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Adaptive robust control based on single neural network approximation for a class of uncertain strict-feedback discrete-time nonlinear systems

Xin Wang [a], Tieshan Li [a,*], C.L. Philip Chen [b,c], Bin Lin [d]

[a] Navigation College, Dalian Maritime University, Dalian 116026, PR China
[b] Faculty of Science and Technology, University of Macau, Macau, PR China
[c] UMacau Research Institute, Zhuhai, Guangdong, PR China
[d] Department of Information Science and Technology, Dalian Maritime University, Dalian 116026, PR China

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ABSTRACT

In this paper, based on single neural network approximation, a novel adaptive robust control algorithm is proposed for a class of uncertain discrete-time nonlinear systems in the strict-feedback form. In order to solve the noncausal problem, the original system is transformed into a predictor form. Different from the existing methods for the investigated system, all unknown parts at internship steps are passed down in the discrete-time backstepping design procedure, and only one single neural network is used to approximate the lumped unknown function in the system at the last step. Following this approach, the designed controller contains only one actual control law and one adaptive law. Compared with the existing results for discrete-time systems, the proposed controller is simpler and the computational burden is lighter. The stability of the closed-loop system is proven to be uniformly ultimately bounded and the tracking error converges to a small neighborhood of zero by choosing the control parameters appropriately. Simulation examples are employed to illustrate the effectiveness of the proposed approach.

1. Introduction

The last decade has witnessed an ever increasing research in neural networks (NNs) since it has been proven to be particularly useful for controlling nonlinear systems with nonlinearly parameterized uncertainties owing to their universal approximation property [1]. In general, NNs are primarily used as online function approximators for the unknown nonlinearities. Specifically in the past decades, the adaptive neural network control for identification and control of nonlinear dynamic systems has obtained many attentions due to the excellent universal approximation ability of the neural network [2–4]. For example, to overcome the matching condition in the expanded class of strict-feedback nonlinear systems, still relax some restrictive assumptions during the design procedure, an adaptive neural-network control design methodology was developed in [3] based on the backstepping technique. Afterward, an adaptive control approach utilizing nonlinearly parameterized NN approximators was developed in [4]. Through years of progress, a lot of adaptive NN control schemes have been proposed based on Lyapunov stability theory and the backstepping technique [5–10]. However, there exists a main drawback of complexity of the designed controllers in the aforementioned results.

One reason for the complexity problem of the designed controllers is the computational expansion [11]. As well known in the conventional backstepping design procedure for the continuous-time area, the complexity problem is caused by the repeated differentiations of the certain nonlinear functions in the controller design process. Especially, the complexity of the designed controller grows drastically as the system order increases [11]. Fortunately, by introducing first-order filters of the synthetic inputs at each intermediate steps of the traditional backstepping approach, a dynamic surface control (DSC) technique was proposed to solve the complexity growing problem in [12]. Recently, this technique was widely used in adaptive control literature such as [13–16].

The other one reason is the use of multiple approximators. For above control approaches in [3–16], many approximators were still employed to construct virtual control laws and actual control law during the process of control design. The use of multiple approximators makes the controller design process still complex and the controller computational burden grows drastically. In order to eliminate the problem derived from the use of multiple approximators, only one NN was used to approximate the lumped unknown function of the system at the last step for a class of strict-feedback nonlinear systems in [17]. In [18], a single neural network (SNN) approximation based adaptive control design

* Corresponding author.
E-mail addresses: tieshanli@126.com, tieshanlidmu@gmail.com (T. Li).

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method was presented for uncertain pure-feedback nonlinear systems. For the nonlinear systems in presence of uncertainties which could be come from measurement noise, modeling errors, external disturbances, etc., the robust adaptive single neural network (SNN) control design approach was studied in [19,20].

It is well-known that the discrete-time systems can be more veritably to describe practical problem in control systems than the continuous-time systems. Nevertheless, comparing to the nonlinear continuous-time systems indicated in the above mentioned papers, adaptive control is less developed for nonlinear discrete-time systems. The reason lies in that the linearity property of the derivative of a Lyapunov function in continuous-time is not present in the difference of Lyapunov function in the discrete-time [21]. As a consequence, many elegant control schemes for continuous-time systems may be not suitable for discrete-time systems. However, there are still considerable advances in adaptive NN control for discrete-time systems [22–27]. For systems with general relative degree, multilayer NN control was studied through back-propagation [22], and for nonlinear discrete-time systems in normal form, NN control with filtered tracking error was proposed in [23]. To solve the noncausal problem, the approach that “looks ahead” and chooses the control law to force the states to acquire their desired values was proposed in [24] for parameter strict-feedback discrete-time systems. But it is hard to extend this technique to more general systems. Recently, a novel method using prediction functions of future states for discrete-time nonlinear systems transformation was studied in [25], in which adaptive NN backstepping design has been applied to the transformed strict-feedback discrete-time systems without noncausal problem. Subsequently, many elegant adaptive control schemes were studied in [28–37] for discrete-time nonlinear systems based on the approximation property of the neural network.

