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Robust synchronization of coupled neural networks with mixed delays and uncertain parameters by intermittent pinning control

Cong Zheng [a,b], Jinde Cao [a,b,c,*]

[a] Research Center for Complex Systems and Network Sciences, Southeast University, Nanjing 210096, China
[b] Department of Mathematics, Southeast University, Nanjing 210096, China
[c] Department of Mathematics, King Abdulaziz University, Jeddah 21589, Saudi Arabia

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ABSTRACT

This paper investigates robust synchronization problem for the coupled neural networks with mixed delays and uncertain parameters. By utilizing the intermittent pinning control idea, a novel controller is designed to pin the coupled networks to reach the synchronization state. Some sufficient criteria are derived in matrix inequalities form by resorting to the generalized Halanay inequality, which guarantee the subnetworks synchronizing exponentially. Two numerical examples are finally exploited to show the effectiveness of the obtained results.

1. Introduction

Neural networks have been extensively studied in various fields because of their broad applications such as image processing, solving certain optimization problems, and detecting speed of moving objects. In reality, many neural networks complete a task through interaction or communication with each other. Hence, neural networks with coupling have attracted more and more attention from researches in different fields and have become a focus topic with wide applications, in which synchronization of coupled networks is an important phenomenon. For example, a lot of neural networks rely on a synchronous behavior for a proper functioning, such as pattern recognition, information transmission, and learning [1,2]. Moreover, neural networks with linear coupling are easy to physically implement and hence have promising application especially in secure communications based on synchronization [3,4].

Time delays are ubiquitous in many biological systems because of a variety of axon sizes and lengths and the finite signal propagation time, and they may cause instability and oscillation, etc. Hence, time delays will be introduced into the models of neural networks. Time delays have many types such as discrete delays, time-varying delays and distributed delays. Especially, distributed delays have received many research attention. The main reason is that continuously distributed delays on a certain duration of time can model the spatial nature of neural networks, such that the distant past has less influence compared to the recent behavior of the state [5]. Hence, both discrete delays and distributed delays should be considered in the models of neural networks [6–9]. On the other hand, parameter uncertainties are unavoidable when modeling and implementing real neural networks due to measure errors, the parameter fluctuation and external disturbance, etc. Therefore, parametric uncertainties will also be introduced into the models of neural networks [10,11]. In this paper, we consider neural networks with mixed delays and uncertainty parameters.

Some coupled systems can synchronize by themselves, but others cannot attain synchronization by themselves. In this case, some controllers need designing and are applied to force the systems to synchronize. However, it is impossible to add controllers to all nodes in a large-scale network. To reduce the number of controlled nodes, some local feedback injections may be applied to a fraction of network nodes, which is known as pinning control [4,12–17]. Chen et al. proposed a fundamental work for pinning control of complex dynamical networks in [13], in which a single controller is designed for the pinning control of connected networks if all the eigenvalues of the controlled coupling matrix are negative. Yu et al. [17] investigated synchronization via pinning control on general complex dynamical networks. Moreover, it is

* Corresponding author at: Department of Mathematics, Southeast University, Nanjing 210096, China. Tel.: +86 25 83792315; fax: +86 25 83792316.
E-mail addresses: zhengconglly@gmail.com (C. Zheng), jdcao@seu.edu.cn (J. Cao).

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difficult to manipulate for controlling continuously a system and it need high cost. An interesting intermittent control was introduced and studied [18–28], that is, the control time is periodic, and in any period the time is composed of work time and rest time. It is an engineering approach that has been widely used in engineering fields, such as manufacturing, transportation, and communication. Zochowski [18] introduced intermittent control to control nonlinear dynamical systems. In [19,20], the stabilization problems of chaotic systems with or without delays by periodically intermittent control were discussed. In [25], the authors discussed synchronize coupled delayed dynamical networks using pinning control and intermittent control, in which delay is only the discrete constant delay. In [28], Yuan et al. discussed the synchronization of coupled networks with mixed delays by employing Lyapunov functional method and intermittent control.

