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Delay-dependent robust stability and stabilization of uncertain memristive delay neural networks

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ABSTRACT

In this paper, a general class of uncertain memristive neural networks with time-delay is formulated and studied. And the problems of robust stability analysis and robust controller designing of the new model are derived. The uncertainty is assumed to be norm-bound and appears in all the matrices of the state-space model. There are few studies concerned the robust analysis of the memristive neural networks, so these conditions are improvements and extensions of the existing results in the literature. The proposed methods are dependent on the size of the delay and are given in terms of linear matrix inequalities. Finally, the validity of the theoretical results are demonstrated via some numerical examples.

1. Introduction

Motivated by many systems in science and humanities [1,2,5–15,17–31], memristive neural networks which consist of bio-inspired neuron oscillatory circuits with nanoscale memristors have attracted extensive interest in both modeling studies and neurobiological research in the past few years due to their feasibility to achieve the large connectively and highly parallel processing power of biological systems [3,4,16,32–41]. Since memristive neural networks were introduced by Hu and Wang [3], many researchers have paid attention to the new neural networks model. Research shows that memristive neural networks have many important applications in the fields of pattern recognition, signal processing, optimization and associative memories, especially since they can remember their past dynamical history, store a continuous set of states, and be plastic according to the pre-synaptic and post-synaptic neuronal activity. With more and more successful research results about memristive neural networks being published, we find that it will help us build a brain-like machine to implement the synapses of biological systems. However, the existing memristive neural networks which many researchers had constructed have been found to be computationally restrictive. In these circumstances, the applicability of these memristive neural networks in this area only has limited success.

Recently, we note that many researchers have turned their attention to the dynamical analysis of memristive neural networks [3,16,32–41]. Furthermore, an interesting issue is to investigate the dynamic behavior of memristor-based recurrent neural networks, an ideal model for the case where the memristor-based circuit networks exhibit complex switching phenomena. Hu and Wang [3] considered the global asymptotic stability of memristor-based recurrent neural networks. Wu et al. [32,33] considered the synchronization control of a class of memristor-based neural networks. Wen [35–41] presented dynamic behaviors of a class of memristor-based recurrent neural networks with time-varying delay. Zhang et al. [16] presented some sufficient conditions for exponential stability for memristor-based recurrent neural networks. However, most of the studies overlook the problem that the noise may destroy the stability of the memristive neural networks. So we should further study the memristive neural networks, i.e. conduct robust stability analysis. To the best of authors' knowledge, there are few results in the open literature dealing with the delay-dependent asymptotical stability and robust control design for the memristive neural networks. More importantly, further studies of the dynamic analysis of memristive neural networks will make us build a brain-like model more precisely.

We also note that noise and some other external disturbances are often the sources of instability and they may destabilize stable neural networks. Generally, these disturbances could easily cause the neural network to be unstable. Although various stability properties of neural networks have been analyzed extensively in recent years [8,9,28–31,43,44] the robustness of the uncertain memristive neural networks is rarely investigated directly. Motivated by the aforementioned discussion, this paper is concerned with the problem of delay-

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dependent robust stability analysis and robust control design for uncertain memristive neural networks with delay state and norm-bounded parameter uncertainty. We consider the case of a single constant time-delay. The focal point of this paper is on developing methods for robust stability analysis and robust stabilization based on linear matrix inequalities [28] and depend on the size of time-delay.

The paper is organized as follows: In the next section, the problems investigated in this paper are formulated and some preliminaries are presented. In Section 3, the delay-dependent robust stability and robust control design are derived. Several numerical examples and simulations including comparison analysis of conservatism are presented in Section 4. Finally,some conclusions are drawn in Section 5.