Similarly, the problem of complexity growing is also existed in the above discrete-time nonlinear control design methods [21–37]. The reason is that the control design procedure for discrete-time system is much more complicated. There are too many adaptive parameters needed to be tuned in the online learning laws, so that the learning time tends to be unacceptably long for the higher-order systems and time-consuming process is unavoidable. More recently, a new adaptive control algorithm with only less parameters to be adjusted online has been developed in [38], it can reduce online computation burden. Then, several results on reducing the number of the adjustable parameters and lightening the online computation burden were studied in [39–43], respectively. However, for above control approaches in [21–43], many approximators were still employed to construct virtual control laws and actual control laws. The use of multiple approximators makes the controller design process still complex and the controller computational burden grows. To the best knowledge of the authors, there is still no report on applying SNN to the control design for discrete-time nonlinear systems.

Based on above observation in this paper, and in order to overcome the complexity and computational burden of the controllers in discrete-time nonlinear systems, an adaptive neural control based on single neural network (SNN) approximation is first proposed for a class of uncertain discrete-time nonlinear systems. The original system is transformed into an equivalent n-step ahead predictor, then the discrete-time backstepping design procedure can be carried out. Comparing to the existing methods for discrete-time nonlinear systems, all the unknown functions of the virtual control laws do not need to be approximated on-line at intermediate steps, and only one high-order neural network (HONN) is employed to approximate the lumped unknown functions of the system at the last step. That is, all the virtual control laws are not necessary to be constructed and implemented via NN, and only the actual control law needs to be constructed and implemented via NN at the last step. Thus, the controller in this paper is much simplified and its computational burden is also much lightened. Stability analysis shows that all the closed-loop system signals are uniformly ultimately bounded, and the steady state tracking error can be made arbitrarily small by appropriately choosing control parameters. Simulation results demonstrate the effectiveness of the proposed approach.

The rest of the paper is organized as follows. Section 2 introduces the systems and the control problem to be investigated, some preliminaries are also given in this section. In Section 3, the single neural network approximation based adaptive robust control design produce for a class of uncertain strict-feedback discrete-time nonlinear systems is carried out. And then, Simulation examples are given to demonstrate the effectiveness of the approach in Section 4. This paper ends with conclusion in Section 5.

Throughout this paper, the following notations are used. $\|M\|$ denotes the Euclidean norm of vector M and induced norm of matrix. M^T represents the transpose of a vector or a matrix M. $|\cdot|$ denotes the absolute value.

2. Problem formulation and preliminaries

2.1. Problem formulation

Consider a class of uncertain single-input single-output (SISO) discrete-time nonlinear dynamical systems in the following strict-feedback form:

$$\begin{aligned} &x_i(k+1) = x_{i+1}(k) + f_i(\bar{x}_i(k)), \quad i = 1,2,\ldots,n-1, \\ &x_n(k+1) = u(k) + f_n(\bar{x}_n(k)), \\ &y_k = x_1(k), \end{aligned} \tag{1}$$

where $\bar{x}_i(k) = [x_1(k), x_2(k), \ldots, x_i(k)]^T \in R^i$, $i = 1,2,\ldots,n$, $u(k) \in R$ and $y_k \in R$ are the system state variables, the system input and output, respectively; $f_i(\bar{x}_i(k))$, $i = 1,2,\ldots,n$ are unknown smooth nonlinear functions.

The control objective is, for a known and bounded reference signal $y_d(k)$, to design an adaptive controller for the system (1), such that all the closed-loop system signals remain uniformly ultimately bounded, and the system output y_k follows the reference signal $y_d(k)$.

Assumption 1. the desired signal $y_d(k) \in \Omega_y$, $\forall k > 0$ is smooth and known, where $\Omega_y := \{\chi | \chi = x_1\}$.

2.2. Function approximation by high-order neural networks (HONNs)

There are many well-developed approaches used to approximate an unknown function. NN is one of the most frequently employed approximation method due to the fact that NN is shown to be capable of universally approximating any unknown continuous function to arbitrary precision [44–46]. HONNs satisfies the conditions of the Stone–Weierstrass theorem. Because of its high order interaction between neurons, HONN can approximate any continuous nonlinear smooth function to any desired accuracy over a compact set. In this paper, a HONN is employed to approximate the lumped unknown function of the system. The structure of HONN is expressed as follows: $\phi(W,z) = W^T S(z)$, W and $S(z) \in R^l$,

$$S(z) = [s_1(z), s_2(z), \ldots, s_l(z)]^T, \tag{2}$$

$$s_i(z) = \prod_{j \in I_i} [s(z_j)]^{d_j(i)}, \quad i = 1,2,\ldots l, \tag{3}$$