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Finite-time boundedness for uncertain discrete neural networks with time-delays and Markovian jumps

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ABSTRACT

This paper is concerned with stochastic finite-time boundedness analysis for a class of uncertain discrete-time neural networks with Markovian jump parameters and time-delays. The concepts of stochastic finite-time stability and stochastic finite-time boundedness are first given for neural networks. Then, applying the Lyapunov approach and the linear matrix inequality technique, sufficient criteria on stochastic finite-time boundedness are provided for the class of nominal or uncertain discrete-time neural networks with Markovian jump parameters and time-delays. It is shown that the derived conditions are characterized in terms of the solution to these linear matrix inequalities. Finally, numerical examples are included to illustrate the validity of the presented results.

1. Introduction

Neural networks have received considerable attention due to a variety of applications, such as signal processing, system recognition, target tracking, static image processing and associative memory [1,2]. Meanwhile, the study on time-delay neural networks has also received great attention since time-delay is an inherent feature of many physical processes, such as chemical processes, nuclear reactors and biological systems, and may lead to instability or significantly deteriorated performances for the corresponding closed-loop systems [3–5]. Many results have been investigated and studied for various types of neural networks with time-delays and parameter uncertainties, such as in [6–8] for a continuous-time case and in [9–12] for a discrete-time case. More detailed results on neural networks could be found in [13–16] and the references therein.

On the other hand, Markovian jump systems were referred to as a special family of hybrid systems and stochastic systems, which can be applied to describe the real-world systems subject to random changes in structure and parameters, possibly caused by phenomena such as component failures, sudden environmental disturbances, changing subsystem interconnections, and so forth; see, e.g., [17–22]. As a special class of hybrid neural networks, Markovian jump neural networks with time-delays have attracted a lot of research interests in mathematics and control communities, and many attracting results have been reported in the literature. For instance, stability analysis was considered in [23,24]. Passivity analysis was discussed in [25], and state estimation was addressed in [26,27]. It is worth pointing out that classical control theory focuses mainly on the asymptotic behavior of the systems, which, as mentioned above, deals with the asymptotic property of system trajectories over an infinite-time interval and does not usually specify bounds on the trajectories. In practice, however, many concerns are practical problems in which the described system state does not exceed a certain threshold over a given finite-time interval. In order to handle the transient performance of control systems, finite-time stability or short-time stability was introduced in [28]. With the help of the Lyapunov function approach and linear matrix inequality (LMI) techniques, varieties of results on finite-time stability, finite-time boundedness and finite-time stabilization were obtained for continuous- or discrete-time systems. For instance, the finite-time control problem of discrete-time linear systems was addressed in [29]. The authors in [30] studied the finite-time H_∞ filtering problem for discrete-time Markovian jump systems. In [31,32], the problem of finite-time

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boundedness was considered for neural networks with Markovian jumping parameters. For more details of the literature related to finite-time stability, finite-time boundedness and finite-time H_∞ control, the reader is referred to [33–38].

However, to date and to the best of our knowledge, finite-time boundedness analysis of discrete-time time-delay neural networks with Markovian jumps has not yet been investigated in the literature. The major contributions of this paper are as follows: (i) by applying the Lyapunov function approach and the LMI technique, the stochastic finite-time boundedness analysis is presented for discrete-time neural networks with Markovian jumps and time-varying delays; (ii) when norm-bounded parameter uncertainties appear in discrete-time delayed neural networks with Markovian jumps, the robust stochastic finite-time boundedness criteria are also obtained in terms of LMIs by using the techniques of the matrix decomposition; (iii) as a special case, we also derive sufficient conditions of stochastic finite-time boundedness for uncertain discrete-time neural networks with Markovian jumps and constant time-delays. Therefore, the main aim of this paper is to make the first attempt to tackle the listed contributions.

