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Letters

Formation control for nonlinear multi-agent systems by robust output regulation

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ABSTRACT

In this paper, the formation control problem of multi-agent systems with nonlinear dynamic is considered. Output regulation problem of nonlinear systems is generalized to the formation control problem of nonlinear multi-agent systems. The reference inputs or disturbance signals are generated by an exo-system, which can be considered as an active leader in the considered multi-agent systems. When some agents cannot obtain the information from the exo-system, the distributed nonlinear feedback controller is designed. Based on the internal model principle, the distributed output regulation formation control problem can be solved by solving the regulator equations. The illustrative example demonstrates the effectiveness of the main results.

1. Introduction

A multi-agent system (MAS) is a very complex system, made up of multiple interacting intelligent agents. Generally speaking, it is difficult for a single agent to finish some large or complex tasks; sometimes these tasks cannot be completed at all. However, multi-agent systems can cooperate to solve the problems that are beyond the capacities of any individual agent. More importantly, multi-agent systems have many advantages such as reliability, flexibility, reducing cost, improving system efficiency, providing some new capabilities and so on.

In recent years, cooperative control of multi-agent systems ([1–7] and the references therein) has been widely studied due to the development of advanced theory of complex systems and its broad applications in many fields. As one of the most important and fundamental problem of coordinated control, formation control of multi-agent systems has attracted much attention and has been widely applied in many fields recently, such as unmanned aerial vehicles (UAVS), autonomous underwater vehicles (AUVS), mobile robot systems (MRS) and so on. Formation control problem is to find a coordinated control scheme for multi-agent systems such that the agents would reach and maintain some desired formation or group configuration.

During the past decade, many different types of formation control methods have been proposed [8–12]: such as leader-following strategy, behavior-based approach and virtual structure method. In addition, some researchers also considered the formation control of multi-agent systems by other methods. Xiao et al. [13] applied the proposed nonlinear consensus protocol to the formation control, including time-invariant formation, time-varying formation and trajectory tracking. It turns out that all agents could achieve the expected formation after a finite time. Chen et al. [14] studied formation control design and stability analysis of double-integrator agents with directed communication links, in which the motions of the agents are restricted to given curves. In ref. [15], variants of a consensus algorithm were used to tackle the formation control problem of second-order multi-agent systems, and the formation control was proved to be asymptotically achieved. Chen et al. [16] studied the formation control problem for systems consisting of multi-agents that are described by first-order and second-order differential equations.

Moreover, since the 1990s, the output regulation problem for nonlinear systems has been studied (see, for example [17–20], and the references therein), owing to its profound theoretical background and practical values. The output regulation problem of a controlled system is that of controlling a plant to track (or reject) reference (or disturbance) signals, which are produced by the exo-system. When considering the tracking and formation problem of the multi-agent system under the framework of leader-following, the follower cannot completely get the information from the dynamic leader. It is difficult to consider the leader-following problem by the existing methods. However, from the perspective of output regulation, the problem will be solved easily. Recently, research on the output regulation problem for multi-agent systems

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has been investigated [21–24]. In [21], the authors formulated the formation control problem as a decentralized output regulation problem with the assumptions that all the agents can get the exogenous signals (leader). However, in practice, some agents cannot obtain the information of the leader, it is required to design a distributed controller, obtain their neighbor external measured state through the information exchange, and then design the distributed controller of the multi-agent system based on the measured state. A very recent work by f Hong and Huang et al. [22–24] designed the distributed controller to solve the output regulation problem of linear multi-agent system, and extended the results to the robust regulation problem of the multi-agent system with uncertainties. However, their studies are mainly about linear systems, the dynamics of the multi-agent being nonlinear is rarely seen, especially formation control by output regulation.

Motivated by the above discussions, we considered the formation control problem of a multi-agent system with nonlinear dynamic using robust output regulation. In other words, designing the controller such that the agents have to acquire a pre-defined geometric shape and track a reference trajectory while maintaining the formation. The reference or disturbance signals are generated by an exo-system, which can be considered as an active leader in the considered multi-agent systems. Based on the internal model principle, the formation control problem can be solved by solving the regulator equations.

The rest of this article is organized as follows. In Section 2, some model formulations and useful preliminaries are given. The main results of distributed robust formation output regulation problem for nonlinear multi-agent systems are discussed in Section 3. The numerical example is given to verify the theoretical results in Section 4. In the end, concluding remarks are provided in Section 5.

2. Problem formulation and preliminaries

Here, some preliminary knowledge of the algebraic graph theory is introduced for the following analysis (referring [25]). Let $G(\upsilon,\varepsilon,A)$ be a weighted digraph of order n with the set of nodes $\upsilon=\{1,2,\cdots n\}$, set of arcs $\varepsilon\subseteq\upsilon\times\upsilon$, and a weighted adjacency matrix $A=[a_{ij}]\in R^{n\times n}$ with nonnegative elements. The node indexes belong to a finite index set $I=\{1,2,\cdots,n\}$. An arc of I is denoted by (i,j), which starts from i to j. The element a_{ij} associated with the arc of digraph is positive, i.e. $a_{ij}>0\Leftrightarrow(i,j)\in\varepsilon$. Moreover, we assume $a_{ii}=0$ for all $i\in I$. The set of neighbors of node i is denoted by $N_i=\{j\in\upsilon:(i,j)\in\varepsilon\}$.

A diagonal matrix $D=diag\{d_1,d_2,\cdots,d_n\}\in R^{n\times n}$ is a degree matrix of G, whose elements $d_i=\sum_{j\in N_i}a_{ij}$ for $i=1,2,\cdots,n$. Then the Laplacian matrix of the weighted digraph G is defined as $L=D-A\in R^{n\times n}$.

