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New stability criteria for recurrent neural networks with interval time-varying delay ☆

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ABSTRACT

This paper is concerned with the problem of stability analysis of recurrent neural networks with time-varying delay belonging to a given interval. By constructing a novel augmented Lyapunov functional which contains some triple-integral terms, improved delay-dependent stability criteria are derived in terms of linear matrix inequality (LMI) by introducing some free-weighting matrices and using integral inequality technique and convex combination method. Numerical examples are given to illustrate the effectiveness of the proposed method.

1. Introduction

During past several decades, recurrent neural networks have received much attention due to their widely applications in pattern recognition, signal processing, image processing, optimization problem and model identification [1,2]. It is well known that time-delay is usually one of the sources of instability and oscillations of control systems such as networked control systems [3]. Rich results for systems with time-delay have been obtained [4–6]. With the rapid development of microelectronic technology, neural networks can be implemented by very large scale integrated circuits. However, as the speed of the information processing is not unlimited, there inevitably exist time-delays in neural networks. Therefore, the problem of stability of recurrent neural networks with delays has been an active research area. Due to its importance in theory and engineering, many results on this topic have been reported in the literature [7–17].

Existing stability results for recurrent neural networks with time-varying delays can be classified into two types. One is delay-independent and the other is delay-dependent. Because delay-dependent stability conditions are usually less conservative, they have received much attention. For example, the problem of global exponential stability for a class of delayed recurrent neural networks was studied in [18]. On the basis of a new Lyapunov–Krasovskii functional and Jensen's inequality, some improved delay-dependent stability criteria were developed. Using delay partitioning technique, delay-dependent stability criteria were proposed to guarantee the asymptotic stability for static recurrent neural networks [19]. Using some new algorithms, delay-independent and delay-dependent stability conditions for static recurrent neural networks were proposed in [20]. By constructing a new augmented Lyapunov–Krasovskii functional, delay-independent and delay-dependent stability criteria were obtained in [21] using the well-known free-weighting matrices method. As for other results of stability for recurrent neural networks refer to [22,23] and references therein.

In this paper, a new augmented Lyapunov–Krasovskii functional is proposed to derive less conservative stability conditions. This Lyapunov–Krasovskii functional contains some triple integral terms similar to [24–26]. Combing free-weighting matrices method [27,28] and Jensen's inequity [29] with convex combination technique [30], some less conservative stability conditions are obtained. Numerical examples are given to show the effectiveness of the proposed method.

Notations: Throughout this paper, the superscripts '-1' and 'T' stand for the inverse and transpose of a matrix, respectively; $\mathbb{R}^n$ denotes an n-dimensional Euclidean space; $\mathbb{R}^{m\times n}$ is the set of all $m\times n$ real matrices; $P>0$ means that matrix P is symmetric positive definite; I is an appropriately dimensional identity matrix.

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2. Problem formulation

Consider the following delayed recurrent neural network:

$$\dot{u}(t) = -Au(t) + g(Wu(t-\tau(t)) + J) \tag{1}$$

where $u(\cdot) = [u_1(\cdot)\ u_2(\cdot)\ \cdots\ u_n(\cdot)]^T$ is the neuron state vector, $g(Wu(\cdot)) = [g_1(W_1u(\cdot))\ g_2(W_2u(\cdot))\ \cdots\ g_n(W_nu(\cdot))]^T$ is the neuron activation function. $A = \text{diag}\{a_1, a_2, \dots, a_n\}$ with $a_i > 0$, $i = 1, 2, \dots, n$, is a diagonal matrix representing self-feedback term. $W = [W_1^T\ W_2^T\ \dots\ W_n^T]^T$ is the delayed connection weight matrix. $J = [j_1, j_2, \dots, j_n]^T$ is a constant input. $\tau(t)$ is a time-varying differentiable function and satisfies

$$h_1 \le \tau(t) \le h_2 \tag{2}$$

and

$$\dot{\tau}(t) \le \mu \tag{3}$$

where $h_2 > h_1 > 0$ and μ are constants.

