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Distributed output feedback consensus of discrete-time multi-agent systems

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A B S T R A C T

This paper investigates the problem of distributed output feedback consensus for multi-agent systems. Both fixed topology and stochastic switching topology are considered. It is assumed that each agent updates its state according to the output information of itself and its neighbors. We obtain some necessary and sufficient conditions of consensusability based on algebraic graph theory and Markov jump linear system theory. Algorithms are given to derive the allowable control gains. Simulation results are presented to show the effectiveness of the theoretical results.

1. Introduction

Recently, a lot of research effort has been put into the study of multi-agent systems, especially the distributed consensus problem. Applications of the distributed consensus include unmanned air vehicles, formation control of mobile robots, distributed computation and estimation, wireless sensor network [1–10]. The main idea of consensus problem is to make a group of agents converge to a common value by designing some protocols or algorithms. In [1], two consensus protocols for multi-agent systems with or without time-delays were introduced and some necessary and sufficient or sufficient conditions of consensus were obtained. In [5], the authors studied some distributed consensus problems and their applications systematically, where the results are mainly based on algebraic graph theory and matrix analysis. The containment control problem for linear multi-agent systems was addressed in [8], where all the internal agents converge to the convex hull spanned by some boundary agents. While in [9], the authors studied the output consensus problem for heterogeneous uncertain linear multi-agent systems based on the output regulation theory. More recent progress in distributed multi-agent coordination can be found in [10].

In the aforementioned work on distributed consensus of multi-agent systems, the interaction topologies are fixed or switching in a deterministic framework. However, the stochastic phenomena are frequently appear in the practice due to the disturbances. Some stochastic models, such as Markov chain, have been used to describe the practical systems [11,12]. For multi-agent systems, the stochastic model, especially the Markov chain, is often used to describe the interaction topologies among the agents. It is assumed that there are finite topologies among which the interaction topology at each moment switches stochastically with some probabilities [13–15]. In [13], the authors studied the static stabilization problem for a decentralized discrete-time single-integrator network with Markovian switching topologies. In [14], the mean square consensusability problem for a network of double-integrator agents with Markovian switching topologies was studied. While in [15], the authors studied the distributed discrete-time coordinated tracking problem for multi-agent systems with Markovian switching topologies where the transition probabilities are equal.

In the early literature, the researchers analyzed the consensus problem are mainly based on algebraic graph theory and matrix theory. When the structure of topology becomes complicated or the interaction topologies are stochastic switching, it is difficult to analyze the conditions of consensus from the perspective of the topology structure directly. The linear transformation is employed to overcome this difficult, which transfers the consensusability problem of multi-agent system to the stability problem of linear systems [16–18]. In [16], the authors studied the consensusability of multi-agent systems with input delays by the model transformation, where the methods to compute the maximum input delay

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were given. In [17], a sufficient condition of consensus for multi-agent system with random delays governed by a Markov chain was obtained, where the results given are in terms of a set of linear matrix inequalities. While in [18], some necessary and sufficient conditions for consensusability of linear multi-agent systems were obtained, where the consensus protocol is based on the outputs of the agents. However, the interaction topology in [18] is fixed. It is necessary to extend the results in [18] to the case of stochastic switching topology.

In this paper, we deal with the distributed consensus problem for discrete-time multi-agent systems. Both fixed topology and stochastic switching topologies are considered. The outputs are used by the agents to update their state information. Some necessary and sufficient conditions of consensusability are obtained based on linear system theory, algebraic graph theory and matrix theory. Algorithms are given to design the allowable control gains.

Notation: Let $\mathbb{R}$ and $\mathbb{N}$ represent, respectively, the real number set and the nonnegative integer set. Denote the spectral radius of the matrix M by $\rho(M)$. Suppose that $A, B \in \mathbb{R}^{p\times p}$. Let $A \succcurlyeq B$ (respectively, $A \succ B$) denote that $A-B$ is symmetric positive semi-definite (respectively, symmetric positive definite). Denote the determinant of the matrix A by $|A|$. For a scalar α, $|\alpha|$ represents the modulus of α (α is a complex number) or the absolute value of α (α is a real number). "$\otimes$" represents the Kronecker product of matrices. I_n denotes the $n\times n$ identity matrix. $\Re(\cdot)$ and $\Im(\cdot)$ represent the real part and imaginary part of a complex number, respectively. Let $\mathbf{1}_n$ and $\mathbf{0}_{m\times n}$ denote, respectively, the $n\times 1$ column vector with all components equal to 1 and $m\times n$ zero matrix.

