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Sampled-data state estimation for complex dynamical networks with time-varying delay and stochastic sampling

Hongjie Li

College of Mathematics and Information and Engineering, Jiaxing University, Zhejiang 314001, PR China

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ABSTRACT

The paper investigates state estimation for complex dynamical networks with time-varying delay and stochastic sampling. Only two different sampling periods are considered which occurrence probabilities are given constants and satisfy Bernoulli distribution. By applying an input-delay approach, the probabilistic sampling state estimator is transformed into a continuous time-delay system with stochastic parameters in the system matrices, where the purpose is to design a state estimator to estimate the network states through available output measurements. Delay-dependent asymptotically stability condition is established for the system of the estimation error, which can be readily solved by using the LMI toolbox in MATLAB, the solvability of derived conditions depends on not only the size of the delay and the sampling period, but also the probability of taking values of the sampling period. Finally, a numerical example is provided to demonstrate the effectiveness of the obtained theoretical results.

1. Introduction

Complex dynamical networks are becoming increasingly important in contemporary society both in science and technology [1–6]. In the real world, a large number of practical systems can be represented by models of complex networks, such as the World Wide Web, a network of web site; the brain, a network of neurons; food webs; telephone cell graphs and electricity distribution networks. Many of these networks exhibit complexity in the overall topological properties and dynamical properties of the network nodes and the coupled units. The complex nature of complex networks has results in a series of important research problems [7–16].

Due to the complexity of high-order and large-scale networks, it is often the case that only partial information about the states of key nodes is available in network outputs and it becomes necessary to estimate the states of key nodes through available measurements. Therefore, state estimation has become one of the popular topics, some profound results are established [17–20]. In [17], synchronization and state estimation are investigated for discrete-time complex networks with distributed delays, the addressed problem of synchronization and state estimation could be converted into a feasibility problem of a set of LMIs. Liang et al. [18] study the state estimation for a class of discrete-time coupled uncertain stochastic complex networks with missing measurements and time-varying delays, the parameter uncertainties are assumed to be norm-bounded and enter into both network states and network outputs. The distributed state estimation is investigated for a class of sensor networks described by discrete-time stochastic systems with randomly varying non-linearities and missing measurements [19]. In [20], H_∞ synchronization and state estimation are considered for an array of coupled discrete time-varying stochastic complex networks over a finite horizon. With the rapid development of high-speed computers, modern control systems tend to be controlled by digital controllers, only the samples of the control input signals at discrete time instants will be employed, the sampled-data control theory has attracted much attention because modern control systems usually employ digital technology for controller implementation [21,22]. Recently, a new approach is proposed to solve the problem of sampled-data H_∞ control, where the sampled-data system is modeled as a continuous-time one with an input delay [22]. By following this idea, in [23], the exponential synchronization sampled-data control problem has been studied for neural networks with time-varying mixed delay. Li et al. [24] have studied the sampled-data synchronization control problem, which has first been transformed to the problem of stability analysis for a differential equation by converting the sampling period into a bounded time-varying delay. Note that the above sampling methods are assumed to be implemented in a deterministic way, but in practical engineering within a networked environment, the sampling process itself might be subject to random abrupt changes, namely, stochastic sampling, has been overlooked in area of coupled dynamical networks [25,26], in [25], the problem of robust H_∞ control for sampled-data systems with probabilistic sampling has been investigated via a delay system approach. In [26], the sampled-data synchronization control problem is addressed for a class of dynamical networks, where the sampling period considered here is time-varying that is allowed to switch

E-mail address: lhjymlly@163.com

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between two different values in a random way. Unfortunately, although sampled-data control technologies have been developed well in control theory, the stochastic sampled-data state estimation problem for complex dynamical networks has so far received very little attention, the purpose of this paper is to close this gap. The main contributions of this paper can be summarized as follows: (1) The state estimation is investigated for complex dynamical networks, where the sampling period considered here is assumed to satisfy Bernoulli distribution, which can be further extended to the case with multiple stochastic sampling periods. (2) The solvability of derived conditions depends on not only the size of the delay and the sampling period, but also the probability of taking values of the sampling period.