It is assumed that each neuron activation function, $g_i(\cdot)$, $i = 1, 2, \dots, n$ is nondecreasing, bounded and satisfies

$$0 \le \frac{g_i(s_1) - g_i(s_2)}{s_1 - s_2} \le l_i \quad \forall s_1, s_2 \in \mathbb{R},\ s_1 \ne s_2,\ i = 1, 2, \dots, n \tag{4}$$

where l_i, $i = 1, 2, \dots, n$ are known real constants.

The above assumption guarantees that there exists an equilibrium point of system (1). Denote $u^* = [u_1^*\ u_2^*\ \cdots\ u_n^*]$ is the equilibrium point. Using the transformation $x(\cdot) = u(\cdot) - u^*$, system (1) can be converted to the following error system:

$$\dot{x}(t) = -Ax(t) + f(Wx(t-\tau(t))) \tag{5}$$

where $x(\cdot) = [x_1(\cdot)\ x_2(\cdot)\ \cdots\ x_n(\cdot)]^T$ is the state vector, $f(Wx(\cdot)) = [f_1(W_1x(\cdot))\ f_2(W_2x(\cdot))\ \cdots\ f_n(W_nx(\cdot))]^T$ with $f(Wx(\cdot)) = g(Wx(\cdot) + u^* + J) - g(Wu^* + J)$. It is easy to see that $f_i(\cdot)$, $i = 1, 2, \dots, n$, satisfies

$$0 \le \frac{f_i(s_1) - f_i(s_2)}{s_1 - s_2} \le l_i, f_i(0) = 0 \quad \forall s_1, s_2 \in \mathbb{R},\ s_1 \ne s_2,\ i = 1, 2, \dots, n \tag{6}$$

Before moving on, the following lemma is introduced which has an important role in the derivation of the main results.

Lemma 1 (*Sun et al. [24], Gu [29]*). *For any constant matrix $Z > 0$ and scalars $\tau_2 > \tau_1 > 0$ such that the following integrations are well defined, then*

$$(1) -\int_{t-h_2}^{t-h_1} x^{\mathrm{T}}(s) Z x(s)\, ds \le -\frac{1}{h_{12}} \int_{t-h_2}^{t-h_1} x^{\mathrm{T}}(s)\, ds\, Z \int_{t-h_2}^{t-h_1} x(s)\, ds$$

$$(2) -\int_{-h_2}^{-h_1} \int_{t+\theta}^{t} x^{\mathrm{T}}(s) Z x(s)\, ds\, d\theta$$

$$\le -\frac{2}{h_2^2 - h_1^2} \int_{-h_2}^{-h_1} \int_{t+\theta}^{t} x^{\mathrm{T}}(s)\, ds\, d\theta\, Z \int_{-h_2}^{-h_1} \int_{t+\theta}^{t} x(s)\, ds\, d\theta$$

where $h_{12} = h_2 - h_1$.

3. Main results

In this section, less conservative delay-dependent stability criteria are obtained based on a novel augmented Lyapunov–Krasovskii functional.

Theorem 1. *For given scalars $0 < h_1 < h_2$ and μ, system (5) is asymptotically stable for any time-varying delay satisfying (2) and (3) if there exist matrices $P = [P_{ij}]_{4\times4} > 0$, $Q = [Q_{ij}]_{3\times3} \ge 0$, $Z = [Z_{ij}]_{3\times3} \ge 0$, $R = [R_{ij}]_{3\times3} \ge 0$, $M = [M_{ij}]_{2\times2} \ge 0$, $N = [N_{ij}]_{2\times2} \ge 0$, non-negative diagonal matrices K, T and S and any matrices $H = [H_1^T\ \ H_2^T\ \dots\ H_6^T]^T$, $F = [F_1^T\ F_2^T\ \dots\ F_6^T]^T$, with appropriate dimensions such that the following LMIs hold:*

$$\begin{bmatrix} \Xi & \Theta & A_c^T Y & h_{12}\hat{H} \\ * & \Omega & 0 & 0 \\ * & * & -Y & 0 \\ * & * & * & -h_{12}N \end{bmatrix} < 0 \tag{7}$$

$$\begin{bmatrix} \Xi & \Theta & A_c^T Y & h_{12}\hat{F} \\ * & \Omega & 0 & 0 \\ * & * & -Y & 0 \\ * & * & * & -h_{12}N \end{bmatrix} < 0 \tag{8}$$