In this paper, the problem of stochastic finite-time boundedness is investigated for a class of discrete-time Markovian jump neural networks with time-delays. The concepts of stochastic finite-time stability and stochastic finite-time boundedness are first given for neural networks. Then, sufficient criteria on stochastic finite-time boundedness are derived for the class of nominal or uncertain discrete-time neural networks with Markovian jump parameters and time-delays. The conditions are reduced to LMIs-based feasibility problems. Finally, numerical examples are presented to show the effectiveness of the proposed results. The rest of this paper is organized as follows. Section 2 provides the problem statement and preliminaries. The main results are provided in Section 3. Numerical examples are presented in Section 4, and the conclusions are drawn in Section 5.

Notations: In the paper, $\mathbb{R}^n$, $\mathbb{R}^{n\times m}$ and $\mathbb{Z}_{k\geq 0}$ denote the sets of n component real vectors, $n\times m$ real matrices, and the set of non-negative integers, respectively. $\mathbf{E}\{\cdot\}$ denotes the expectation operator with respect to some probability measure. The superscript T stands for matrix transposition or vector, the symbol $*$ denotes the transposed elements in the symmetric positions of a matrix. In addition, diag$(\cdot)$ denotes a block-diagonal matrix and I stands for an identity matrix of appropriate dimension. Matrices, not specially stated, are assumed to be compatible for algebraic operations.

2. Problem formulation

Consider the following discrete-time Markovian jump neural network with time-delays:

$$x(k+1)=\mathcal{C}(r_k)x(k)+\mathcal{A}(r_k)f(x(k))+\mathcal{B}(r_k)f(x(k-h(k)))+\varpi(r_k), \tag{1}$$

where $x(k)=[x_1(k),x_2(k),\ldots,x_n(k)]^T\in\mathbb{R}^n$ is the neuron state vector, $h(k)$ represents the transmission delay satisfying $0<h_m\leq h(k)\leq h_M$, where h_m and h_M are prescribed positive integers representing the lower and upper bounds of the delay, respectively. $f(x(k))=[f_1(x_1(k)),f_2(x_2(k)),\ldots,f_n(x_n(k))]^T$ is the neuron activation function, and $\varpi(r_k)=[\varpi_1(r_k),\varpi_2(r_k),\ldots,\varpi_n(r_k)]^T$ is a constant external input vector. $\mathcal{C}(r_k),\mathcal{A}(r_k)$ and $\mathcal{B}(r_k)$ are coefficient matrices and satisfy

$$[\mathcal{C}(r_k),\mathcal{A}(r_k),\mathcal{B}(r_k)]=[C(r_k),A(r_k),B(r_k)]+F(r_t)\Delta(r_k,k)[E_1(r_k),E_2(r_k),E_3(r_k)], \tag{2}$$

where $\Delta(r_k,k)$ is an unknown, time-varying matrix function and satisfies $\Delta^T(r_k,k)\Delta(r_k,k)\leq I$ for all $k\in\mathbb{Z}_{k\geq 0}$. $C(r_k)=\text{diag}(c_1(r_k), c_2(r_k),\ldots,c_n(r_k))$ is the known mode-dependent diagonal matrix with $|c_j(r_k)|<1$ $(\forall j\in\{1,2,\ldots,n\})$. The mode-dependent matrices $A(r_k)$ and $B(r_k)$ are the connection weight matrix and the delayed connection weight matrix, respectively. $F(r_t),E_1(r_k),E_2(r_k)$ and $E_3(r_k$ are known mode-dependent matrices. The matrices are functions of the stochastic jump process $\{r_k,k\geq 0\}$, which is a discrete-time, discrete-state Markov chain taking values in a finite set $\Lambda=\{1,2,\ldots,s\}$ with transition probabilities

$$\Pr\{r_{k+1}=j|r_k=i\}=\pi_{ij}, \tag{3}$$

where $\pi_{ij}\geq 0$ and $\sum_{j=1}^{s}\pi_{ij}=1$ for all $i\in\Lambda$.

