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Exponential stability of stochastic memristor-based recurrent neural networks with time-varying delays

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ABSTRACT

In real nervous systems and in the implementation of very large-scale integration (VLSI) circuits, noise is unavoidable, which leads to the stochastic model of the memristor-based recurrent neural networks. Exponential stability of stochastic memristor-based recurrent neural networks with time-varying delays is studied and some sufficient conditions in terms of inequalities are derived. Numerical examples are given to demonstrate the effectiveness of the proposed stability criteria.

1. Introduction

Memristor (a contraction of "memory resistor"), which can be realized in laboratory experiments [1,2], was first postulated by Chua in 1971 [3]. The memristor can be applied to build new neural networks model [4–11] to emulate the human brain. Memristor-based recurrent neural networks can be implemented by very large-scale integration (VLSI) circuits (a graphic representation can be found in Refs. [9,10]). The potential applications are in next generation computer and powerful brain-like neural computer.

The dynamics of recurrent neural networks (RNNs) have been extensively investigated since they play an important role in applications such as classification of patterns, associative memories and optimization (eg, see [14–23]). Recently, some initial study work has been carried out for the dynamics of memristor-based recurrent neural networks [4–11]. By taking the memristor into consideration in modelling, Hu and Wang considered the global uniform asymptotic stability of memristor-based recurrent neural networks with time delays [8]. The dynamic behaviors of memristor-based Hopfield networks in [4], the dynamic behaviors of memristor-based recurrent neural networks with time-varying delays in [5] and the exponential stabilization of memristor-based neural networks in [6] have been studied by the research team of Zeng. Almost at the same time, the exponential stability of the equilibrium [7,9] and the periodic solution [10] of memristor-based recurrent neural networks have been investigated. Guo et al. [11] studied the global exponential dissipativity of memristor-based recurrent neural networks with time-varying delays. To the best of our knowledge, the memristor-based neural networks models proposed and studied in the literature are deterministic. However, in real nervous systems and in the implementation of artificial neural networks, noise is unavoidable and should be taken into consideration in modelling. Therefore, it is of practical importance to study the stochastic neural networks [24–31].

Motivated by the above discussion, in this paper, we attempt to propose a stochastic memristor-based recurrent neural networks model and concern the exponential stability problem. To our knowledge, there are very few studies regarding this issues. The structure of this paper is outlined as follows. In Section 2, the stochastic memristor-based recurrent neural networks model and some preliminaries are introduced. In Section 3, some algebraic conditions concerning exponential stability of the proposed model are derived. In Section 4, numerical simulations are given to demonstrate the effectiveness of the proposed approach. Finally, the paper ends with a conclusion.

Notations: Throughout this paper, $\mathbb{R}^n$ denotes the n-dimensional Euclidean space and $\mathbb{R}_+$ stands for the set of nonnegative real numbers. $|\cdot|$ is the Euclidean norm in $\mathbb{R}^n$, i.e. $|y| = \sqrt{\sum_{i=1}^n y_i^2}$

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for vector $y=(y_1,y_2,\dots,y_n)^T\in\mathbb{R}^n$. $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},P)$ is a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ satisfying the usual conditions (i.e., the filtration contains all P-null sets and is right continuous). Let $L^p_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$ be the family of all $\mathcal{F}_0$-measurable $C([-\tau,0];\mathbb{R}^n)$-valued random variables $\xi=\{\xi(s):-\tau\leq s\leq 0\}$ such that $\sup_{-\tau\leq s\leq 0}\mathbb{E}|\xi(s)|^p<\infty$, where $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator with respect to the given probability measure P. $\max\{\cdot\}$ and $\min\{\cdot\}$ denote the maximum and minimum values, respectively.

