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$\mathcal{H}_\infty$ consensus performance for discrete-time multi-agent systems with communication delay and multiple disturbances

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ABSTRACT

This paper proposes a new $\mathcal{H}_\infty$ consensus criterion for discrete-time multi-agent systems with communication-delay and disturbances. By constructing a suitable Lyapunov–Krasovskii (L–K) functional, which fractionizes the delay interval into two subsections, and utilizing reciprocally convex approach, a new $\mathcal{H}_\infty$ consensus criterion for the concerned systems is established in terms of linear matrix inequalities (LMIs) which can be easily solved by various effective optimization algorithms. One numerical example is given to illustrate the effectiveness of the proposed method.

1. Introduction

During the last few years, the stability [1–10], passivity [11–13], synchronization [14,15], state estimation [16–18] and other problems are being put to treat in the various dynamic systems. Of this, one pays close attention to the $\mathcal{H}_\infty$ control problem for the following reason: $\mathcal{H}_\infty$ control problem has been used to minimize the effects of the external disturbances. It is the aim of this theory to design the controller such that the closed-loop system is internally stable and its $\mathcal{H}_\infty-$norm of the transfer function between the controlled output and the disturbances will not exceed a given $\mathcal{H}_\infty$ performance level γ. Naturally, $\mathcal{H}_\infty$ control problem was issued in the various dynamic systems [19–23]. The goal of this problem is to design an $\mathcal{H}_\infty$ controller to robustly stabilize the systems while guaranteeing a prescribed level of disturbance attenuation γ in the $\mathcal{H}_\infty$ sense for the systems with external disturbances. Within this framework, the controller law will ensure an $\mathcal{H}_\infty$ performance for the systems in the face of the disturbances.

On the other hand, the multi-agent systems (MASs) [24] are the network with the interconnection topology between each agent and have a prime concern of the agreement of a group of agents on their states of leader by interaction; namely, the concern is a leader-following consensus problem. During recent years, MASs have received considerable attentions due to their extensive applications in many fields such as distributed sensor networks [25], vehicle systems [26,27], groups of mobile autonomous agents [28], multi-agent robotic systems [29], and other applications [30–32]. Before handling this system, since modern systems use information between each agent in networks, these days, we need to pay keen attention to the three following considerations: (a) During the information exchange between each agent in networks, there exists external disturbance. Also, in implementation of many practical systems such as aircraft and electric circuits, there exist occasionally stochastic perturbations. The perturbations have influence on the random occurrence of the disturbance. (b) It is well known that the time-delay often causes undesirable dynamic behaviors such as performance degradation and instability of various systems. Therefore, the study on various problems for systems with time-delay has been widely investigated [1–18]. (c) Most systems use microprocessor or microcontrollers, which are called digital computer, with the necessary input/output hardware to implement the systems. A little more to say, the fundamental character of the digital computer is that it takes

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compute answers at discrete steps [33]. Therefore, discrete-time modeling for MASs with time-delay plays an important role in many fields of science and engineering applications. Unfortunately, to the best of authors' knowledge, this problem of $\mathcal{H}_\infty$ consensus protocol for discrete-time MASs with time-delay and the disturbances has not been investigated yet. Moreover, in the case of continuous-time, the $\mathcal{H}_\infty$ consensus problems for directed networks of agents with external disturbances and model uncertainty on fixed and switching topologies are addressed in [34].

With this motivation mentioned above, in this paper, firstly, the problem to get a consensus protocol for a class of discrete-time MASs with interval time-varying delays is considered. Here, stability or stabilization of system with interval time-varying delays has been a focused topic of theoretical and practical importance [35] in very recent years. The system with interval time-varying delays means that the lower bounds of time-delay which guarantees the stability of system is not restricted to be zero. A typical example of dynamic systems with interval time-varying delays is networked control system. Secondly, a new model of discrete-time MASs with multiple disturbances is constructed and its $\mathcal{H}_\infty$ consensus protocol is proposed for the first time. At this time, because we do not know the subliminal influence of disturbance in practice, the occurrence property of each disturbance is assumed with the property of Bernoulli sequence. This concept of model is shown in Fig. 1. Here, a Bernoulli sequence is used to model the presence of the random nonlinearity which mimics the packet dropping scenario in networked world. After introducing the Bernoulli sequence to engineering, very recently, Bernoulli distributed variables have been widely used in the concept of randomly occurring which has various types such as randomly occurring nonlinearities, randomly occurring delays, randomly occurring sensors saturations and so on [36,37]. To do this, by construction of a suitable augmented L–K functional, which fractionize the delay interval into two subsections, and utilization of the reciprocally convex approach [5] with some added decision variables, a consensus protocol design method for discrete-time MASs without external disturbances is derived in Theorem 1. Based on the result of Theorem 1, new $\mathcal{H}_\infty$ consensus conditions are proposed in Theorem 2 with the LMI framework. The LMIs can be formulated as convex optimization algorithms which are amenable to computer solution [38]. Finally, one numerical example is included to show the effectiveness of the proposed methods.

