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New approaches to finite-time stability and stabilization for nonlinear system

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ABSTRACT

This paper is concerned with the problems of finite-time stability (FTS) and finite-time stabilization for both continuous and discrete nonlinear systems, which can be represented by affine fuzzy system. Some new FTS conditions are provided and applied to the design problem of finite-time fuzzy regulators. The common Lyapunov function (CLF) approach is first used to analyze the FTS of affine fuzzy system. Then, a piecewise Lyapunov function (PLF), which is less conservative than CLF, is adopted to analyze the FTS of affine fuzzy systems and design corresponding finite-time fuzzy controller. In analysis, the FTS conditions are formulated in terms of linear matrix inequalities (LMIs). In synthesis, the conditions of finite-time stabilization turn out to be in the formulation of nonconvex matrix inequalities for discrete affine fuzzy system (DAFS) and bilinear matrix inequalities (BMIs) for continuous affine fuzzy system (CAFS). Thus, iterative LMI (ILMI) approach is applied to obtain the feasible solutions. Two examples are provided to illustrate the validity of the proposed results.

1. Introduction

Fuzzy control has been proved to be a successful control method to deal with certain complex nonlinear control problems [1]. T–S fuzzy model [2], which is also called the Type-III fuzzy model suggested by Takagi and Sugeno, is based on using a set of local linear models which are smoothly connected by fuzzy membership functions [3,4]. This fuzzy modelling approach can offer an alternative method to describe complex nonlinear systems [5,6].

In general, the T–S fuzzy system can be divided into two categories: homogeneous fuzzy system (HFS) and affine fuzzy system (AFS). The main difference between HFS and AFS is that the AFS has a constant bias. Most of the works, however, focus on the homogeneous fuzzy system due to the ease of analysis. It is not until recently that some interesting results on affine fuzzy system were reported. In [7], the stability analysis and controller synthesis methodology for a discrete affine fuzzy system are investigated, and the stabilizability condition is solved numerically in an iterative manner. In [8], a continuous version of the results in [7] is studied and the fuzzy local controllers are obtained in a numerical manner simultaneously together with the gains. In [9] and [10], stability analysis and H_∞ controller design are investigated for continuous and discrete affine fuzzy systems, respectively.

It is well known that the stability of fuzzy system can be determined by checking a Lyapunov equation or a linear matrix inequality (LMI). A common positive definite matrix is required to satisfy the Lyapunov equation or the LMI for all the local models. In [11], new stability conditions are obtained by relaxing the stability conditions and LMI-based design procedures for fuzzy regulators and fuzzy observers, which are constructed using the parallel distributed compensation and the relaxed stability conditions. In [12], new stability conditions, which relax the stability conditions further, are proposed by taking the interactions into consideration. In [13], new stability conditions that provide better or at least the same results for fuzzy regulators and fuzzy observers are presented. The problem of state feedback control for continuous-time T–S fuzzy systems via switched fuzzy controllers is investigated in [14]. In [15], staircase membership functions are introduced to facilitate the stability analysis of fuzzy model-based control systems. A novel control design on discrete-time T–S fuzzy systems with time-varying delays is proposed in [16]. The problem of analysis of a T–S fuzzy control system in the frequency domain is studied in [17].

However, a common Lyapunov function may not exist for many fuzzy systems, especially for highly nonlinear complex systems [9]. To deal with this problem, a piecewise Lyapunov functions (PLFs) method is presented in [18]. In [19], a new H_∞ controller design method for the discrete fuzzy systems based on the piecewise Lyapunov functions is proposed. A generalised H_2 controller synthesis method for discrete fuzzy systems based on a piecewise

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Lyapunov function is studied in [20]. The problem of robust H_∞ output feedback control fuzzy affine dynamic systems with parametric uncertainties and input constraints is investigated in [21]. Some other results related to piecewise Lyapunov functions can be found in [22–27].

It is worth pointing out that most of the existing results related to stability of T–S fuzzy systems focus on Lyapunov asymptotic stability, which is defined over an infinite time interval. However, in practice, we may have interest in a bound of system trajectories over a fixed short time. The finite-time stability is a different stability concept which admits that the state does not exceed a certain bound during a fixed finite-time interval. Finite-time stability and finite-time bounded problems of linear systems subject to parametric uncertainties are discussed in [28]. A dynamic output feedback controller is designed for the finite-time stabilization problem in [29]. The problem of finite-time quantized guaranteed cost fuzzy control for continuous-time nonlinear systems is studied in [30]. In [31], the finite-time H_∞ control problem for time-delay nonlinear jump systems via dynamic observer-based state feedback is investigated. However, most of the existing results are focused on linear system.

Thus, in this paper, both the finite-time stability and finite-time stabilization problem of the nonlinear systems, which can be represented by affine fuzzy system, are presented in both continuous and discrete cases. Some new FTS conditions are provided and applied to the synthesis problem of finite-time fuzzy regulators. The main contribution of this paper is that a new method, which is based on fuzzy control, is provided to deal with the finite-time stability and finite-time stabilization problem of nonlinear system.

The remainder of the paper is organized as follows: following the introduction, both continuous and discrete affine fuzzy systems are modeled in Section 2. The new finite-time stability and finite-time stabilization conditions for both continuous and discrete affine fuzzy systems based on CLF and PLF are presented in Sections 3 and 4, respectively. In Section 5, two examples are provided to illustrate the validity of the proposed results. Finally, some conclusions are drawn in Section 6.

