Neurocomputing 121 (2013) 365–378

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Effects of leakage time-varying delays in Markovian jump neural networks with impulse control ☆

R. Rakkiyappan [a], A. Chandrasekar [a], S. Lakshmanan [b], Ju H. Park [b,*], H.Y. Jung [b]

[a] Department of Mathematics, Bharathiar University, Coimbatore 641 046, Tamilnadu, India

[b] Department of Electrical Engineering/Information and Communication Engineering, Yeungnam University, 214-1 Dae-dong, Kyongsan 712-749, Republic of Korea

ARTICLE INFO

Article history:
Received 10 August 2012
Received in revised form
18 March 2013
Accepted 3 May 2013
Communicated by N. Ozcan
Available online 10 June 2013

Keywords:
Global exponential stability
Leakage time-varying delays
Lyapunov–Krasovskii functional
Markovian jumping parameters

ABSTRACT

In this paper, the stability analysis problem is investigated for delayed neural networks with mixed time-varying delays, impulsive control and Markovian jumping parameters. The mixed time-varying delays include leakage, discrete and distributed time-varying delays. Sufficient conditions for the global exponential stability in the mean square are derived by using Lyapunov–Krasovskii functional having triple integral terms and model transformation technique. The stability criterion that depends on the upper bounds of the leakage time-varying delay and its derivative is given in terms of linear matrix inequalities (LMIs), which can be efficiently solved via standard numerical softwares. Finally, three numerical examples and simulations are given to demonstrate the usefulness and effectiveness of the presented results.

1. Introduction

During the last two decades, neural networks (NNs) are widely studied, because of their massive potentials of application in modern society of science and technology. Its potential applications are signal processing, pattern recognition, static image processing, associative memory, and combinatorial optimization [1–5]. In such applications, it is major importance to ensure that the designed neural network is stable. Further, when the neural networks are implemented with the help of very large-scale integrated electronic circuits, the finite switching speed of amplifiers and the communication time of neurons may induce time delays in the interaction between the neurons. It has been shown that time delays may cause undesirable dynamic network behaviors such as oscillation and instability. Thus, it is important to investigate the stability analysis of delayed neural networks in the recent years [6–14].

It is well known that the stochastic modeling plays an important role in the branches of science and technology. A particular area of interest is neural networks with Markovian jumping parameters. Markovian jumping systems can be considered as a special class of hybrid systems, and it is sometimes the case that a neural network has finite state representations (also called modes, patterns, or clusters), and the modes that may switch (or jump) from one to another at different times. In [15,16], it has been shown that the network states may switch (or jump) between different neural networks modes according to a Markovian chain. Further, these structures are subjected to random abrupt changes due to some unexpected factors such as component failures or repairs, sudden environmental disturbance and failures that occurred in components or interconnections and executor faults, etc. Hence, the stability analysis problem for neural networks with Markovian jumping becomes more and more significant. Moreover, many relevant results have been reported in the literature [17–26].

Recently, the effect of leakage delay in dynamical neural networks is one of the research topics and it has been studied by many researchers in the literature. As correctly pointed out in the literature [27,28], the time delay in stabilizing negative feedback term has a tendency to destabilize the system. Furthermore, sometimes it has more significant effect on dynamics of neural networks than other kinds of delays. Hence, the leakage term also has great impact on the dynamical behavior of neural networks. Therefore, there are many authors considering the problem of stability analysis of neural networks and system involving time delay in the leakage term, see for example [28–32]. In addition, the authors in [33,34] discussed the problem of global stability

☆The work was supported by Basic Science Research Program through the National Research Foundation of Korea (NRF) funded by the Ministry of Education, Science, and Technology (2010-0009373).

* Corresponding author. Tel.: +82 53 8102491.
E-mail addresses: rakkigru@gmail.com (R. Rakkiyappan), lakshm85@gmail.com (S. Lakshmanan), jessie@ynu.ac.kr (J.H. Park).

0925-2312/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.neucom.2013.05.018

analysis of equilibrium of neural networks with time delay in the leakage term under impulsive perturbations. The problem of synchronization of chaotic neural networks with time delay in the leakage term and parametric uncertainties based on sampled-data control has been presented in [35]. Also, it is well known that impulsive effects are likely to exist in the neural network systems [36–40]. For instance, in the implementation of electronic networks, the state is subjected to instantaneous perturbations and also it experiences abrupt change at certain moments. These abrupt changes might be caused by switching phenomenon, frequency changes or other sudden noises, which exhibit impulsive effects [41,42]. Hence, it is necessary to consider the impulsive control to the stability problem of neural networks with delays to reflect a more realistic dynamics. In the meanwhile, effect of leakage time-varying delay on stability of nonlinear differential systems has been investigated in [43]. To the best of authors' knowledge, so far, no results on the effects of leakage time-varying delays in Markovian jump neural networks with impulse control are available in the existing literature. This motivates our current research.

In this paper, we consider a class of Markovian jumping neural networks with leakage time-varying delays and impulsive effects. Then, by using a suitable Lyapunov Krasovskii's functional, model transformation and some analysis techniques, some delay-dependent criteria ensuring the global exponential stability in the mean square of the unique equilibrium point are obtained, which depend on the upper bounds of the leakage time-varying delay and its derivative. The criteria are given in terms of LMIs and can be efficiently solved via standard numerical softwares such as the MATLAB LMI toolbox [44]. It should be mentioned that Zhu and Cao [45] obtained some interesting and important results about the stability of Markovian jump system based on similar methods. However, we focus on the discussion of the leakage time-varying delays, which was not investigated in [45]. Thus, our development result is more general than the recent work [45].

