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Anti-periodic solutions for high-order Hopfield neural networks with impulses ☆

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ABSTRACT

In this paper, we consider anti-periodic solutions of high-order Hopfield neural networks (HHNNs) with time-varying delays and impulses. Sufficient conditions for the existence and exponential stability of anti-periodic solutions are established by using Krasnoselski's fixed point theorem and Lyapunov functions with inequality techniques. In the end, example and numerical simulations are given to illustrate our main results.

1. Introduction

It is well known that the existence of anti-periodic solutions plays a key role in characterizing the behavior of nonlinear differential equations (see [1–10]). Due to the fact that high-order Hopfield neural networks have stronger approximation property, faster convergence rate, greater storage capacity, and higher fault tolerance than lower-order neural networks, high-order Hopfield neural networks have been the object of intensive analysis by numerous authors in recent years. In particular, there have been extensive results on the problem of the existence and stability of equilibrium points, periodic solutions, almost periodic solutions and anti-periodic solutions of high-order Hopfield neural networks (HHNNs)

$$x_i'(t) = -c_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t)g_j(x_j(t-\tau_{ij}(t))) + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(t)g_j(x_j(t-\sigma_{ijl}(t))) \times g_l(x_l(t-\upsilon_{ijl}(t))) + I_i(t),\ i=1,2,\ldots,n \tag{1.1}$$

in the literature [11–18,29–33,36–39] and the references therein.

Impulsive differential equations are mathematical apparatus for simulation of process and phenomena observed in control theory, physics, chemistry, population dynamics, biotechnologies, industrial robotics, economics, etc. [19–21]. Consequently, many neural networks with impulses have been studied extensively, and a great deal of literature is focused on the existence and stability of an equilibrium point [22–25]. In [26–28,40], the authors discussed the existence and global exponential stability of periodic solution of a class of neural networks with impulse. In [29], the authors discussed the existence and global exponential stability of anti-periodic solution of a class of cellular neural networks with impulse

$$\begin{cases} x_i'(t) = -c_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t)f_j(x_j(t)) + \sum_{j=1}^{n} b_{ij}(t)g_j(x_j(t-\tau_{ij}(t))) + u_i(t), \\ \quad t \ge 0,\ t \ne t_k, \\ \Delta x_i(t_k) = I_{ik}(t_k, x_i(t_k)), \\ x_i(t) = \varphi_i(t), \quad t \in [-\tau, 0],\ k=1,2,\ldots,\ i=1,2,\ldots,n. \end{cases} \tag{1.2}$$

However, to the best of our knowledge, there are little results for the existence and stability of anti-periodic solutions of HHNNs (1.1) with impulses. Moreover, HHNNs can be analog voltage transmission, and the voltage transmission process is often an anti-periodic process. Thus, it is worthwhile to continue the investigation of the existence and stability of anti-periodic

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$$\begin{cases} x_i'(t) = -c_i(t)x_i(t) + \sum_{j=1}^{n} a_{ij}(t)g_j(x_j(t-\tau_{ij}(t))) \\ \qquad + \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(t)g_j(x_j(t-\sigma_{ijl}(t)))g_l(x_l(t-\upsilon_{ijl}(t))) + I_i(t), \\ \quad t \ge 0,\ t \ne t_k,\ k = 1,2,\dots, \\ \Delta x_i(t_k) = I_{ik}(t_k, x_i(t_k)), \\ x_i(t) = \varphi_i(t),\ t \in [-\tau, 0],\ i = 1,2,\dots,n, \end{cases} \tag{1.3}$$

where n is the number of units in a neural network, $x_i(t)$ corresponds to the state vector of the i-th unit at time t, $c_i(t) > 0$ represents the rate with which the i-th unit will reset its potential to the resting state in isolation when disconnected from the network and external inputs, $a_{ij}(t)$ and $b_{ijl}(t)$ are the first- and second-order connection weights of the neural network, $\tau_{ij}(t) \ge 0$, $\sigma_{ijl}(t) \ge 0$ and $\upsilon_{ijl}(t) \ge 0$ correspond to the transmission delays, $I_i(t)$ denotes the external inputs at time t, and g_j is the activation function of signal transmission. c_i, a_{ij}, b_{ijl}, g_j, τ_{ij}, σ_{ijl}, υ_{ijl} are continuous functions on R, $I_{ik}: R^2 \to R$ are continuous. $\tau = \max_{t\in[0,\omega]}\{\tau_{ij}(t), \sigma_{ijl}(t), \upsilon_{ijl}(t)\}$ is a positive constant. $\Delta x_i(t_k) = x_i\ (t_k^+) - x_i(t_k)$, $x_i(t_k^+) = \lim_{h\to 0^+} x_i(t_k+h)$, $x_i(t_k) = \lim_{h\to 0^-} x_i(t_k+h)$, $i = 1,2,\dots,$ $k = 1,2,\dots,n$, $t_k \ge 0$ are impulsive moments satisfying $t_k < t_{k+1}$ and $\lim_{k\to+\infty} t_k = +\infty$, I_{ik} characterizes the impulsive function at time t_k for i-th unit.

