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Novel stability criteria for recurrent neural networks with time-varying delay ☆

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ABSTRACT

This paper is concerned with the problem of stability analysis of recurrent neural networks with time-varying delay. An augmented Lyapunov–Krasovskii functional containing a triple integral term and considering more information of activation functions is constructed. Then, Wirtinger-based inequality and two zero-value free-weighting matrix equations are used to deal with the derivative of the Lyapunov–Krasovskii functional. Those treatments lead to less conservatism. A numerical example is given to verify the effectiveness and benefit of the proposed criteria.

1. Introduction

In recent years, neural networks have been frequently applied into various fields, such as pattern recognition, signal processing, associative memories and other scientific areas [1]. As we all know, a time delay which may cause oscillation even instability inevitably exists in any system. Therefore, the stability problem of the delayed neural networks (DNNs) has attracted a lot of attention [2–4].

The stability criteria of the DNNs can be classified into a delay-dependent one and a delay-independent one [5,6]. As the former always has less conservatism, we consider the delay-dependent stability of recurrent neural networks in this paper. Based on the Lyapunov theory, there are two key points to reduce the conservatism in this field, one is the construction of a suitable Lyapunov–Krasovskii functional (LKF) and the other is the estimation of its derivative.

For the construction of the LKFs, the simple LKF was firstly employed to investigate the stability of DNNs, and rich results have been reported [7–9]. Especially, an efficient delay-dependent stability criterion was established in [22] for the recurrent neural networks with a time-varying delay. However, the stability criteria are very conservative by using the simple LKFs as the delay information is not fully taken into account in the LKFs. To reduce the conservatism, some new techniques, such as using the delay-decomposition idea [11,12], introducing the triple integral terms [18,20], considering more information of the activation functions [21] and augmenting the terms in the simple LKFs [19], were employed to construct the LKFs. The delay–partitioning approach considers the delay information by dividing the delay interval into smaller subintervals and studying the stability based on the subintervals, while, other methods add more terms considering delay information to the LKF to reduce the conservatism in a different way. As a result, they are also adopted to investigate the stability of the recurrent neural networks with a time-varying delay. For example, a complete quadratic LKF augmenting all the terms of the simple LKF and a LKF including two triple integral terms have been given in [23,24], respectively. Unfortunately, there are few results combining these terms together to study the stability of the recurrent neural networks with a time-varying delay.

On the other hand, for the derivative of the LKFs, the works focus on the estimation of the integral terms. As we know, the free weighting matrix (FWM) approach [14–16] and the integral inequality method [10,11] are the most popular methods reported in the literature. In addition, equivalent conditions can be obtained for the systems with a time-invariant delay. However, it is not easy to handle the case of a time-varying delay by using the latter method. As a result, the convex combination approach [13,17–20] was presented to avoid this limitation. Combining with the above techniques, many results have been derived for the recurrent neural networks with a time-varying delay. For example, delay-dependent

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stability criteria were derived following the FWM approach in [22] and Jensen's inequality combined with the convex combination method in [24]. However, as reported in [28], there is a gap encompassing Jensen's inequality and a new Wirtinger-based integral inequality was introduced. As the Wirtinger-based integral inequality contains Jensen's inequality, it can be desired to derive the improved criteria by using this inequality to estimate the derivative of the LKFs for the recurrent neural networks with a time-varying delay.

In this paper, a new augmented LKF containing a triple integral term and considering more information of the activation functions is proposed. Less conservative stability conditions are established by adopting Jensen's inequality with a convex combination approach, Wirtinger-based integral inequality and two zero-value FWM equations to estimate the derivative of the constructed LKF. Finally, a numerical example is given to show the effectiveness and benefit of the proposed method.