2. Problem formulation and preliminaries

Consider uncertain memristive time-delay neural networks described by the following systems:

$$\dot{x}_i(t) = -x_i(t) + \sum_{j=1}^{n} (\xi_{ij}(x_i(t)) + \Delta a_{ij}(t)) f_j((x_j(t))) + \sum_{j=1}^{n} (\zeta_{ij}(x_i(t)) + \Delta b_{ij(t)}) f_j((x_j(t-\tau_j)) + \sum_{j=1}^{n} C_{ij} u_i(t) \quad (1)$$

where $x(t) = [x_1(t), \ldots, x_n(t)]^{\mathrm{T}} \in R^{\mathrm{T}}$ denotes the state of memristive neuron, $\hat{A} = (\xi_{ij})_{n\times n}, \hat{B} = (\zeta_{ij})_{n\times n}, C = (C_{ij})_{n\times n}, f(\cdot)$ and $g(\cdot)$ are feedback activation functions; $u_i(t)$ denotes the control inputs and τ_j is time delay satisfying $0 \le \tau \le \overline{\tau}$, where $\overline{\tau}$ is a real constant. $[\Delta A(t), \Delta B(t)] = [\Delta a_{ij}(t)_{n\times n}, \Delta b_{ij}(t)_{n\times n}]$ are unknown real norm-bound matrix functions which represent time-varying parameter uncertainties. The admissible uncertainties are assumed to be of the form

$$[\Delta A(t), \Delta B(t)] = DF(t)[E_a, E_b] \quad (2)$$

where $F(t) \in R^{i\times j}$ are unknown real time-varying matrices and Lebegue measurable elements satisfying

$$\| F(t) \| \le 1 \quad \forall t \quad (3)$$

and D, E_a, E_b are known real constant matrices which characterize how uncertain parameters in$F(t)$ enter the norm matrix $\hat{A}$ and $\hat{B}$.

Now from the literature [3,16,32–41], by applying the theories of set-valued maps and differential inclusions [42] to model (1), we have

$$\dot{x}_i(t) \in -x_i(t) + \sum_{j=1}^{n} (co\{\overline{\xi}_{ij}, \underline{\xi}_{ij}\} + \Delta a_{ij}(t)) f_j(x_j(t)) + \sum_{j=1}^{n} (co\{\overline{\zeta}_{ij}, \underline{\zeta}_{ij}\} + \Delta b_{ij}(t)) g_j(x_j(t-\tau_j)) + \sum_{j=1}^{n} C_{ij} u_i(t) \quad t \ge 0, \quad i = 1,2,\ldots,n \quad (4)$$

or equivalently, for $i, j = 1,2,\ldots,n$, there exist $a_{ij}(x_i(t)) \in co\{\overline{\xi}_{ij}, \underline{\xi}_{ij}\}$ and $b_{ij}(x_i(t)) \in co\{\overline{\zeta}_{ij}, \underline{\zeta}_{ij}\}$, such that

$$\dot{x}_i(t) = -x_i(t) + \sum_{j=1}^{n} (a_{ij}(x_i(t)) + \Delta a_{ij}(t)) f_j(x_j(t)) + \sum_{j=1}^{n} (b_{ij}(x_i(t)) + \Delta b_{ij}(t)) g_j(x_j(t-\tau_j)) + \sum_{j=1}^{n} C_{ij} u_i(t) \quad t \ge 0, \quad i = 1,2,\ldots,n \quad (5)$$

in this paper as a matter of convenience, let

$$a_{ij}(x_i(t)) = \begin{cases} \hat{a}_{ij} & |x_i(t)| < \gamma \\ \check{a}_{ij} & |x_i(t)| > \gamma \end{cases}, \quad b_{ij}(x_i(t)) = \begin{cases} \hat{b}_{ij} & |x_i(t)| < \gamma \\ \check{b}_{ij} & |x_i(t)| > \gamma \end{cases} \quad (6)$$

where the switching jumps vectors $\gamma > 0, \hat{a}_{ij}, a\check{a}_{ij}, \hat{b}_{ij}, \check{b}_{ij}$ are constant vectors. Transforming model (5) into the matrix-vector notation. we have

$$\dot{x}(t) = -x(t) + [A(x(t)) + \Delta A(t)] f(x(t)) + [B(x(t)) + \Delta B(t)] g(x(t-\tau)) + Cu(t) \quad (7)$$

where

$$A(x(t)) = (a_{ij}(x_i(t)))_{n\times n}, B(x(t)) = (b_{ij}(x_i(t)))_{n\times n}$$
$$x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^{\mathrm{T}}$$
$$f(x(t)) = [f_1(x_1(t)), f_2(x_2(t)), \ldots, f_n(x_n(t))]^{\mathrm{T}}$$
$$g(x(t-\tau)) = [g_1(x_2(t-\tau)), g_2(x_2(t-\tau)), \ldots, g_n(x_n(t-\tau))]^{\mathrm{T}} \quad (8)$$

