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Perceptual grouping through competition in coupled oscillator networks

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ABSTRACT

In this paper we present a novel approach to model perceptual grouping based on phase and frequency synchronization in a network of coupled Kuramoto oscillators. Transferring the grouping concept from the Competitive Layer Model (CLM) to a network of Kuramoto oscillators, we preserve the excellent grouping capabilities of the CLM, while dramatically improving the convergence rate, robustness to noise, and computational performance, which is verified in a series of artificial grouping experiments and with real-world data.

1. Introduction

The ability to robustly group related perceptual items to form higher-order concepts is crucial for many cognitive tasks. Exploiting the recurrent dynamics of neurons, the Competitive Layer Model (CLM) [1] has proven to solve a broad spectrum of complex grouping tasks in a very robust fashion – even in the presence of strong noise. Among others, these tasks include segmentation of cell images [2], grouping of object contours in edge images [3], as well as motion segmentation [4]. However, a major drawback of the CLM for real-world applications, is its high demand for computational resources: the network converges slowly and each update step is costly. Furthermore, the network can only hardly escape from a reached optimum, when the underlying grouping dynamics is changed.

Hence, inspired by the fast synchronization ability of coupled oscillator networks [5–7], we transfer the grouping principles of the CLM to a network of Kuramoto oscillators [5] in order to improve the computational performance. In the presented model, each oscillator represents a distinct input feature from an arbitrary feature domain. The coupling strengths between the oscillators are based on the compatibility of the corresponding features. Similar features have a positive compatibility causing the corresponding oscillators to phase-lock and thus form a perceptual group. Conversely, dissimilar features induce negative couplings causing a repelling of oscillator phases.

The Kuramoto model has been investigated in many variations. It was shown that large enough positive couplings between oscillators induce a phase synchronization where the oscillators converge towards a mean phase. This *critical coupling strength* to achieve synchrony has been heavily investigated. For example the authors in [8] derived a lower bound of the required coupling strength and validated their results in simulations. The authors in [9] examined a network having both, positive and negative couplings, and discovered not only that the positively connected proportion of oscillators phase-lock, but also that the remaining negatively coupled oscillators shift into an state of opposing phase.

In [10], the synchronization properties of the Kuramoto model were exploited to find strongly coupled communities in graph structures. By introducing a correlation measurement based on the cosine similarity among the phases of oscillator pairs, they were able to identify community structures by analyzing oscillator correlations over time. Building on this correlation measure, the authors in [11] were able to trace the formation of communities over time in real world graph structures like social networks.

The use of coupled oscillator networks to solve segmentation tasks has been exhaustively studied, with one of the most prominent examples being the LEGION model [12]. This model uses a combination of local excitatory and global inhibitory couplings of relaxation oscillators. To determine clusters of related features, the model also employs the correlation of oscillator phases. The model has been adapted and extended along various dimensions. In [13], the authors use a LEGION based oscillator network to extract the most salient features from input images, whereas [14] extends the model with excitatory and inhibitory shortcuts, relaxing the original coupling topology which overall boosts the synchronization process. Besides the domain of image

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processing, the model was for example successfully used to separate speech from background noise in [15].

Using a network with diffusely connected FitzHugh–Nagumo oscillators, the system presented in [16] was able to solve various clustering and segmentation tasks on artificial datasets and in real-world images. Exploiting chaotic Ro ssler oscillators, Breve et al. [17] were able to increase the discrimination sensitivity of oscillator clusters in order to better distinguish multiple objects within a scene for an object selection task.

In the following sections, we shortly outline the principles of the CLM and introduce our approach to transfer them to a network of coupled Kuramoto oscillators. Particularly, we augment existing models with a frequency dynamics boosting the synchronization process and avoiding the costly phase-correlation analysis to determine oscillator clusters. In Section 4, both approaches are evaluated with artificial data and with regard to the grouping quality, convergence speed and perturbations in the presence of increasing levels of noisy connections. In the ensuing section, we exploit properties of the Kuramoto model to gain insights into the behavior of the oscillator network during grouping tasks. Section 6 presents an extension of the oscillator model, which allows a more robust handling of spurious input features. Subsequently, this extension is evaluated within a texture grouping task and compared to the basic model. Finally, the results are discussed.

2. The CLM for perceptual grouping

The CLM consists of $N \times L$ neurons which are arranged in L layers. Neurons are indexed column wise with $m = 1, \ldots, N$ describing the position in each layer and $\alpha = 1, \ldots, L$ denoting the layer index. A single neuron's activity is therefore denoted as $x_{m,\alpha}$. The neurons in each layer are coupled with a symmetric interaction function $f(v_m, v_n) = f(v_n, v_m) = f_{m,n}$ which describes the compatibility between two features v_m and v_n. They are additionally coupled with a *winner-takes-all* (WTA) circuit in each column to assure that only one neuron in each column becomes active. Combining the lateral interaction and columnar WTA circuit, the recurrent CLM dynamics can be written as

$$\dot{x}_{m,\alpha} = -x_{m,\alpha} + \sigma\left(J\left(1 - \sum_{\beta=1}^{L} x_{m,\beta}\right) + \sum_{n=1}^{N} f_{m,n} x_{n,\alpha}\right). \tag{1}$$

Here $J(1 - \sum_\beta x_{m,\beta})$ represents the WTA competition weighted by the constant J, and $\sigma(x) = \max(0, x)$ is a linear threshold function. The lateral interaction is expressed as $\sum_n f_{m,n} x_{n,\alpha}$, which calculates the support for the feature at position m from all other features n in a given layer α. A graphical representation of the Competitive Layer Model is shown in Fig. 1(a). For a more comprehensive overview, we refer to [3].