Notations: Throughout this paper, the superscripts T and -1 mean the transpose and the inverse of a matrix, respectively; $\mathcal{R}^n$ denotes the n-dimensional Euclidean space; $\mathcal{R}^{n\times m}$ is the set of all $n\times m$ real matrices; $P>0$ (≥ 0) means that P is a real symmetric and positive-definite (semi-positive-definite) matrix; $\mathrm{diag}\{\cdots\}$ denotes a block-diagonal matrix; symmetric term in a symmetric matrix is denoted by $\star$; and $\mathrm{Sym}\{X\}=X+X^T$.

2. Problem formulation

Consider the following recurrent neural networks with a time-varying delay:

$$\dot{y}(t)=-Ay(t)+g(Wy(t-d(t))+J) \tag{1}$$

where $y(\cdot)=[y_1(\cdot)\ y_2(\cdot)\ \cdots\ y_n(\cdot)]^T$ is the state vector; $g(\cdot)=[g_1(\cdot)\ g_2(\cdot)\ \cdots\ g_n(\cdot))]^T$ represents the neutron activation function; $A=\mathrm{diag}\{a_1,a_2,\ldots,a_n\}>0$ with $a_i>0$, $i=1,2,\ldots,n$; $W=[W_1\ W_2\ \cdots\ W_n]^T$ is the connection weight matrix; $J=[J_1\ J_2\ \cdots\ J_n]^T$ is a vector representing the bias; and $d(t)$ is a time-varying delay satisfying

$$0\leq d(t)\leq h,\quad \mu_1\leq \dot{d}(t)\leq \mu_2 \tag{2}$$

where h, μ_1 and μ_2 are known constants.

The neuron activation function $g(\cdot)$ is assumed to be bounded and satisfies the following condition:

$$l_i^-\leq \frac{g_i(u)-g_i(v)}{u-v}\leq l_i^+,\quad u\neq v,\ i=1,2,\ldots,n \tag{3}$$

where l_i^- and l_i^+ are known real constants.

Based on the assumption on the activation function, there exists an equilibrium point y^* for the neural network. Using transformation $x(t)=y(t)-y^*$, one can shift the equilibrium point y^* of (1) to the origin and rewrite system (1) as

$$\dot{x}(t)=-Ax(t)+f(Wx(t-d(t))) \tag{4}$$

where $f(\cdot)=[f_1(\cdot)\ f_2(\cdot)\ \cdots\ f_n(\cdot)]^T$ and $f(Wx(\cdot))=g(Wx(\cdot)+y^*+J)-g(Wy^*+J)$ with $f_i(0)=0$. Thus, it follows from (3) and $f_i(0)=0$ that

$$l_i^-\leq \frac{f_i(s_1)-f_i(s_2)}{s_1-s_2}\leq l_i^+,\quad s_1\neq s_2 \tag{5}$$

$$l_i^-\leq \frac{f_i(s)}{s}\leq l_i^+,\quad s\neq 0 \tag{6}$$

The following lemmas to establish the main results are introduced at first.

Lemma 1 (*Jensen's inequality, Gu et al. [25], Sun et al. [26]*). *For any matrix $R\in\mathcal{R}^{n\times n}$, $R=R^T>0$, scalars $\beta<\alpha$, vector $\omega:[\beta,\alpha]\mapsto\mathcal{R}^n$ such that the integration concerned is well defined, then*

$$(\alpha-\beta)\int_\beta^\alpha \omega^T(s)R\omega(s)\,ds\geq\left(\int_\beta^\alpha\omega(s)\,ds\right)^T R\left(\int_\beta^\alpha\omega(s)\,ds\right) \tag{7}$$

$$\begin{aligned}&\frac{(\alpha-\beta)^2}{2}\int_\beta^\alpha\int_s^\alpha \omega^T(s)R\omega(s)\,ds\,d\theta\\&\geq\left(\int_\beta^\alpha\int_s^\alpha\omega(s)\,ds\,d\theta\right)^T R\left(\int_\beta^\alpha\int_s^\alpha\omega(s)\,ds\,d\theta\right)\end{aligned} \tag{8}$$