2. Problem formulation and preliminaries

We introduce the graph theory notions similar to [19], firstly. Let $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ be a directed graph of order n, where $\mathcal{V}$ and $\mathcal{E}$ denote, respectively, the node set and the edge set. An edge $(i,j)\in\mathcal{E}$ if agent j can obtain the information from agent i. We say agent i is a neighbor of agent j. $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{n\times n}$ is the adjacency matrix associated with $\mathcal{G}$, where $a_{ij}>0$ if $(i,j)\in\mathcal{E}$, otherwise, $a_{ij}=0$. The (nonsymmetrical) Laplacian matrix $\mathcal{L}$ associated with $\mathcal{A}$ and hence $\mathcal{G}$ is defined as $\mathcal{L}=[l_{ij}]\in\mathbb{R}^{n\times n}$, where $l_{ii}=\sum_{j=1,j\neq i}^{n} a_{ij}$ and $l_{ij}=-a_{ij}$, $\forall i\neq j$. A directed path is a sequence of edges in a directed graph in the form of (i_1,i_2), (i_2,i_3), …, where $i_k\in\mathcal{V}$. A directed tree is a directed graph, where every node has exactly one parent except for one node, called the root, which has no parent, and the root has a directed path to every other node. A directed spanning tree of $\mathcal{G}$ is a directed tree that contains all nodes of $\mathcal{G}$. A directed graph has or contains a directed spanning tree if there exists a directed spanning tree as a subset of the directed graph, that is, there exists at least one node having a directed path to all of the other nodes. The union of graphs $\mathcal{G}_1$ and $\mathcal{G}_2$ is the graph $\mathcal{G}_1\bigcup\mathcal{G}_2$ with vertex set $\mathcal{V}(\mathcal{G}_1)\bigcup\mathcal{V}(\mathcal{G}_2)$ and edge set $\mathcal{E}(\mathcal{G}_1)\bigcup\mathcal{E}(\mathcal{G}_2)$.

In this paper, we also consider the stochastic switching topologies governed by a finite Markov chain. Let $\{\theta[k], k\in\mathbb{N}\}$ be a homogeneous, discrete-time Markov process which takes values in a finite set $S\triangleq\{1,\dots,m\}$. Let $\Pi=[\pi_{ij}]\in\mathbb{R}^{m\times m}$ be the probability transition matrix. We assume that the Markov process is ergodic throughout this paper.[1] The switching topology set is $\overline{\mathcal{G}}=\{\mathcal{G}^1,\dots,\mathcal{G}^m\}$, where $\mathcal{G}^i$, $i=1,\dots,m$, are the directed graph with order n.

[1] We give the initial state $\theta[0]$ and probability transition matrix Π. According to probability transition matrix Π, we separate the interval $[0,1]$ into r subintervals. At the next time, the system will generate a number on interval $[0,1]$ randomly. The interval which the number belongs to will determine the corresponding topology.

Suppose that the multi-agent systems considered consist of n agents indexed by $1,\dots,n$. The ith agent's dynamics is given by

$$\begin{cases} x_i[k+1]=Ax_i[k]+Bu_i[k] \\ y_i[k]=Cx_i[k] \end{cases} \tag{1}$$

where $x_i[k]\in\mathbb{R}$ is the state, $u_i[k]\in\mathbb{R}$ and $y_i[k]\in\mathbb{R}$ represent the input and the output, respectively. $A,B,C\in\mathbb{R}$ are the system coefficients. Similar to [18], in this paper, the output information will be used by the agents to update their states. So we consider the following algorithm:

$$\begin{aligned} u_i[k] &= K\sum_{j\in N_i} a_{ij}^{\theta[k]}(y_j[k]-y_i[k]) \\ &= KC\sum_{j\in N_i} a_{ij}^{\theta[k]}(x_j[k]-x_i[k]), \end{aligned} \tag{2}$$

here N_i is the neighbor set of the ith agent. Each agent will update its information according to the outputs of itself and its neighbors. Substituting (2) to (1), we get that

$$x_i[k+1]=Ax_i[k]+BKC\sum_{j\in N_i} a_{ij}^{\theta[k]}(x_j[k]-x_i[k]). \tag{3}$$

