelet analysis approaches to detect, localize and classify different types of power system disturbances, including harmonic and inter-harmonic distortions in a power system [19,20]. Those kind of approaches are based on decomposing the disturbed signal into the other components which represent smoothed and detailed versions of the original signal [3].

Recently, several parametric methods have been also suggested for EDSs signal analysis [3,5]. Among these methods, subspace approaches such as estimation of signal parameters via rotational invariance techniques (ESPRIT) become very popular for their lower complexity [5]. The principle is to first separate the data

* Corresponding author at: School of Information Science and Technology, Sun Yat-sen University, Guangzhou 510006, PR China. Tel.: +86 13168313819.
*E-mail addresses:* springxxc@163.com (X. Xiao), stsljh@mail.sysu.edu.cn (J.-H. Lai), changdongwang@hotmail.com (C.-D. Wang).

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into signal and noise subspaces via eigenvalue decomposition (EVD) of the sample covariance matrix or singular value decomposition (SVD) of the raw data matrix, then the parameters of interest are calculated from the corresponding eigenvectors and eigenvalues, or singular vectors and singular values [15]. The implementation of the algorithm in power systems was first proposed in [21]. Maximum likelihood (ML) and iterative quadratic ML (IQML) can also be used for estimating EDSs signal [22,15]. However, due to their extremely high computational requirement, the ML-based methods are only feasible for 2-D harmonic retrieval. Prony analysis has been shown to be a very appropriate technique to model a linear sum of exponentially damped sinusoids signal that are uniformly sampled [23–26]. In fact, it can exactly fit the typical EDSs signal in the sense of the least-squared error (LSE) technique.

In these years, neural network methods are becoming popular in the field of parameter estimation problem for their high accuracy and good performance in resistance to noise [16,27–29]. Nevertheless, though neural network can be successfully applied in estimating the parameters of some harmonic or inter-harmonic distortions, it cannot be directly applied in EDSs signal analysis for the reason that it is not easy to construct an appropriate topology and a good training algorithm of a general neural network such as an Adaline network because of the complicated nonlinear formulation of the EDSs signal which contain exponential functions, sine functions and cosine functions [16,29] (see Subsections 2.2 and 2.3 for more details).

To solve the aforementioned problems, we construct a specific topology for a certain feedforward neural network strictly following the mathematic formulation of EDSs signal. We termed this specific neural network EDSNN. Similar to some common neural network model, the EDSNN has three layers and the most important layer is the hidden-layer. However, unlike the traditional neural network, the hidden-layer of EDSNN is mainly composed of some different kinds of neurons and some different operating units, i.e., each neuron employs one of the three distinct activation functions: exponential function, sine function or cosine function, and the operating units can perform addition or multiplication operations. By using various kinds of weights to connect all the distinct neurons, the mathematical formulas of EDSNN are consistent with the EDSs signal. Thus, we can estimate the parameters of EDSs signal by effectively training the EDSNN to force its weights to converge to stable values so as to approximate the given EDSs signal. In order to achieve this purpose, we defined an appropriate objective function, then a modified Levenberg–Marquardt algorithm is carefully derived to optimize the objective function. Finally, the damped factors, frequencies, also amplitudes and initial phases of all the EDSs can be estimated from the weights of the converged EDSNN.

The remainder of the paper is organized as follows: in Section 2, we introduce the problem formulation and the proposed neural network for parameter estimation of exponentially damped sinusoids signal. As to the proposed neural network model, we divide it into two main parts: one is how to construct the EDSNN with specific topology and the other is to derive the modified Levenberg–Marquardt algorithm to train the proposed EDSNN. Section 3 reports experimental results of parameter estimation of some computer simulation experiments, and each experiment including noise free and noise cases. Finally, we conclude our paper in Section 4.

It is worth pointing out that the main results in this paper were first presented at the IScIDE 2013 conference [29]. We have extended and revised the paper with some new and unpublished materials to give more details of the deduction of the proposed improved Levenberg–Marquardt algorithm (see Subsection 2.3) and to better verify that the proposed algorithm can achieve a very high precision and has good performance in resistance to noise (see Subsections 3.2 and 3.3).

## 2. Proposed approach

In this section, we will discuss the proposed approach in detail. Firstly, we present the mathematical formulation of the practical problem of estimating the parameters of exponentially damped sinusoids (EDSs) signal, then a specific topology of neural network termed EDSNN is constructed according to this formulation. In order to effectively solve the weights of the proposed EDSNN, an adaptive learning algorithm based on improved Levenberg–Marquardt is derived. As a result, the parameters of each exponentially damped sinusoid (EDS) component can be directly calculated using the converged weights of EDSNN.

### 2.1. Problem formulation

In general, an actual signal consisting of $n$ distinct exponentially damped sinusoid components can be represented by their respective unknown damping factors, angular frequencies, amplitudes and initial phases. Assuming $m$ samples drawn from the signal $y(t)$ with uniformly sampling interval time $\Delta t$ are recorded as

$$y(t_j) := y(j\Delta t), \quad j = 0, 1, 2, \ldots, m-1, \tag{1}$$

then, $y(t_j)$ can be formulated as follows:

$$y(t_j) = \sum_{i=1}^{n} A_i e^{\sigma_i t_j} \sin(\omega_i t_j + \varphi_i), \quad i = 1, 2, \ldots, n, \; j = 0, 1, 2, \ldots, m-1, \tag{2}$$

where $y(t_j)$ denotes the signal sampled at $t_j$; $i$ denotes the component order; $A_i$ denotes the amplitude of component $i$; $\sigma_i$ denotes the damping factor of component $i$; $\omega_i$ denotes the angular frequency of component $i$; $\varphi_i$ denotes the initial phase of component $i$; $n$ denotes the number of EDS components; $m$ denotes the size of sample set.

A typical EDSs signal can be given as Ref. [3],

$$y(t) = 1.0e^{-0.025t} \sin(2\pi \times 0.4t) + 0.5e^{0.037t} \sin(2\pi \times 0.5t). \tag{3}$$

As the author [3] declared, the signal formulated in Eq. (3) represents two superposed low-frequency transient oscillations in power system. However, the parameters of each component in the signal are commonly unknown, thus, how to estimate these parameters from the data-set of measurements from real signal is very important in some practical applications.

In fact, if the damping factors $\sigma_i, i = 1, 2, \ldots, n$ are all equal to zero, then the corresponding EDS components are periodic and the problem of estimating the parameters of EDSs signal would degenerate to estimating the parameters of harmonic and inter-harmonic distortions, which can be solved by a neural network method with a general topology [18,16]; conversely, if the damping factors $\sigma_i, i = 1, 2, \ldots, n$ are not all equal to zero, the corresponding EDS components will be aperiodic and decay to zero as time goes by [13], and the problem of estimating the parameters of EDSs signal in this case cannot be easily solved by a neural network method with a general topology [27,28].

In order to construct an appropriate topology of the proposed neural network, we should do some mathematical transformations with Eq. (2). By using the identity trigonometric given by the following well-known equation:

$$\sin(\alpha + \beta) = \sin(\alpha)\cos(\beta) + \cos(\alpha)\sin(\beta). \tag{4}$$