The outline of the paper is as follows. In Section 2, some preliminaries and basic results are established. In Section 3, we give sufficient conditions for the existence and exponential stability of anti-periodic solutions for system (1.3). In Section 4, we shall give an example to illustrate our results.

2. Preliminaries and basic results

For the sake of convenience, we introduce the following notations:

$$c_i^- = \min_{t\in[0,\omega]}|c_i(t)|,\quad c_i^+ = \max_{0\le i\le n}\max_{t\in[0,\omega]}|c_i(t)|,\quad a_{ij}^+ = \max_{t\in[0,\omega]}|a_{ij}(t)|,$$

$$b_{ijl}^+ = \max_{t\in[0,\omega]}|b_{ijl}(t)|,\quad u_i^+ = \max_{0\le i\le n}\max_{t\in[0,\omega]}|u_i(t)|,$$

$$\kappa_i = \exp\left(\int_0^\omega c_i(\theta)\,d\theta\right).$$

Throughout this paper, we have the following assumptions:

(H_1) $i, j, l = 1,2,\dots,n,\ k \in N$, there exists $\omega > 0$ such that for $u \in R$

$$c_i(t+\omega) = c_i(t),\quad a_{ij}(t+\omega)g_j(u) = -a_{ij}(t)g_j(-u),$$
$$b_{ijl}(t+\omega)g_j(u)g_l(u) = -b_{ijl}(t)g_j(-u)g_l(-u),\quad \tau_{ij}(t+\omega) = \tau_{ij}(t),$$
$$\sigma_{ijl}(t+\omega) = \tau_{ijl}(t),\quad \upsilon_{ijl}(t+\omega) = \upsilon_{ijl}(t),\quad I_{ik}(t+\omega) = -I_{ik}(t),\ t, u \in R. \tag{2.1}$$

(H_2) For $i = 1,2,\dots,n,\ k \in N$, there exists a positive integer q such that $I_{i(k+q)} = I_{ik},\ t_{k+q} = t_k + \omega$.

(H_3) For each $j \in \{1,2,\dots,n\}$, there are nonnegative constants L_j, $j = 1,2,\dots,n$ and ν such that $|g_j(u)| \le \nu$, $|g_j(u) - g_j(v)| \le L_j|u-v|$, $u, v \in R$.

(H_4) For each $i \in \{1,2,\dots,n\}$, $k \in N$, there exist nonnegative constants $d_{ik} > 0$ such that

$$|I_{ik}(t,u) - I_{ik}(t,v)| \le d_{ik}|u-v|,\quad t \in [0,\omega],\ u, v \in R.$$

(H_5) There is $r > 1$ such that $H = \sum_{k=1}^{q}\left[\sum_{i=1}^{n}\left((\kappa_i/(1+\kappa_i))d_{ik}\right)^r\right]^{1/r} < 1.$

(H_6) There exist constants $\eta > 0$ and $\lambda > 0$ such that for all $t \ge 0$

$$\lambda - c_i(t) + \sum_{j=1}^{n}|a_{ij}(t)|L_j e^{\lambda\tau} + \sum_{j=1}^{n}\sum_{l=1}^{n}|b_{ijl}(t)|(L_j + L_l)\nu e^{\lambda\tau} < -\eta < 0.$$

Let $x = (x_1, x_2, \dots, x_n)^T \in R^n$, where T denotes the transposition. The initial conditions associated with system (1.3) are given by the function $x(t) = \varphi(t), t \in [-\tau, 0]$, where $\varphi(t) = (\varphi_1, \varphi_2, \dots, \varphi_n)^T$, $\varphi_i(t): [-\tau, 0] \to (0, +\infty)$, $i = 1,2,\dots,n$, are continuous with the norm $\|\varphi\| = \sup_{-\tau\le t\le 0}(\sum_{i=1}^{n}|\varphi_i(t)|^r)^{1/r}$, where $r > 1$ is a constant.