To study a leader-following problem, we also use another graph $\overline{G}$ which consists of n agents and one leader (labeled by 0). For $\overline{G}$, if there is a path in $\overline{G}$ from each node i in G to node 0, we say that node 0 is globally reachable in $\overline{G}$. Denote b_i as the linked weight between agent i and the leader and there are positive constants $\beta_i(i=1,2,\cdots,n)$, such that

$$b_i=\begin{cases}\beta_i, & \textit{if the agent } i \textit{ is connected to the } \text{leader},\\ 0, & \textit{otherwise}.\end{cases}$$

In this paper, the dynamics of the considered multi-agent system can be described as follows:

$$\begin{aligned}&\dot{x}_i=f_i(x_i,u_i,w,\mu),\\&\dot{w}=s(w),\\&y_o=q(w),\\&e_i=x_i-r_i-y_o,\quad i=1,2,\cdots,N,\end{aligned}\tag{1}$$

where $x_i\in R^n$ and $u_i\in R^m$ represent the state and control input of the i-th agent, respectively.$\mu\in R^p$ represents the unknown plant parameter and $w\in R^q$ is an exogenous signal which may represent the reference signal to be tracked and (or) the disturbance to be rejected, and is generated by the exo-system $\dot{w}=s(w)$. $y_0\in R^n$ is the measured output, and $r_i\in R^n$ is the desired formation vector from the i-th agent to the leader. Here $e_i\in R^n$ is the regulated output for the i-th agent, which describes the control target. It is assumed that $f_i(\cdot),s(w),q(w)$ are sufficiently smooth and known $\mathbf{C}^k(k\ge2)$ functions, and $s(0)=0,\quad q(0)=0$.

In this paper, the exo-system can be considered as an active leader. In practice, only some of the agents can obtain signal of the exo-system. It also means that the state of the exo-system w cannot be measured by all the agents, so it cannot be used in the control design. Moreover, e_i may not be used directly too. Therefore, we introduce a virtual regulated error as follows:

$$e_{vi}=\sum_{j\in N_i}a_{ij}(e_i-e_j)+b_ie_i\tag{2}$$

Now we define the dynamic feedback distributed controller based on the virtual regulated error as follows:

$$\begin{aligned}&u_i=\theta_i(z_i,e_{vi}),\\&\dot{z}_i=\eta_i(z_i,e_{vi}).\end{aligned}\tag{3}$$

where $\theta_i(\cdot,\cdot)$ and $\eta_i(\cdot,\cdot)$ are $\mathbf{C}^k(k\ge2)$ functions. For convenience, we assume that $\theta_i(0,0)=0$ and $\eta_i(0,0)=0,\quad i=1,2,\cdots,N$.

Remark 1. Different from conventional output regulation problem, distributed output regulation problem is mainly based on the virtual regulated error of the multi-agent system, and this method does not require each agent and the active leader to have identical dynamics. In [21], the author assumed that each agent can obtain the signal from the leader. In fact, each agent has to collect the information in a distributed way from its neighbor agents. In addition, the considered multi-agent systems in [22–24] are linear, while in this paper, the dynamic of the considered system is nonlinear, which can be more applicable in practice.

Denote

$$\begin{aligned}&x=\begin{pmatrix}x_1 & x_2 & \cdots & x_N\end{pmatrix}^T,\qquad \overline{x}_i=x_i-r_i,\\&r=\begin{pmatrix}r_1 & r_2 & \cdots & r_N\end{pmatrix}^T,\quad u=\begin{pmatrix}u_1 & u_2 & \cdots & u_N\end{pmatrix}^T,\\&e=\begin{pmatrix}e_1 & e_2 & \cdots & e_N\end{pmatrix}^T,\quad z=\begin{pmatrix}z_1 & z_2 & \cdots & z_N\end{pmatrix}^T,\\&e_v=\begin{pmatrix}e_{v1} & e_{v2} & \cdots & e_{vN}\end{pmatrix}^T,f(\cdot)=\begin{pmatrix}f_1(\cdot) & f_2(\cdot) & \cdots & f_N(\cdot)\end{pmatrix}^T,\\&\theta(z,e_v)=\begin{pmatrix}\theta_1(z_1,e_{v1}) & \theta_2(z_2,e_{v2}) & \cdots & \theta_N(z_N,e_{vN})\end{pmatrix}^T,\\&\eta(z,e_v)=\begin{pmatrix}\eta_1(z_1,e_{v1}) & \eta_2(z_2,e_{v2}) & \cdots & \eta_N(z_N,e_{vN})\end{pmatrix}^T.\end{aligned}$$

Then system (1) and controller (3) can be rewritten as

$$\begin{aligned}&\dot{\overline{x}}=f(\overline{x}+r,u,w,\mu),\\&\dot{w}=s(w),\\&e=\overline{x}-\mathbf{1}\otimes q(w),\end{aligned}\tag{4}$$

and

$$\begin{aligned}&u=\theta(z,e_v),\\&\dot{z}=\eta(z,e_v).\end{aligned}\tag{5}$$

Denoting system (4) in the following form:

$$\begin{aligned}&\dot{\overline{x}}=\overline{f}(\overline{x},u,w,\mu),\\&\dot{w}=s(w),\\&e=\overline{x}-\mathbf{1}\otimes q(w).\end{aligned}\tag{6}$$

where we use the notation $\overline{f}(\cdot)$ since we use $\overline{x}$ instead of $\overline{x}+r$ in $f(\cdot)$. Let $B_0=diag\{b_1,b_2,\cdots,b_N\}$, then the virtual regulated error (2) can be rewritten as $e_v=H\overline{x}-(B_0\mathbf{1}_N)\otimes y_o$, where $H=(L+B_0)\otimes I_n$, $\mathbf{1}_N=[1,1,\cdots,1]^T\in R^N$.