Motivated by the above discussions, in this paper, by using pinning control and intermittent control, we study robust synchronization of the coupled neural networks with mixed delays and uncertain parameters. Some sufficient conditions are derived to ensure robust exponential synchronization of the presented coupled neural network. The obtained results extend the existed ones. Finally, two numerical examples are worked out to illustrate the effectiveness of the obtained results. The main contributions of this paper lie in three aspects: (i) mixed delays contain discrete delay and distributed delay; (ii) our model of coupled neural network constants uncertain parameters; (iii) by using pinning control and intermittent control, we give some sufficient conditions to make the coupled neural network synchronizing.

The rest of this paper is organized as follows. In Section 2, the discussed model is given and some preliminaries are briefly outlined. In Section 3, some criteria are derived for the robust exponential synchronization of the proposed coupled neural network by pinning intermittent control. In Section 4, two numerical examples are provided to show the effectiveness of the obtained results. Some conclusions are finally drawn in Section 5.

2. Preliminaries

Consider a coupled neural network with mixed delays and uncertain parameters as follows:

$$\dot{x}_i(t) = -(D+\Delta D(t))x_i(t)+(A+\Delta A(t))f(x_i(t))+(B+\Delta B(t))g(x_i(t-\tau))+(E+\Delta E(t)) \times \int_{t-\tau}^{t} h(x_i(\nu))\,d\nu + J(t) + c\sum_{j=1}^{N} G_{ij}\Gamma x_j(t)+u_i(t), \quad i=1,2,\ldots,N, \tag{1}$$

where $x_i(t)=(x_{i1}(t),x_{i2}(t),\ldots,x_{in}(t))^T \in \mathbb{R}^n$ represents the state vector of the ith neural network at time t; $D=diag\{d_1,d_2,\ldots,d_n\}>0$; $A=(a_{jk})_{n\times n}$, $B=(b_{jk})_{n\times n}$ and $E=(e_{jk})_{n\times n}$ are the connection synaptic matrix, the discretely delayed synaptic weight matrix and the distributively delayed synaptic weight matrix, respectively; $\Delta D(t)$, $\Delta A(t)$, $\Delta B(t)$ and $\Delta E(t)$ represent the time varying parameter uncertainties; $f(\cdot)$, $g(\cdot)$, $h(\cdot):\mathbb{R}^n\times\mathbb{R}^n$ are activation functions, and $f(x_i(t))=(f_1(x_{i1}(t)),f_2(x_{i2}(t)),\ldots,f_n(x_{in}(t)))^T$, $g(x_i(t-\tau))=(g_1(x_{i1}(t-\tau)),g_2(x_{i2}(t-\tau)),\ldots,g_n(x_{in}(t-\tau)))^T$, $h(x_i(\nu))=(h_1(x_{i1}(\nu)),h_2(x_{i2}(\nu)),\ldots,h_n(x_{in}(\nu)))^T$; $J(t)=(J_1(t),J_2(t),\ldots,J_n(t))^T$ is the input vector of each neural network. The dynamics of the isolated neural network can be written as follows:

$$\dot{s}(t) = -(D+\Delta D(t))s(t)+(A+\Delta A(t))f(s(t))+(B+\Delta B(t))g(s(t-\tau)) +(E+\Delta E(t))\int_{t-\tau}^{t} h(s(\nu))\,d\nu+J(t). \tag{2}$$

The constant $c>0$ denotes the coupling strength; Γ is the inner coupling matrix between neural networks; $G=(G_{ij})_{N\times N}$ denotes the outer coupling matrix, the elements of G are defined as follows: $G_{ij}>0$ $(i\neq j)$ if there is a connection from neural network j to neural network i; otherwise, $G_{ij}=0$, $G_{ii}=-\sum_{j=1,j\neq i}^{N} G_{ij}$, $i=1,2,\ldots,N$.

In this paper, we want to control the system (1) such that the states $x_i(t)$ $(i=1,2,\ldots,N)$ can synchronize to the state $s(t)$, which satisfies Eq. (2) and may be an equilibrium point, a periodic solution or a chaotic attractor.