Definition 2.1. A function $x(t): [-\tau, \alpha) \to R^n, \alpha > 0$ is said to be a solution of system (1.3), if

(i) $x(t) = \varphi(t)$ for $-\tau \le t \le 0$;
(ii) $x(t)$ satisfies system (1.3) for $t \ge 0$;
(iii) $x(t)$ is continuous everywhere except for some t_k and left continuous at $t = t_k$, and the right limit $x(t_k^+)$ exist for $k = 1,2,\dots$.

Definition 2.2. A solution $x(t)$ of (1.3) is said to be ω-anti-periodic solution of (1.3), if

$$\begin{cases} x(t+\omega) = -x(t), & t \ne t_k; \\ x((t_k+\omega)^+) = -x(t_k^+), & k = 1,2,\dots, \end{cases}$$

where the smallest positive number ω is called the anti-periodic of function $x(t)$.

Definition 2.3. Let $x(t) = (x_1, x_2, \dots, x_n)^T \in R^n$ be an ω-anti-periodic solution of system (1.3) with initial value $\varphi(t) = (\varphi_1, \varphi_2, \dots, \varphi_n)^T$, $\varphi_i(t): [-\tau, 0] \to (0, +\infty)$, $i = 1,2,\dots,n$. If there exist constants $\lambda > 0$ and $M > 1$ such that for every solution $\overline{x}(t) = (\overline{x}_1, \overline{x}_2, \dots, \overline{x}_n)^T \in R^n$ of system (1.3) with any initial value $\overline{\varphi}(t) = (\overline{\varphi}_1, \overline{\varphi}_2, \dots, \overline{\varphi}_n)^T$, $\overline{\varphi}_i(t): [-\tau, 0] \to (0, +\infty)$, $i = 1,2,\dots,n$

$$|x_i(t) - \overline{x}_i(t)| \le M\|\varphi - \overline{\varphi}\|e^{-\lambda t},\quad \forall\ t > 0,\ i = 1,2,\dots,n,$$

where $\|\varphi - \overline{\varphi}\| = \sup_{-\tau\le s\le 0}\max_{1\le i\le n}|\varphi_i(s) - \overline{\varphi}_i(s)|$. Then $x(t)$ is said to be globally exponentially stable.

Let $PC(R^n) = \{x = (x_1, x_2, \dots, x_n)^T: R \to R^n, x|_{(t_k, t_{k+1}]} \in C((t_k,\ t_{k+1}], R^n),\ x(t_k^+)$ and $x(t_k)$ exist and $x(t_k^-) = x(t_k), k = 1,2,\dots\}$. Set $X = \{x: x \in PC(R^n), x(t+\omega) = -x(t), x((t_k+\omega)^+) = -x(t_k^+), t \in R\}$. Then X is a Banach space with the norm $\|x\| = \sup_{0\le t\le\omega}(\sum_{i=1}^{n}|x_i(t)|^r)^{1/r}$.

The proof of the following lemma is similar to [29], for the completeness, we list it as follows.

Lemma 2.1. *Let $x = (x_1, x_2, \dots, x_n)^T$ be an ω-anti-periodic solution of system (1.3). Then*

$$\begin{aligned} x_i(t) = &\int_t^{t+\omega} G_i(t,s)\left[\sum_{j=1}^{n} a_{ij}(s)g_j(x_j(s-\tau_{ij}(s)))\right. \\ &\left.+ \sum_{j=1}^{n}\sum_{l=1}^{n} b_{ijl}(t)g_j(x_j(s-\sigma_{ijl}(s)))g_l(x_l(s-\upsilon_{ijl}(s))) + I_i(s)\right]ds \\ &+ \sum_{t\le t_k<t+\omega} G_i(t,t_k)I_{ik}(t_k, x_i(t_k)), \\ &i = 1,2,\dots,n, \end{aligned} \tag{2.2}$$

where

$$G_i(t,s) = -\exp\left(\int_t^s c_i(\theta)\,d\theta\right)/(1+\exp\left(\int_0^\omega c_i(\theta)\,d\theta\right),\ s \in [t, t+\omega],$$

$i = 1,2,\dots,n.$