Motivated by the above analysis, state estimation problem with the sampled-data is investigated for a class of complex dynamical networks with time-varying delay. The sampling period considered here is assumed to be time-varying that switches between two different values in a random way with given probability. By applying an input-delay approach, the probabilistic sampling state estimator is transformed into a continuous time-delay system with stochastic parameters in the system matrices, where the purpose is to design a state estimator to estimate the network states through available output measurements. The solvability of derived conditions depends on not only the size of the delay and the sampling period, but also the probability of taking values of the sampling period. Finally, a numerical example is provided to demonstrate the effectiveness of the obtained theoretical results.

The rest of this paper is organized as follows. In Section 2, problem formulation and preliminaries are briefly outlined. In Section 3, main results are derived in the form of LMIs. In Section 4, a simulation example is provided to show the advantages of the obtained results, and some conclusions are drawn in Section 5.

Notation: The notation used in the paper is fairly standard. $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}^{n\times m}$ is a set of real $n\times m$ matrices. The notation $X>0$ (respectively, $X<0$), for $X\in\mathbb{R}^{n\times n}$ means that the matrix X is real symmetric positive definite (respectively, negative definite). $diag\{\cdots\}$ stands for a block-diagonal matrix. $\|\cdot\|$ denotes the Euclidean norm in $\mathbb{R}^n$. The superscript 'T' stands for matrix transposition. I_n denotes $n\times n$ identity matrix. $E\{\cdot\}$ stands mathematical expectation. The Kronecker product of matrices $Q\in\mathbb{R}^{m\times n}$ and $R\in\mathbb{R}^{p\times q}$ is a matrix in $\mathbb{R}^{mp\times nq}$ and denoted as $Q\otimes R$. In this paper, if not explicitly stated, matrices are assumed to have compatible dimensions.

2. Problem formulation and preliminaries

Consider the following delayed complex dynamical networks consisting of N coupled nodes of the form

$$\begin{cases} \dot{x}_i(t)=Ax_i(t)+f(x_i(t))+g(x_i(t-\tau(t)))+\sum_{j=1}^{N}G_{ij}\Gamma x_j(t) \\ y_i(t)=C_ix_i(t) \qquad (i=1,2,\ldots,N), \end{cases} \tag{1}$$

where $x_i(t)\in\mathbb{R}^n$ is the state vector of the ith node. $y_i(t)\in\mathbb{R}^m$ is the output of the ith node. $A\in\mathbb{R}^{n\times n}$ and $C_i\in\mathbb{R}^{m\times n}$ are some constant matrices. $f(\cdot),g(\cdot)\in\mathbb{R}^n$ are two unknown but sector-bounded nonlinear vector functions. $G=(G_{ij})_{N\times N}$ is the outer-coupling matrix of the networks representing the coupling strength and topological structure of complex networks, in which $G_{ij}\ge 0$ $(i\ne j)$, $G_{ii}=-\sum_{j=1,j\ne i}G_{ij}$. $\Gamma\in\mathbb{R}^{n\times n}$ denotes the inner-coupling matrix. The function $\tau(t)$ denotes time-varying delay satisfying $\tau(t)\in[0,\tau]$, where τ is a known positive scalar.

The state estimator is constructed as follows:

$$\begin{aligned}\dot{\hat{x}}_i(t)&=A\hat{x}_i(t)+f(\hat{x}_i(t))+g(\hat{x}_i(t-\tau(t)))\\&\quad+\sum_{j=1}^{N}G_{ij}\Gamma\hat{x}_j(t)+K_i(y_i(t_k)-C_i\hat{x}_i(t_k))\end{aligned} \tag{2}$$

where $t_k\le t\le t_{k+1}$ $(k=0,1,2,\ldots)$, and $\lim_{k\to\infty}t_k=\infty$. Noting that $t_k=t-(t-t_k):=t-d(t)$, then the above state estimator can be rewritten as follows:

$$\begin{aligned}\dot{\hat{x}}_i(t)&=A\hat{x}_i(t)+f(\hat{x}_i(t))+g(\hat{x}_i(t-\tau(t)))\\&\quad+\sum_{j=1}^{N}G_{ij}\Gamma\hat{x}_j(t)+K_i(y_i(t-d(t))-C_i\hat{x}_i(t-d(t)))\end{aligned} \tag{3}$$