2. Preliminaries

The T–S fuzzy model proposed by Takagi–Sugeno, which is described by IF-THEN rules, can represent local linear input–output relations of a nonlinear system. If a nonlinear dynamic system is modeled by the T–S fuzzy system, then it can be described by the following continuous affine fuzzy system (CAFS) or discrete affine fuzzy system (DAFS).

(C1) Continuous affine fuzzy system (CAFS)

$$R_i: \quad \text{IF } x_1 \text{ is } F_i^1 \text{ AND } \cdots x_n \text{ is } F_i^n$$
$$\text{THEN } \dot{x}(t) = A_i x(t) + B_i u(t) + \mu_i \tag{1}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $\mu_i \in \mathbb{R}^n$ is the bias term. $F_i^1, F_i^2, \ldots, F_i^n$ are fuzzy variables. It is worth noting that the local subsystem includes a constant bias term. Given a pair of (x, u), the input–output form of affine fuzzy system (1) can be represented as follows:

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(x)(A_i x(t) + B_i u(t) + \mu_i) \tag{2}$$

where

$$h_i(x) = \frac{\omega_i(x)}{\sum_{i=1}^{r} \omega_i(x)}, \quad \omega_i(x) = \prod_{j=1}^{n} F_i^j(x_j)$$
$$h_i(x) \ge 0, \quad \sum_{i=1}^{r} h_i(x) = 1 \tag{3}$$

(D1) Discrete affine fuzzy system (DAFS)

$$R_i: \quad \text{IF } x_1 \text{ is } F_i^1 \text{ AND } \ldots x_n \text{ is } F_i^n$$
$$\text{THEN } x(t+1) = A_i x(t) + B_i u(t) + \mu_i \tag{4}$$

where $x(t) \in \mathbb{R}^n$ is the state vector, $u(t) \in \mathbb{R}^m$ is the input vector, $\mu_i \in \mathbb{R}^n$ is the bias term. $F_i^1, F_i^2, \ldots, F_i^n$ are fuzzy variables. Given a pair of (x, u), the input–output form of DAFS (2) can be represented as follows:

$$x(t+1) = \sum_{i=1}^{r} h_i(x)(A_i x(t) + B_i u(t) + \mu_i) \tag{5}$$

where

$$h_i(x) = \frac{\omega_i(x)}{\sum_{i=1}^{r} \omega_i(x)}, \quad \omega_i(x) = \prod_{j=1}^{n} F_i^j(x_j)$$
$$h_i(x) \ge 0, \quad \sum_{i=1}^{r} h_i(x) = 1 \tag{6}$$

Definition 1 (*Finite-time stability*). Given a positive definite matrix R and three positive constants c_1, c_2, T_f (or δ, ϵ, N), with $c_1 < c_2$ (or $\delta < \epsilon$), the CAFS (2) (or DAFS (5)) with $u(t) \equiv 0$ is said to be finite-time stable with respect to (c_1, c_2, T_f, R) (or (δ, ϵ, N, R)), if $x^T(0)Rx(0) < c_1 \Rightarrow x(t)^T Rx(t) < c_2$ (or $x^T(0)Rx(0) < \delta \Rightarrow x(t)^T Rx(t) < \epsilon$), $\forall t \in [0, T_f]$ (or $\forall t \in \{1, \ldots, N\}$).

Definition 2 (*Finite-time stabilizable*). Given a positive definite matrix R and three positive constants c_1, c_2, T_f (or δ, ϵ, N), with $c_1 < c_2$ (or $\delta < \epsilon$), the CAFS (2) (or DAFS (5)) is said to be finite-time stabilizable with respect to (c_1, c_2, T_f, R) or (δ, ϵ, N, R)), if there exists a control input $u(t)$ such that $x^T(0)Rx(0) < c_1 \Rightarrow x(t)^T Rx(t) < c_2$ (or $x^T(0)Rx(0) < \delta \Rightarrow x(t)^T Rx(t) < \epsilon$), $\forall t \in [0, T_f]$ (or $\forall t \in \{1, \ldots, N\}$).

Remark 1. In view of Definition 1, it can be seen that the concept of finite-time stability is different from Lyapunov asymptotic stability [32]. A Lyapunov asymptotically stable system may not be finite-time stable if its states exceed the prescribed bounds.

3. Finite-time stability and stabilization based on CLF

In this section, the problems of finite-time stability and stabilization of affine fuzzy system are addressed based on CLF. The following assumption is needed to facilitate the analysis.

Assumption 1. Let I_0 be the set of indexes for the fuzzy rules that contain the origin $x=0$ and I_1 be the set of indexes for the fuzzy rules that do not contain the origin.

$$I_0 = \{k | h_k(0) \ne 0\} \tag{7}$$

For $i \in I_0$, the constant bias term μ_i in (2) or (5) is assumed to be zero, i.e.,

$$\mu_i = 0, \quad i \in I_0 \tag{8}$$

This assumption can guarantee that the origin $x=0$ is the equilibrium point of the given affine fuzzy system.

3.1. Finite-time stability based on CLF

Consider the CAFS (2) and DAFS (5) without input $u(t)$, i.e.,

$$\dot{x}(t) = \sum_{i=1}^{r} h_i(x)(A_i x(t) + \mu_i) \tag{9}$$

$$x(t+1) = \sum_{i=1}^{r} h_i(x)(A_i x(t) + \mu_i) \tag{10}$$