Notation: The notations used throughout this paper are fairly standard. $\mathbb{R}^n$ is the n-dimensional Euclidean space, and $\mathbb{R}^{m\times n}$ denotes the set of all $m\times n$ real matrices. For real symmetric matrices X and Y, $X>Y$ (resp., $X\geq Y$) means that the matrix $X-Y$ is positive (resp., nonnegative) definite. $X^\perp$ denotes a basis for the null-space of X. I_n, 0_n and $0_{m\cdot n}$ denote $n\times n$ identity matrix, $n\times n$ and $m\times n$ zero matrices, respectively. $\mathbb{E}\{\cdot\}$ stands for the mathematical expectation operator. $\|\cdot\|$ refers to the Euclidean vector norm or the induced matrix norm. diag$\{\cdots\}$ denotes the block diagonal matrix. For square matrix X, sym$\{X\}$ means the sum of X and its symmetric matrix X^T; i.e., sym$\{X\}=X+X^T$. For any vectors $x_i\in\mathbb{R}^m$ $(i=1,2,\ldots,n)$, col$\{x_1,x_2,\ldots,x_n\}\in\mathbb{R}^{m\times n}$ means the column vector, i.e., $[x_1^T,x_2^T,\ldots,x_n^T]^T$. For any matrix $X\in\mathbb{R}^{m\times n}$, $[\kappa_{ij}X]\in\mathbb{R}^{Nm\times Nn}$ means that the matrix with elements $\kappa_{ij}X$, where κ_{ij} denotes the Kronecker symbol with $\kappa_{ij}=1$ for $i=j$ and $\kappa_{ij}=0$, otherwise; i.e., $[\kappa_{ij}X]=I_N\otimes X$ for $i,j\in\{1,2,\ldots,N\}$.

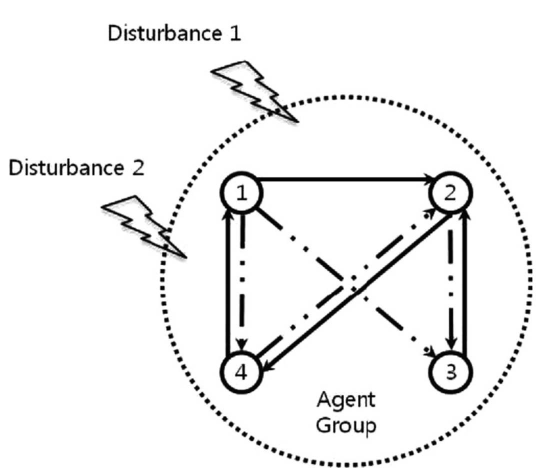

Fig. 1. The concept of proposed model.

2. Problem statements

The interaction topology of a network of agents is represented using a directed graph (digraph) $\mathcal{G}=(\mathcal{V},\mathcal{E},\mathcal{A})$ with the set of nodes $\mathcal{V}=\{1,2,\ldots,N\}$ and edges $\mathcal{E}=\{(i,j):i,j\in\mathcal{V}\}\subset\mathcal{V}\times\mathcal{V}$. An adjacency matrix $\mathcal{A}=[a_{ij}]_{N\times N}$ of the digraph $\mathcal{G}$ is the matrix with nonnegative elements satisfying $a_{ii}=0$ and $a_{ij}\geq 0$. If there is an edge between i and j, then the elements of matrix A described as $a_{ij}>0\Leftrightarrow(i,j)\in\mathcal{E}$. The digraph $\mathcal{G}$ is said to be undirected if $(i,j)\in\mathcal{E}\Leftrightarrow(j,i)\in\mathcal{E}$. A set of neighbors of agent i is denoted by $\mathcal{N}_i=\{j\in\mathcal{V}:(i,j)\in\mathcal{E}\}$. A degree of node i is denoted by $deg(i)=\sum_{j\in\mathcal{N}_i}a_{ij}$. A degree matrix of digraph $\mathcal{G}$ is diagonal and defined as $\mathcal{D}=diag\{deg(1),\ldots,deg(N)\}$. The Laplacian matrix $\mathcal{L}$ of graph $\mathcal{G}$ is defined as $\mathcal{L}=\mathcal{D}-\mathcal{A}$. More details can be seen in [39].

Consider the following discrete-time MASs with the dynamics of agent i [24]:

$$p_i(k+1)=p_i(k)+u_i(k)\quad(i=1,2,\ldots,N),\tag{1}$$

where N is the number of agents, n is the number of states of agent i, $p_i(t)\in\mathbb{R}^n$ and $u_i(t)\in\mathbb{R}^n$ are the state vector and the target-concurring protocol vector of agent i, respectively.

According to the work [40], a leader-following algorithm in agent can be described as

$$u_i(k)=-K\sum_{j\in\mathcal{N}_i}a_{ij}(p_i(k)-p_j(k))-Kb_i(p_i(k)-p_0)\quad(i=1,2,\ldots,N),\tag{2}$$

where $K\in\mathbb{R}^{n\times n}$ is a protocol gain matrix which will be chosen, $p_0\in\mathbb{R}^n$ is the state vector of leader, a_{ij} and b_i are the interconnection weights defining $a_{ij}=1$ if agent i is connected to agent j, $a_{ij}=0$, otherwise, and $b_i=1$ if agent i is connected to the leader, $b_i=0$, otherwise.

With understanding the communication delay, a consensus algorithm can be

$$u_i(k)=-K\sum_{j\in\mathcal{N}_i}a_{ij}(p_i(k)-p_j(k-h(k)))-Kb_i(p_i(k)-p_0)\quad(i=1,2,\ldots,N)\tag{3}$$

where $h(k)$ is the time-varying function satisfying

$$h_m\leq h(k)\leq h_M,$$

where h_m and h_M are positive integers.

Moreover, the randomly occurring multiple distributions are considered as follows:

$$w_i(k)=[\rho_1(k)\ \rho_2(k)]\begin{bmatrix}w_{1i}(k)\\ w_{2i}(k)\end{bmatrix}\quad(i=1,2,\ldots,N),\tag{4}$$

here it is assumed that the disturbances are randomly occurring. This means that $\rho_1(k)$ and $\rho_2(k)$ are the probabilistic processes representing the disturbance processes; that is, let $\rho_1(k)$ and $\rho_2(k)$