2. Model description and preliminaries

Based on some relevant works in [4–11], which deal with the detailed construction of some general classes of memristor-based recurrent neural networks from the aspects of circuit analysis and memristor physical properties, and some studies in [24–31], which take noise or stochastic disturbances into consideration in modelling neural networks, we propose a class of stochastic memristor-based recurrent neural networks model described by the following stochastic differential equations:

$$\begin{aligned}dy_i(t)=&\left\{-d_i(y_i)y_i(t)+\sum_{j=1}^n a_{ij}(y_i)f_j(y_j(t))\right.\\&\left.+\sum_{j=1}^n b_{ij}(y_i)g_j(y_j(t-\tau_{ij}(t)))\right\}dt\\&+\sigma_i(t,y_i(t),y_i(t-\tau_{ij}(t)))\,d\omega_i(t),\quad t\geq 0,\quad i=1,2,\dots,n,\end{aligned}\tag{1}$$

where n is the number of neurons, y_i denotes the ith state variable (or the voltage of the capacitor $\mathbf{C}_i$ in very large-scale integration (VLSI) circuits. One can refer to Refs. [4–11] for more detailed information). $d_i(y_i), a_{ij}(y_i)$ and $b_{ij}(y_i)$ are state-dependent bounded functions for the existence of memristor. According to the feature of memristor, we let

$$\begin{aligned}&d_i(y_i)=\begin{cases}d_i^*, & |y_i(t)|<T_i,\\ d_i^{**}, & |y_i(t)|>T_i,\end{cases}\quad a_{ij}(y_i)=\begin{cases}a_{ij}^*, & |y_i(t)|<T_i,\\ a_{ij}^{**}, & |y_i(t)|>T_i,\end{cases}\\&b_{ij}(y_i)=\begin{cases}b_{ij}^*, & |y_i(t)|<T_i,\\ b_{ij}^{**}, & |y_i(t)|>T_i,\end{cases}\end{aligned}\tag{2}$$

for $i,j=1,2,\dots,n$, in which switching jumps $T_i>0$, $d_i^*>0, d_i^{**}>0, a_{ij}^*, a_{ij}^{**}, b_{ij}^*, b_{ij}^{**}$ are constant numbers. $\tau_{ij}(t)$ denotes the time-varying delays and w_i is a scalar Brownian motion defined on the probability space $(\Omega,\mathcal{F},\{\mathcal{F}_t\}_{t\geq 0},P)$. f_i and g_i are the activation functions, and σ_i is nonlinear function.

Remark 1. Various types of functions satisfy the feature of memristor. In this paper, a simple type of functions (2) of memristor is chosen as Refs. [5–7,11]. Some other types of memristor models can be found in Refs [4,8,9]. In fact, the main results in this paper also can be made some parallel promotions.

For convenience, we make the following preparations. Throughout this paper, solutions of all the systems considered in the following are intended in Filippov's sense. In Banach space of all continuous functions $C([-\tau,0],\mathbb{R}^n)$, where $0\leq\tau=\max_{1\leq i,j\leq n}\{\tau_{ij}(t)\}$. Let $\overline{d}_i=\max\{d_i^*,d_i^{**}\}$, $\underline{d}_i=\min\{d_i^*,d_i^{**}\}$, $\overline{a}_{ij}=\max\{a_{ij}^*,a_{ij}^{**}\}$, $\underline{a}_{ij}=\min\{a_{ij}^*,a_{ij}^{**}\}$, $\overline{b}_{ij}=\max\{b_{ij}^*,b_{ij}^{**}\}$, $\underline{b}_{ij}=\min\{b_{ij}^*,b_{ij}^{**}\}$ for $i,j=1,2,\dots,n$. $\mathrm{co}[\underline{\xi}_i,\overline{\xi}_i]$ denotes closure of the convex hull generated by real numbers $\underline{\xi}_i$ and $\overline{\xi}_i$. Clearly, in this paper, we have $[\underline{\xi}_i,\overline{\xi}_i]=\mathrm{co}[\underline{\xi}_i,\overline{\xi}_i]$. Let matrices $\underline{D}=diag(\underline{d}_i)$, $\hat{A}=(\hat{a}_{ij})_{n\times n}$, $\hat{B}=(\hat{b}_{ij})_{n\times n}$. The initial condition of (1) are given by $y(s)=\phi(s)=(\phi_1(s),\phi_2(s),\dots,\phi_n(s))^T\in C([-\tau,0],\mathbb{R}^n), i=1,2,\dots,n$.

In order to obtain our main theorems, the following assumptions for the system (1) are always made throughout this paper.

Assumption 1. For $i=1,2,\dots,n$, $\forall k\in\mathbb{R}$, the neuron activation functions $f_i(y_i), g_i(y_i)$ with $f_i(0)=g_i(0)=0$ in (1) are bounded and satisfy

$$0\leq\frac{f_i(k)}{k}\leq\sigma_i,\quad 0\leq\frac{g_i(k)}{k}\leq\rho_i\tag{3}$$

where σ_i, ρ_i are nonnegative constants.