In order to realize the synchronization, we use the intermittent pinning control strategy. Without loss of generality, the first l neural networks are controlled, the controllers $u_i(t)$, $1\le i\le N$, can be described by

$$u_i(t)=\begin{cases} -k_i(x_i(t)-s(t)), & 1\le i\le l, \quad t\in[mT,mT+\delta];\\ 0, & l+1\le i\le N, \quad t\in[mT,mT+\delta];\\ 0, & l\le i\le N, \quad t\in(mT+\delta,(m+1)T),\end{cases} \tag{3}$$

where $k_i>0$ is the control gain, $T>0$ is the control period, $0<\delta<T$ denotes the control width. Let $e_i(t)=x_i(t)-s(t)$, then the following error dynamical system is obtained:

$$\begin{cases} \dot{e}_i(t) = -(D+\Delta D(t))e_i(t)+(A+\Delta A(t))[f(x_i(t))-f(s(t))]+(B+\Delta B(t)) \\ \quad\times[g(x_i(t-\tau))-g(s(t-\tau))]+(E+\Delta E(t))\int_{t-\tau}^{t}[h(x_i(\nu))-h(s(\nu))]\,d\nu \\ \quad+c\sum_{j=1}^{N}G_{ij}\Gamma e_j(t)-k_ie_i(t), \quad 1\le i\le l, \ t\in[mT,mT+\delta]; \\ \dot{e}_i(t) = -(D+\Delta D(t))e_i(t)+(A+\Delta A(t))[f(x_i(t))-f(s(t))]+(B+\Delta B(t)) \\ \quad\times[g(x_i(t-\tau))-g(s(t-\tau))]+(E+\Delta E(t))\int_{t-\tau}^{t}[h(x_i(\nu))-h(s(\nu))]\,d\nu \\ \quad+c\sum_{j=1}^{N}G_{ij}\Gamma e_j(t), \quad l+1\le i\le N, \ t\in[mT,mT+\delta]; \\ \dot{e}_i(t) = -(D+\Delta D(t))e_i(t)+(A+\Delta A(t))[f(x_i(t))-f(s(t))]+(B+\Delta B(t)) \\ \quad\times[g(x_i(t-\tau))-g(s(t-\tau))]+(E+\Delta E(t))\int_{t-\tau}^{t}[h(x_i(\nu))-h(s(\nu))]\,d\nu \\ \quad+c\sum_{j=1}^{N}G_{ij}\Gamma e_j(t), \quad 1\le i\le N, \ t\in(mT+\delta,(m+1)T). \end{cases} \tag{4}$$

Our objective is to design suitable T, δ and k_i such that the system (4) is exponential stable, i.e., the states $x_i(t)$ can synchronize to $s(t)$.

In addition, the following assumptions, definition and lemmas are needed.

Assumption (H1). For activation functions $f(\cdot)$, $g(\cdot)$, $h(\cdot)$ are Lipschitz continuous, that is, there exist constants $l_f>0, l_g>0$, $l_h>0$ such that

$$\|f(x)-f(y)\|\le l_f\|x-y\|, \quad \|g(x)-g(y)\|\le l_g\|x-y\|, \quad \|h(x)-h(y)\|\le l_h\|x-y\|$$

hold for any $x,y\in\mathbb{R}^n$.

Assumption (H2). The parametric uncertainties $\Delta D(t)$, $\Delta A(t)$, $\Delta B(t)$ and $\Delta E(t)$ are of the forms

$$[\Delta D(t)\ \Delta A(t),\Delta B(t),\Delta E(t)]=MF(t)[H_D,H_A,H_B,H_E],$$

where M,H_D,H_A,H_B,H_E are known real constant matrices with appropriate dimensions and the uncertain matrix $F(t)$ is unknown real time vary matrix satisfying $F^T(t)F(t)\le I$.

Definition 1. The neural network (1) is said to be robustly globally exponentially synchronized if there exist constants $\varepsilon>0$, $\overline{T}$ and $M>0$, such that

$$\|e(t)\|\le Me^{-\varepsilon t}, \quad t>\overline{T}$$

holds for all uncertain parameters satisfying assumption (H2) and any initial conditions, where $e(t)=(e_1^T(t),e_2^T(t),\ldots,e_N^T(t))^T$.

Lemma 1 (*Xu et al. [29]*). *For any vectors x, $y\in\mathbb{R}^n$, and positive-definite matrix $Q\in\mathbb{R}^{n\times n}$, the following inequality holds:*

$$2x^Ty\le x^TQx+y^TQ^{-1}y.$$

Lemma 2 (*Halanay [31], Jensen's inequality*). *For any positive definite symmetrical matrix $M\in\mathbb{R}^{m\times m}$, positive constant $r>0$ and vector function $u(\cdot):[0,r]\to\mathbb{R}^m$ such that the integrations concerned*