For notational simplicity, in the sequel, for each possible $r_k=i, i\in\Lambda$, a matrix $M(r_k)$ will be denoted by M_i; for instance, $A(r_k)$ will be denoted by A_i, $B(r_k)$ by B_i, and so on. In addition, $\check{P}_i$ denotes $\sum_{j=1}^{s}\pi_{ij}P_j$.

Assumption 1. The neuron state-based nonlinear function $f(\cdot)$ in (1) is bounded for all $a,b\in\mathbb{R}^n$, $a\neq b$, satisfying

$$0\leq\frac{f_j(a)-f_j(b)}{a-b}\leq\gamma_j,\quad j=1,2,\ldots,n, \tag{4}$$

where γ_j is a known real scalar for all $j=1,2,\ldots,n$.

Note that by using the Brouwers fixed-point theorem, it can be easily proven that there exists one equilibrium point for the neural network (1). Assuming that $x^*(k)=[x_1^*(k),x_2^*(k),\ldots,x_n^*(k)]^T\in\mathbb{R}^n$ is the equilibrium point of (1) and using the transformation $e(k)=x(k)-x^*(k)$, the neural network (1) can be converted to the following form:

$$e(k+1)=\mathcal{C}(r_k)e(k)+\mathcal{A}(r_k)g(e(k))+\mathcal{B}(r_k)g(e(k-h(k))), \tag{5}$$

where $e(k)=[e_1(k),e_2(k),\ldots,e_n(k)]^T$ and $g(e(k))=[g_1(e_1(k)),g_2(e_2(k)),\ldots,g_n(e_n(k))]^T$ with $g_j(e_j(k))=f_j(e_j(k)+x_j^*(k))-f_j(x_j^*(k)),j=1,2,\ldots,n$. According to Assumption 1, one can obtain that

$$g_j^T(e_j(k))g_j(e_j(k))\leq\gamma_j^2,\quad g_j(0)=0,\ \ j=1,2,\ldots,n. \tag{6}$$

The main purpose of this paper is to deal with the problem of stochastic finite-time boundedness analysis of discrete-time neural network with time-delays and Markovian jumps. Throughout the paper, we need the following definitions and lemmas.

Definition 1 (*Stochastic finite-time stability*). The neural network (1) with $\varpi_i=0$ is said to be stochastically finite-time stable with respect to (δ,ϵ,R_i,N), where $0<\delta<\epsilon$, $R_i>0$ and $N\in\mathbb{Z}_{k\geq 0}$, if

$$\mathbf{E}\{x^T(k_1)R_ix(k_1)\}\leq\delta^2\Rightarrow\mathbf{E}\{x^T(k_2)R_ix(k_2)\}<\epsilon^2,$$
$$\forall\ k_1\in\{-h_M,\ldots,-1,0\},\ \ \forall\ k_2\in\{1,2,\ldots,N\}. \tag{7}$$

Definition 2 (*Stochastic finite-time boundedness*). The neural network (1) is said to be stochastically finite-time bounded with respect to (δ,ϵ,R_i,N), where $0<\delta<\epsilon$, $R_i>0$ and $N\in\mathbb{Z}_{k\geq 0}$, if the neural network (1) satisfies the constraint condition (7).

Remark 1. It is noted that the concept of stochastic finite-time boundedness reduces to stochastic finite-time stability if the external input $\varpi_i=0$ for all $i\in\Lambda$. Thus, stochastic finite-time boundedness implies stochastic finite-time stability, but the converse is not true.

Lemma 1 (Zhang et al. [33,35]). *For matrices Ω, Φ and Ψ of appropriate dimensions, where Ω is a symmetric matrix,*

$$\Omega+\Phi\Delta(k)\Psi+[\Phi\Delta(k)\Psi]^T<0 \tag{8}$$

holds for all time-varying matrix function $\Delta(k)$ satisfying $\Delta^T(k)\Delta(k)\leq I$ for all $k\in\mathbb{Z}_{k\geq 0}$, if and only if there exists a positive constant ϵ, such that $\Omega+\epsilon\Phi\Phi^T+\epsilon^{-1}\Psi^T\Psi<0$ holds.