Assumption 2. $\sigma_i:\mathbb{R}_+\times\mathbb{R}\times\mathbb{R}\longrightarrow\mathbb{R}$ is locally Lipschitz continuous and satisfies the linear growth condition with $\sigma_i(t,0,0)=0$ and

$$\sigma_i^2(t,x_i(t),x_i(t-\tau(t)))\leq\alpha_i^2x_i^2(t)+\beta_i^2x_i^2(t-\tau(t)),\tag{4}$$

where α_i and β_i $i=1,2,\dots,n$ are constants.

Now, as the literature [4–13], by applying the theories of set-valued maps and stochastic differential inclusions, from (1), we have

$$\begin{aligned}dy_i(t)\in&\left\{-co[\underline{d}_i,\overline{d}_i]y_i(t)+\sum_{j=1}^n co[\underline{a}_{ij},\overline{a}_{ij}]f_j(y_j(t))\right.\\&\left.+\sum_{j=1}^n co[\underline{b}_{ij},\overline{b}_{ij}]g_j(y_j(t-\tau_{ij}(t)))\right\}dt\\&+\sigma_i(t,y_i(t),y_i(t-\tau_{ij}(t)))\,d\omega_i(t),\quad t\geq 0\end{aligned}\tag{5}$$

or equivalently, for $i,j=1,2,\dots,n$, there exist $\hat{d}_i\in co[\underline{d}_i,\overline{d}_i]$, $\hat{a}_{ij}\in co[\underline{a}_{ij},\overline{a}_{ij}], \hat{b}_{ij}\in co[\underline{b}_{ij},\overline{b}_{ij}]$, which are dependent on the initial condition of network (1) and time t [12,13], such that

$$\begin{aligned}dy_i(t)=&\left\{-\hat{d}_iy_i+\sum_{j=1}^n\hat{a}_{ij}f_j(y_j(t))+\sum_{j=1}^n\hat{b}_{ij}g_j(y_j(t-\tau_{ij}(t)))\right\}dt\\&+\sigma_i(t,y_i(t),y_i(t-\tau_{ij}(t)))\,d\omega_i(t),\quad t\geq 0\end{aligned}\tag{6}$$

Remark 2. Because of taking stochastic disturbances into consideration, the main difference between our proposed model (1) and the previous memristor-based recurrent neural networks model [4–11] is that the deduced equation (5) is stochastic differential inclusions, called multi-valued stochastic differential equations, which comes from the set-valued functions $d_i(y_i), a_{ij}(y_i), b_{ij}(y_i)$ and stochastic term $\sigma_i(t,y_i(t),y_i(t-\tau_{ij}(t)))\,d\omega_i(t)$. Some relations regarding differential inclusions and stochastic differential inclusions can be found in reference [32].

It is obvious from Assumptions 1 and 2 that $(0,0,\dots,0)^T$ is an equilibrium point of Eq. (5) or (6). So, there exists at least one equilibrium point of Eq. (1).

Definition 1 (*Wang et al. [31]*). For any initial conditions $\phi\in L^2_{\mathcal{F}_0}([-\tau,0];\mathbb{R}^n)$, the trivial solution (equilibrium point) of system (1) is robustly, globally, exponentially stable in the mean square, if there exist constants $\alpha>0$ and $\mu>0$ such that every solution $y(t;\phi)$ of (1) satisfies

$$\mathbb{E}\{|y(t,\phi)|^2\}\leq\mu e^{-\alpha t}\sup_{-\tau\leq s\leq 0}\mathbb{E}\{|\phi(s)|^2\},\quad\text{for all } t>0.$$

For further deriving the global exponential stability conditions, the following lemmas are needed.

Lemma 1. *For any $a,b\in\mathbb{R}^n$ and any positive matrix Y satisfying*

$$\pm 2a^Tb\leq a^TYa+b^TY^{-1}b.$$

Lemma 2 (*Zhou and Cao [33]*). *Let a,b be constants with $0<a<b$. $x(t)$ is a continuous nonnegative function on $t\geq t_0-\tau$ and satisfy the*