The initial condition of (5) is assumed to be

$$x(t) = \phi(t), \quad \forall t \in [-\tau, 0] \quad (9)$$

where $\phi(t) \in C([-\tau, 0], R^n)$ with $\| \phi(t) \| = \max \| \phi(t) \|$.

For convenience, we assume that the feedback functions $f_j(\cdot), g_j(\cdot)$ satisfying the following condition

(H1). $f_i, g_i (i = 1,2,\ldots,n)$ are bounded on R and for any $u, v \in R$,

$$0 \le \frac{f_i(u) - f_i(v)}{u - v} \le 1, 0 \le \frac{g_i(u) - g_i(v)}{u - v} \le 1. \quad (10)$$

Recalling the assumption on the activation functions, we define, for $i = 1,2,\ldots,n$,

$$s_i(t) = \begin{cases} \dfrac{f_i(x_i(t))}{x_i(t)} & |x_i(t)| \ne 0 \\ 0 & |x_i(t)| \equiv 0 \end{cases}, \quad h_i(t) = \begin{cases} \dfrac{g_i(x_i(t))}{x_i(t)} & |x_i(t)| \ne 0 \\ 0 & |x_i(t)| \equiv 0 \end{cases} \quad (11)$$

Obviously, $s_i(t), h_i(t)$ is piecewise continuous on R. From (10) and the assumption (H1), we have $0 \le s_i \le 1, 0 \le h_i \le 1$. □

Furthermore, system (7) can be rewritten as

$$\dot{x}_i(t) = -x_i(t) + \sum_{j=1}^{n} (a_{ij}(x_i(t)) + \Delta a_{ij}(t)) s_j(t) x_j(t) + \sum_{j=1}^{n} (b_{ij}(x_i(t)) + \Delta b_{ij}(t)) h_j(t-\tau) x_j(t-\tau) + \sum_{j=1}^{n} (c_{ij}) u_i(t), \quad i = 1,2,\ldots,n \quad (12)$$

or in the matrix-vector notation

$$\dot{x}(t) = -x(t) + \overline{A}(t) S(t) x(t) + \overline{B}(t) H(t-\tau) x(t-\tau) + Cu(t) \quad (13)$$

where

$$\overline{A}(t) = A + DF(t)E_a, \overline{B}(t) = B + DF(t)E_b$$
$$S(t) = diag(s_i(t)), \quad H(t-\tau) = diag(h_i(t-\tau)).$$

Applying the following model transformation above to the considered system

$$x(t-\tau) = x(t) - \int_{-\tau}^{0} \dot{x}(t+\theta)\, d\theta = x(t) - \int_{-\tau}^{0} [-x(t+\theta) + \overline{A}(t+\theta) S(t+\theta) x(t+\theta) + \overline{B}(t+\theta) H(t+\theta-\tau) x(t+\theta-\tau) + Cu(t+\theta)]\, d\theta$$

we derive

$$\dot{x}(t) = (-I + \overline{A}(t) S(t) + \overline{B}(t) H(t-\tau)) x(t) + (\overline{B}(t) H(t-\tau) \eta(y, t) + Cu(t) \quad (14)$$

where

$$\eta(x, t) = -\int_{-\tau}^{0} [-x(t+\theta) + \overline{A}(t+\theta) S(t+\theta) x(t+\theta) + \overline{B}(t+\theta) H(t+\theta-\tau) x(t+\theta-\tau) + Cu(t+\theta)]\, d\theta,$$