Denote $x[k]\triangleq[x_1[k],\dots,x_n[k]]^T$, then the whole multi-agent system can be written as

$$x[k+1]=Ax[k]+BKC\mathcal{L}^{\theta[k]}x[k]. \tag{4}$$

Set $e_i[k]\triangleq x_i[k]-x_n[k]$, $i=1,\dots,n-1$, and $e[k]\triangleq[e_1[k],\dots,e_{n-1}[k]]^T$, then we get that

$$e[k+1]=(AI_{n-1}+BKC\tilde{\mathcal{L}}^{\theta[k]})e[k] \tag{5}$$

where

$$\tilde{\mathcal{L}}^{\theta[k]}=\begin{bmatrix} l_{11}^{\theta[k]}-l_{n1}^{\theta[k]} & l_{12}^{\theta[k]}-l_{n2}^{\theta[k]} & \cdots & l_{1(n-1)}^{\theta[k]}-l_{n(n-1)}^{\theta[k]} \\ l_{21}^{\theta[k]}-l_{n1}^{\theta[k]} & l_{22}^{\theta[k]}-l_{n2}^{\theta[k]} & \cdots & l_{2(n-1)}^{\theta[k]}-l_{n(n-1)}^{\theta[k]} \\ \cdots & \cdots & \cdots & \cdots \\ l_{(n-1)1}^{\theta[k]}-l_{n1}^{\theta[k]} & l_{(n-1)2}^{\theta[k]}-l_{n2}^{\theta[k]} & \cdots & l_{(n-1)(n-1)}^{\theta[k]}-l_{n(n-1)}^{\theta[k]} \end{bmatrix}.$$

By the above transformation, the consensus problem of (4) is transformed into the stability problem of the error system (5). According to the Markov jump system theory [21], we know that $\{e[k], k\in\mathbb{N}\}$ is not a Markov process, but the joint process $\{e[k],\theta[k], k\in\mathbb{N}\}$ is.

Remark 1. For the high-order scenario, we can get the similar results by using Kronecker product under the assumption that A, B, C are scalar matrices. However, when there is no assumption of the coefficient matrices, we failed to obtain the desired results in this paper. We will consider this problem in our future work.

Definition 1 (*Zhang and Tian [14]*). For algorithm (2), the multi-agent system (1) achieves mean-square consensus if for any $i\neq j$, $|x_i[k]-x_j[k]|\to 0$ as $k\to\infty$ holds in mean-square sense for any initial distribution and initial state.

Property 1 (Horn and Johnson [20]). *Some basic properties of the Kronecker product which will be used in this paper are as follows*:

(1) $(\alpha A)\otimes B=A\otimes(\alpha B)$ *for all* $\alpha\in\mathbb{F}$, $A\in M_{m,n}(\mathbb{F})$, $B\in M_{p,q}(\mathbb{F})$;
(2) $(A+B)\otimes C=A\otimes C+B\otimes C$ *for* $A\in M_{m,n}(\mathbb{F})$, $B\in M_{p,q}(\mathbb{F})$, $C\in M_{r,s}(\mathbb{F})$;
(3) $(A\otimes B)(C\otimes D)=AC\otimes BD$ *for* $A\in M_{m,n}(\mathbb{F})$, $B\in M_{p,q}(\mathbb{F})$, $C\in M_{n,k}(\mathbb{F})$, $D\in M_{q,r}(\mathbb{F})$.

3. Consensusability analysis

The output feedback consensusability problem for multi-agent systems was studied in [18] where the interaction topology is