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An empirical evaluation of Gravitational Swarm Intelligence for graph coloring algorithm

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ABSTRACT

This paper gives an extensive empirical evaluation of the innovative nature inspired Gravitational Swarm Intelligence (GSI) algorithm solving the Graph Coloring Problem (GCP). GSI follows the Swarm Intelligence problem solving approach, where the spatial position of agents is interpreted as problem solution and agent motion is determined solely by local information, avoiding any central control system. To apply GSI to search for solutions of GCP, we map agents to graph's nodes. Agents move as particles in the gravitational field defined by goal objects corresponding to colors. When the agents fall in the gravitational well of the color goal, their corresponding nodes are colored by this color. Graph's connectivity is mapped into a repulsive force between agents corresponding to adjacent nodes. We discuss the convergence of the algorithm, testing it over an extensive suite of well-known benchmarking graphs. Comparison of this approach to state-of-the-art approaches in the literature shows improvements in many of the benchmark graphs.

1. Introduction

The Graph Coloring Problem (GCP) is a classical combinatorial NP-complete optimization problem [1–5]. The GCP consists in assigning a color to the nodes of a graph with the restriction that any pair of adjacent nodes cannot have the same color. The chromatic number K is the minimum number of colors needed to color the graph. Classical search algorithms to solve GCP are deterministic [6–8]. Heuristics and random search allow to obtain approximations to the optimal solutions in bounded time. Some recent approaches have applied Ant Colony Optimization (ACO) [9], Particle Swarm Optimization (PSO) [10], more accurate PSO approaches like Liu et al. [11], or hybrid PSO like Sun et al. [12], Simulated Annealing [2] and Reynolds' Boid swarms [13,14]. The most famous algorithm giving an upper bound of the chromatic number is the DSATUR, a fast and very accurate algorithm, though suboptimal because it does not guarantee to return the chromatic number. Tabu search has been the foundation for several graph coloring approaches, such as the bare application of [15,16], or hybridized with other methods like Genetic Algorithms [17] or Swarm Intelligence methods [18].

We consider a nature inspired strategy to solve this problem following a Swarm Intelligence (SI) [19] approach. The bee hives [20], ant colonies [21] and flocking birds [22–24] are examples of such swarm computational metaphors. In SI models, the collective behavior emerges from a self-organization process of agents evolving autonomously according to a set of internal rules specifying its motion patterns and interaction with the environment and other agents, such that intelligent collective behavior arises from simple individual behaviors. An important feature of SI is that there is no leader agent or central control. SI allows a high level of scalability, dividing the problem to be solved into small problems, each one solved by an individual agent. This feature also provides SI with robustness against individual failure.

The proposed algorithm is called Gravitational Swarm Intelligence (GSI) [25], which was further tested in [26]. This paper progresses beyond [26] adding an exhaustive experimental assessment of the algorithm over more graph instances of benchmarking classes, while improving the algorithm behavior. In GSI, agents correspond to graph nodes placed in a torus shaped space. Agents are attracted to specific space places (color goals) where the corresponding graph node acquires a color. Such attraction is modeled as a gravitational field reaching the entire space, thus the name of GSI. When a GSI agent reaches a color goal, it remains there unless pushed out by repulsive forces of antagonistic agents. The repulsive relation between GSI agents is determined by the topology of the graph to be colored. Nodes connected by an arc correspond to antagonistic GSI agents exerting mutually repulsive forces. When an agent is impeded to reach any color goal because of these repulsive forces, its "discomfort" grows increasing the force that it can exert on repelling agents to push them out of the

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color goals. This discomfort reaction allows the system to escape local minima that do not correspond to solutions of the GCP. The SI dynamics reach a termination stable state when all the agents are in a color goal and there is no conflict among them.

The rest of the paper is organized as follows: Section 2 presents the Gravitational Swarm Intelligence algorithm. Section 3 gives experimental results comparing the GSI algorithm performance with other methods over well-known benchmarking graphs. Finally, Section 4 gives some conclusions and lines for future work.

2. Gravitational Swarm Intelligence

Following natural inspiration, such as bird flocks [24], ant colonies [27] or bees [28], we focus on the gravitational attraction between objects. We construct a world where agents navigate through the space attracted by the gravitational pull of specific objects, the color goals, and may suffer specific repulsion forces, activated by a friend-or-foe quality of the relation between agents.

Let be $G=(N,E)$ a graph with a set of nodes $N=\{1,\ldots,n\}$ and edges $E\subseteq N\times N$. We define the group of GSI agents $B=\{b_1,b_2,\ldots,b_n\}$, each one corresponding to a graph node navigating inside a square planar toric world. Each GSI agent moves through this space according to a speed vector $\overrightarrow{v}_i$. At any time we know the position of each agent $p_i(t)=(x_i,y_i)$ where x_i and y_i are the cartesian coordinates in the space. When $t=0$ we have the initial position of the agents $p_i(0)=(x_{0i},y_{0i})$. Suppose that we want to color the graph with K colors, so that $C=\{1,2,\ldots,K\}$ is the set of colors. The chromatic number of the graph K^* is the minimum number of colors allowing to color the graph. We assign to these colors, K static points in space, which are the color goals $CG=\{g_1,\ldots,g_K\}$, endowed with a gravitational attraction resulting in a velocity component $\overrightarrow{v}_{gc}$ in the computation of the agents' speed. The attraction force increases with the distance, a trick against Newton's law to increase convergence speed, affecting all the agents in the space.