3. Transfer to network of coupled Kuramoto oscillators

The oscillator model replaces each CLM column – composed from L neurons representing the grouping result for a given feature v_m – with a single oscillator of the Kuramoto type [5]. The original Kuramoto model describes a set of oscillators O_m, where an oscillator is described by its phase θ_m and a frequency ω_m. In this model, ω_m is drawn from a random distribution and the phase θ_m evolves according to the update equation

$$\dot{\theta}_m = \omega_m + \frac{K}{N} \sum_{n=1}^{N} \sin(\theta_n - \theta_m) \tag{2}$$

where K is a global coupling constant that controls the strength of the phase-locking of oscillators [18]. If K is too small, the network does not synchronize and remains in a chaotic state. Transferring the idea of feature-dependent coupling strengths from the CLM model to the Kuramoto model, we also employ individual coupling strengths determined by the symmetric matrix $M_{m,n} \equiv f(v_m, v_n)$, such that the strength of synchronization of features will correlate with the strength of feature compatibility. This results in the following, slightly adapted phase update rule, also known as a hierarchical Kuramoto model [19]:

$$\dot{\theta}_m = \omega_m + \frac{K}{N} \sum_{n=1}^{N} f(v_m, v_n) \cdot \sin(\theta_n - \theta_m). \tag{3}$$

Notice the similarity of $\sum f \cdot \sin$ to the appropriate sum in Eq. (1). While in the original CLM model the sum measures the support of all neurons $n = 1 \ldots N$ to the neuron m, in the Kuramoto model it measures the drive to adapt the phase of oscillator m based on the weighted phase-asynchrony (the sine term) to all oscillators.

The interaction function f is limited to the interval $[-1, 1]$, where -1 and $+1$ represent strongest dissimilarity and similarity of features respectively. Negative couplings among dissimilar features assure a large phase spread, as pointed out in [9]. Assuming, that several features group into clusters, such that intra-cluster couplings are significantly stronger than inter-cluster couplings, the oscillator network will naturally form corresponding clusters of phase-synchronized oscillators as can be seen from Fig. 9(e). Phases corresponding to different clusters tend to separate as far as possible.

The authors in [10] introduced a correlation measure $\rho_{mn} = \langle \cos(\theta_n - \theta_m) \rangle$ based on the cosine similarity of oscillator phases. Using this measure, they were able to trace the formation of clusters, also called communities, of similar nodes in graphs composed of Kuramoto oscillators. However, due to a continuous drift – induced by the oscillator frequencies ω_m – an evaluation and tracking of the grouping result over time is costly and prone to noise in the input data. Also, a threshold would be required to decide whether the correlation among oscillators is strong enough to be interpreted as a cluster. This threshold has to be chosen depending on the grouping task.

To overcome this drawback, we augment the phase dynamics by a frequency dynamics limiting oscillator frequencies to discrete values (inspired by the *layered* CLM architecture), thus facilitating the evaluation of the grouping result. More concretely, oscillator frequencies are limited to discrete values $\omega_\alpha = \alpha \cdot \omega_0$, where $\alpha \in \{1, \ldots, L\}$ denotes the group index – following the CLM notation where α denotes the group/layer index. To achieve an association of similar features to the same discrete frequency level ω_α, the frequency ω_m of each oscillator is adapted employing the cosine similarity between the phases of the oscillators, which is mapped to the interval $[0, 1]$ to preserve the sign of the coupling strengths $f(v_m, v_n)$. The oscillator is then assigned to the frequency from which it gains the most support $S_m(\alpha)$. Hence, the frequencies are updated according to:

$$\begin{aligned} S_m(\alpha) &= \sum_{n \in \mathcal{N}(\alpha)} f_{m,n} \cdot \frac{1}{2}(\cos(\theta_n - \theta_m) + 1) \\ \omega_m &= \omega_0 \cdot \underset{\alpha}{\operatorname{argmax}}(S_m(\alpha)) \end{aligned} \tag{4}$$

Here, $\mathcal{N}(\alpha)$ denotes the set of oscillators with frequency index α, i.e. forming the current perceptual group indexed by α. This update rule associates the oscillator m to the group $\mathcal{N}(\alpha)$ of oscillators, which most strongly supports the mth oscillator (based on strength-weighted synchrony of phases). Hence, it ensures that oscillators representing similar features will both phase-lock and frequency-lock. The grouping result is then readily read from the assigned indices α. Eq. (4) also boosts the phase-locking process, because synchronized phases do not tend to desynchronize anymore due to randomly assigned frequencies. Contrarily, oscillators representing dissimilar features will spread both in phase and frequency.