where

$$\Xi = [\Xi_{ij}]_{6\times6}$$

$$\begin{aligned}\Xi_{11} = {} & -P_{11}A - A^TP_{11} - Q_{12}A - A^TQ_{12}^T - Z_{12}A - A^TZ_{12}^T \\ & -h_1M_{12}A - h_1A^TM_{12}^T - h_{12}N_{12}A - h_{12}A^TN_{12}^T \\ & +P_{14} + P_{14}^T + Q_{11} + Z_{11} + h_1M_{11} + h_{12}N_{11} \\ & -\frac{1}{h_1}M_{22} - 2X_1 - \frac{2h_{12}}{h_1 + h_2}X_2\end{aligned}$$

$$\Xi_{12} = -H_1 + F_1$$

$$\Xi_{13} = -A^TP_{12} + P_{24}^T + \frac{1}{h_1}M_{22} + H_1$$

$$\Xi_{14} = -A^TP_{13} + P_{34}^T - P_{14} - F_1$$

$$\Xi_{15} = -A^TQ_{23} - A^TZ_{23} + Q_{13} + Z_{13} + W^TL^TK^T - A^TW^TS^T$$

$$\Xi_{16} = P_{11} + Q_{12} + Z_{12} + h_1M_{12} + h_{12}N_{12}$$

$$\Xi_{22} = -(1-\mu)Q_{11} - H_2 - H_2^T + F_2 + F_2^T$$

$$\Xi_{23} = H_2 - H_3^T + F_3^T$$

$$\Xi_{24} = -P_{45}^T - H_4^T - F_2 + F_4^T$$

$$\Xi_{25} = -H_5^T + F_5^T$$

$$\Xi_{26} = -H_6^T + F_6^T - (1-\mu)Q_{13} + W^TL^TT^T$$

$$\Xi_{33} = -Z_{11} + R_{11} - \frac{1}{h_1}M_{22} + H_3 + H_3^T$$

$$\Xi_{34} = -P_{24} - F_3 + H_4^T$$

$$\Xi_{35} = H_5^T$$

$$\Xi_{36} = P_{12}^T + H_6^T$$

$$\Xi_{44} = -P_{34} - P_{34}^T - R_{11} - F_4 - F_4^T$$

$$\Xi_{45} = -F_5^T$$

$$\Xi_{46} = P_{13}^T - F_6^T$$

$$\Xi_{55} = Q_{33} + Z_{33} - 2K$$

$$\Xi_{56} = Q_{23}^T + Z_{23}^T + SW$$

$$\Xi_{66} = -(1-\mu)Q_{33} - 2T$$

$$\Theta = \begin{bmatrix} 0 & 0 & P_{12} & P_{13} & 0 & \Theta_{16} & \Theta_{17} \\ 0 & 0 & 0 & 0 & -(1-\mu)Q_{12} & 0 & 0 \\ -Z_{13} + R_{13} & 0 & \Theta_{33} & P_{23} & 0 & \frac{1}{h_1}M_{12}^T & 0 \\ 0 & -R_{13} & P_{23}^T & P_{33} - R_{12} & 0 & -P_{44} & -P_{44} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & -(1-\mu)Q_{23}^T & P_{14} & P_{14} \end{bmatrix}$$

$$\Theta_{16} = -A^TP_{14} + P_{44} - \frac{1}{h_1}M_{12}^T + \frac{2}{h_1}X_1$$

$$\Theta_{17} = -A^TP_{14} + P_{44} + \frac{2}{h_2 + h_1}X_2$$

$$\Theta_{33} = P_{22} - Z_{12} + R_{12}$$

$$\Omega = \begin{bmatrix} -Z_{33} + R_{33} & 0 & -Z_{23}^T + R_{23}^T & 0 & 0 & 0 & 0 \\ * & -R_{33} & 0 & -R_{23}^T & 0 & 0 & 0 \\ * & * & -Z_{22} + R_{22} & 0 & 0 & P_{24} & P_{24} \\ * & * & * & -R_{22} & 0 & P_{34} & P_{34} \\ * & * & * & * & -(1-\mu)Q_{22} & 0 & 0 \\ * & * & * & * & * & \Omega_{66} & 0 \\ * & * & * & * & * & * & -\frac{2}{h_2^2 - h_1^2}X_2 \end{bmatrix}$$