As discussed in [25,26], the sampling period might be a stochastic variable due to unpredictable environmental changes, in this paper, the sampling period is allowed to randomly switch between two different values d_1 and d_2 with $0<d_1<d_2$, and the probability of the occurrence of each is known, that is

$$Prob\{d=d_1\}=\beta,\quad Prob\{d=d_2\}=1-\beta \tag{4}$$

We take the interval $[0,d_2]$ apart into two interval $[0,d_1]$ and $(d_1,d_2]$, and introduce a new random variable $\alpha(t)$ as follows:

$$\alpha(t)=\begin{cases}1 & (0\le d(t)\le d_1)\\ 0 & (d_1<d(t)\le d_2)\end{cases} \tag{5}$$

Similar to [25], we can obtain

$$\begin{aligned}Prob\{\alpha(t)=1\}&=Prob\{0\le d(t)\le d_1\}\\&=\beta+\frac{d_1}{d_2}(1-\beta):=\alpha\end{aligned} \tag{6}$$

$$Prob\{\alpha(t)=0\}=\frac{d_2-d_1}{d_2}(1-\beta):=1-\alpha \tag{7}$$

then we can rewrite the system (3) equivalently as

$$\begin{aligned}\dot{\hat{x}}_i(t)&=A\hat{x}_i(t)+f(\hat{x}_i(t))+g(\hat{x}_i(t-\tau(t)))+\sum_{j=1}^{N}G_{ij}\Gamma\hat{x}_j(t)\\&\quad+\alpha(t)K_iC_i(x_i(t-d_1(t))-\hat{x}_i(t-d_1(t)))\\&\quad+(1-\alpha(t))K_iC_i(x_i(t-d_2(t))-\hat{x}_i(t-d_2(t)))\end{aligned} \tag{8}$$

Remark 1. Eq. (8) is a re-modeling of the probabilistic sampling system with stochastic parameter $\alpha(t)$ and time-varying delays, then we can make use of the input-delay approach to deal with the problem of state estimation for complex dynamical networks.

Remark 2. In [24], the sampled-data synchronization control problem is investigated for a class of general complex networks, where the sampling period is converted into a bound time-varying delay, note that the method is assumed to be implemented in a deterministic way. But in a networked environment, the sampling period itself might be a stochastic variable due to unpredictable environmental changes [25,26]. To reflect such a reality, the sampling period is assumed to be time-varying that switches between two different values in a random way with given probability.

With the matrix Kronecker product, the systems (1) and (8) can be rewritten in the following compact form:

$$\dot{x}(t)=(I_N\otimes A+G\otimes\Gamma)x(t)+F(x(t))+G(x(t-\tau(t))) \tag{9}$$

$$\begin{aligned}\dot{\hat{x}}(t)&=(I_N\otimes A+G\otimes\Gamma)\hat{x}(t)+F(\hat{x}(t))+G(\hat{x}(t-\tau(t)))\\&\quad+\alpha(t)KC(x(t-d_1(t))-\hat{x}(t-d_1(t)))\\&\quad+(1-\alpha(t))KC(x(t-d_2(t))-\hat{x}(t-d_2(t)))\end{aligned} \tag{10}$$

where

$$\begin{aligned}&x(t)=(x_1^T(t),x_2^T(t),\ldots,x_N^T(t))^T,\\&F(x(t))=(f^T(x_1(t)),f^T(x_2(t)),\ldots,f^T(x_N(t)))^T,\\&G(x(t-\tau(t)))=(g^T(x_1(t-\tau(t))),g^T(x_2(t-\tau(t))),\ldots,g^T(x_N(t-\tau(t))))^T,\end{aligned}$$