The agents steer toward the nearest goal by the pull of the gravitational forces. When the Euclidean distance between an agent and a color goal is below a threshold *nearenough*, the agent stops moving and the corresponding node becomes colored. We denote the set of agents whose position is in the region of the space near enough to a color neighborhood of the color as $N(g_k)=\{b_i \text{ s.t. } \|p_i-g_k\|<nearenough\}$. We formalize the node color assignment giving value to an agent's color attribute $b_i\in N(g_k)\Rightarrow c_i=k$. The initial value of the agent color attribute is zero or null. Inside the color goal neighborhood there is no further gravitational attraction. However, there may be a repulsion force from agents whose nodes are adjacent in the graph G. This repulsion is only effective between agents inside the same color goal neighborhood. The repulsive forces experimented by agent b_i inside $N(g_k)$ are computed as follows: $R(b_i,g_k)=\sum_{N(g_k)} repel(b_i,b_j)$. The function *repel* has value 1 if a pair of GSI agents has an edge between them, and 0 otherwise. If $R(b_i,g_k)=0$ then the incoming agent gets the goal color, if not, the agent must wait until the repulsion forces are zero or be expelled.

We can model the GCP problem solving by GSI as a tuple

$$F=(B,CG,\{\overrightarrow{v}_i\},K,\{\overrightarrow{a}_{i,k}\},R) \tag{1}$$

where B is the group of GSI agents, $\{\overrightarrow{v}_i\}$ the set of agent velocity vectors at time instant t, K the hypothesized graph's chromatic number, and $\{\overrightarrow{a}_{i,k}\}$ are the attraction forces the color goals exert on the agents, finally, R denotes the repulsion forces in the neighborhood of color goals. The cost function defined on the global system spatial configuration is

$$f(B,CG)=|\{b_i \text{ s.t. } c_i\in C\wedge R(b_i,g_{c_i})=0\}|. \tag{2}$$

This cost function is the number of graph nodes which have a color assigned suffering no conflict inside the color goal. The agents outside the neighborhood of any color goal cannot be evaluated, so they cannot be a part of the problem's solution. The dimension of the world and the *nearenough* threshold value allow tuning the speed of convergence of the algorithm. If the world is big and the *nearenough* value is small, the algorithm converges slowly but monotonically to the solution. On the other hand, if the world is small and the *nearenough* value is big, the algorithm's convergence is faster but jumpy because the algorithm falls in local minima, needing transitory energy increases to escape them. The reason for this behavior is that the world is not normalized and the magnitude of the velocity vector can be bigger than the color goal spatial influence, so that the agent can walk by a goal neighborhood without falling in it. The dimension of the world can be compensated adjusting the velocity vector of the agents, so reducing the complexity of the algorithm. The dynamics of each GSI agent in the world is specified by the iteration:

$$\overrightarrow{v}_i(t+1)=\begin{cases}0 & c_i\in C\wedge(\lambda_i=1),\\ d\cdot\overrightarrow{a}_{i,k^*} & c_i\notin C,\\ \overrightarrow{v}_r\cdot(p_r-p_i) & (c_i\in C)\wedge(\lambda_i=0),\end{cases} \tag{3}$$

where d is the Euclidean distance between the agent's position p_i and the position of the nearest color goal g_{k^*}, $\overrightarrow{a}_{i,k^*}$ represents the attraction force to approach it, and $\overrightarrow{v}_r$ is a random vector meant to avoid being stuck in spurious unstable equilibrium, toward a random position p_r. Parameter λ_i represents the effect of the degree of $Comfort_i$ of the GSI agent. When a GSI agent b_i reaches a color goal neighborhood at instant t, its velocity becomes 0. Every time step that the GSI agent stays in that color goal neighborhood $N(g_k)$ without being disturbed, its *Comfort* increases, until reaching a maximum value *maxcomfort*. When a GSI agent b_i outside $N(g_k)$ tries to go inside the neighborhood of that color goal, the repulsion force $R(b_i,g_k)$ is evaluated. If the repulsion force is greater than zero then the incoming agent is challenging the stability of the color assignment in $N(g_k)$, and at least one agent must leave $N(g_k)$, which can be the incoming agent itself. The repulsion force is only applied between connected agents. If the $Comfort_i$ values of the challenged agents are bigger than 0, then their *Comfort* decreases. If some $Comfort_i$ reaches 0, then one connected agent is expelled from the color goal toward a random position in space p_r with velocity $\overrightarrow{v}_r$. In Eq. (3) when $Comfort_i$ is positive then $\lambda_i=0$. If the repulsion force is greater than zero and $Comfort_i=0$ of a GSI agent b_i inside that goal, then $\lambda_i=1$ and b_i is expelled from the goal. When all the GSI agents stop, i.e. $\forall i,\overrightarrow{v}_i=0$, then $f(B,CG)=n$, therefore the GCP of assigning K colors to graph G is solved.

Each color goal has an attraction well spanning the entire space, therefore the gravitational analogy. But in our approach the magnitude of the attraction grows proportionally with the Euclidean distance d between the color goal and the GSI agent, and this force disappears when the agent gets inside a color goal. If $\|d\|<nearenough$ then we make $d=0$, and the agent's velocity becomes 0, stopping it. The flowchart of Fig. 1 shows the internal logic guiding the dynamics of each GSI agent. In the flowchart, the colored agents stay in a *Stand By* state until the last uncolored agent is colored. It is not necessary to ask if all the agents have been colored for each agent, because if all the agents are colored then the problem is solved and the cost function has value $f(B,CG)=n$, and the flow diagram reaches the *finish* state.

Convergence: The system as a whole reaches an stationary state only if all the GSI agents' speed becomes zero. Then, the algorithm has converged to some fixed state where all of them are inside a color goal and there is no conflict inside the color goal neighborhoods. If the chromatic number is the hypothesized K or lower,