Lemma 2 (*Wirtinger-based inequality, Seuret and Gouaisbaut [28]*). *For any matrix $R\in\mathcal{R}^{n\times n}$, $R=R^T>0$, any differentiable function ω in $[a,b]\to\mathcal{R}^n$, the following inequality holds:*

$$\int_a^b\dot{\omega}^T(s)R\dot{\omega}(s)\,ds\geq\frac{\varsigma^T\left[W_1^TRW_1+\pi^2W_2^TRW_2\right]\varsigma}{b-a} \tag{9}$$

where $\varsigma=[\omega^T(b),\omega^T(a),\int_a^b\omega^T(s)/(b-a)\,ds]^T$, $W_1=[I\ -I\ 0]$, $W_2=[I/2\ \ I/2\ -I]$.

Lemma 3 (*Reciprocally convex combination lemma, Park et al. [27]*). *Let $p_1,p_2,\ldots,p_M:\mathcal{R}^m\mapsto\mathcal{R}$ have positive values in an open subset D of R^m, and $\alpha_i>0,\sum_{i=1}^M\alpha_i=1$, then a reciprocally convex combination of p_i over D satisfies*

$$\sum_{i=1}^M\frac{p_i(t)}{\alpha_i}\geq\sum_{i=1}^M p_i(t)+\sum_{i=1}^M\sum_{j=1;j\neq i}^M q_{i,j}(t)$$

subject to

$$\left\{q_{i,j}:R^M\mapsto R,q_{i,j}(t)\equiv q_{j,i}(t),\begin{bmatrix}p_i(t)&q_{i,j}(t)\\q_{j,i}(t)&p_j(t)\end{bmatrix}\geq 0\right\} \tag{10}$$

3. Main results

New stability criteria will be derived in this section. At first, some matrices will be defined for simplicity. $e_i\in\mathcal{R}^{12n\times n}$, $i=1,2,\ldots,12$, and $e_{13}=e_{12}$ are defined as block entry matrix (For example, $e_3^T=[0\ 0\ I\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0\ 0]$).

Now, we give a less conservative delay-dependent stability criterion by introducing a triple integral term in the LKF and using the new techniques such as the Wirtinger-based inequality and two zero-value FWM equations.

Theorem 1. *For the given scalars h, μ_1 and μ_2, and diagonal matrices $L^-=\mathrm{diag}\{l_1^-,l_2^-,\ldots,l_n^-\}$ and $L^+=\mathrm{diag}\{l_1^+,l_2^+,\ldots,l_n^+\}$, system (4) with a time-varying delay satisfying (2) and the activation function satisfying (3) are asymptotically stable if there exist positive-definite symmetric matrices $R\in\mathcal{R}^{5n\times 5n}$, $N\in\mathcal{R}^{3n\times 3n}$, $Q_i\in\mathcal{R}^{3n\times 3n}, i=1,2$, $G\in\mathcal{R}^{3n\times 3n}$, $Q_3\in\mathcal{R}^{n\times n}$, $U\in\mathcal{R}^{3n\times 3n}$, positive diagonal matrices $\Lambda_i\in\mathcal{R}^{n\times n}$, $\Delta_i\in\mathcal{R}^{n\times n}, i=1,2,3$, $H_i\in\mathcal{R}^{n\times n}, i=1,2,\ldots,6$, symmetric matrices $P_i\in\mathcal{R}^{n\times n}, i=1,2$, and any matrices $S_i\in\mathcal{R}^{n\times n}, i=1,2$, $T\in\mathcal{R}^{12n\times n}$ satisfying the following LMIs:*

$$\Sigma_1+\Omega+h\Sigma_2<0 \tag{11}$$

$$\Sigma_1+\Omega+h\Sigma_3<0 \tag{12}$$

$$\begin{bmatrix}G_{22}&S_1\\\star&G_{22}\end{bmatrix}>0 \tag{13}$$

$$\begin{bmatrix}G_{22}&S_2\\\star&G_{22}\end{bmatrix}>0 \tag{14}$$

$$\Sigma_4\geq 0 \tag{15}$$

$$\Sigma_5\geq 0 \tag{16}$$