Notations Let $\mathbb{R}^n$ denote the n-dimensional Euclidean space and the superscript "T" denote the transpose of a matrix or vector. I denotes the identity matrix with compatible dimensions. $diag(\cdots)$ denotes a block diagonal matrix. For square matrices, M_1 and M_2, the notation $M_1 > (\geq, <, \leq) M_2$ denotes $M_1 - M_2$ is a positive-definite (positive-semi-definite, negative, negative-semi-definite) matrix. $\lambda_{\min}(\cdot)$ and $\lambda_{\max}(\cdot)$ stand for the minimum and maximum eigenvalues of a given matrix, respectively. Let $(\Omega, \mathfrak{F}, \mathcal{P})$ be a complete probability space with a natural filtration $\{\mathfrak{F}_t\}_{t\geq 0}$ and $\mathbf{E}[\cdot]$ stands for the correspondent expectation operator with respect to the given probability measure $\mathcal{P}$. Also, let $\tau > 0$ and $\mathcal{C}([-\tau, 0]; \mathbb{R}^n)$ denote the family of continuously differentiable function ϕ from $[-\tau, 0]$ to $\mathbb{R}^n$ with the uniform norm $\|\phi\| = \max\{\max_{-\tau\leq\theta\leq 0}|\phi(\theta)|, \max_{-\tau\leq\theta\leq 0}|\phi'(\theta)|\}$. Denote by $\mathcal{C}^2_{\mathfrak{F}_0}([-\tau, 0]; \mathbb{R}^n)$ the family of bounded $\mathfrak{F}_0$-measurable, $\mathcal{C}([-\tau, 0]; \mathbb{R}^n)$-valued stochastic variables $\xi = \{\xi(\theta) : -\tau \leq \theta \leq 0\}$ such that $\int_{-\tau}^0 \mathbf{E}|\xi(\theta)|^2 \, ds < \infty$.

2. Model description and preliminaries

Let $\{r(t), t\geq 0\}$ is a right-continuous Markov chain on the probability space $(\Omega, \mathfrak{F}, \mathcal{P})$ taking values in a finite state space $\mathcal{S} = \{1, 2, \ldots, N\}$ with generator $Q = (q_{ij})_{N\times N}$ given by

$$P\{r(t+\Delta t) = j | r(t) = i\} = \begin{cases} q_{ij}\Delta t + o(\Delta t), & i\neq j, \\ 1 + q_{ii}\Delta t + o(\Delta t), & i = j, \end{cases}$$

where $\Delta t > 0$ and $\lim_{\Delta t\to 0} o(\Delta t)/\Delta t = 0$, $q_{ij} \geq 0$ is the transition rate from i to j, if $i\neq j$ while $q_{ii} = -\sum_{j\neq i} q_{ij}$.

Consider the following delayed recurrent neural networks with Markovian jumping parameters and leakage time-varying delay:

$$\begin{cases} \dot{x}(t) = -C(r(t))x(t-\sigma(t)) + A(r(t))f^{(1)}(x(t)) + B(r(t))f^{(2)}(x(t-\tau(t))) + V, & t\neq t_k \\ x(t_k) - x(t_k^-) = -D_k(r(t))\Big\{x(t_k^-) - C(r(t))\int_{t_k-\sigma(t_k)}^{t_k} x(s)\,ds\Big\}, & k\in\mathbb{Z}_+ \end{cases} \tag{1}$$

where $x(t-\sigma(t)) = [x_1(t-\sigma(t)), x_2(t-\sigma(t)), \ldots, x_n(t-\sigma(t))]^T \in \mathbb{R}^n$ is the state vector associated with the n neurons and leakage time varying delay. The diagonal matrix $c(r(t)) = diag(c_1(r(t)), c_2(r(t)), \ldots, c_n(r(t)))$ has positive entries $c_i(r(t)) > 0$ $(i = 1, 2, \ldots, n)$. The matrices $A(r(t)) = (a_{ij}(r(t)))_{n\times n}$ and $B(r(t)) = (b_{ij}(r(t)))_{n\times n}$ are the interconnection matrices representing the weight coefficients of the neurons. $f^{(1)}(x(t)) = [f_1^{(1)}(x_1(t)), f_2^{(1)}(x_2(t)), \ldots, f_n^{(1)}(x_n(t))]^T$ and $f^{(2)}(x(t)) = [f_1^{(2)}(x_1(t)), f_2^{(2)}(x_2(t)), \ldots, f_n^{(2)}(x_n(t))]^T$ are the neuron activation functions. $\tau(t)$ and $\sigma(t)$ denote the time-varying delay and time-varying leakage delay, respectively, and $V = [V_1, V_2, \ldots, V_n]^T$ denotes a constant external input vector. $D_k(r(t)) \in \mathbb{R}^{n\times n}, k\in\mathbb{Z}_+$ is the impulse gain matrix at the moment of time t_k. The discrete set $\{t_k\}$ satisfies $0 = t_0 < t_1 < \cdots < t_k < \cdots$, $\lim_{k\to\infty} t_k = \infty$. Assume that $x(t_k)$ is right continuous, i.e., $x(t_k^+) = x(t_k)$.

Assumption 1. $\tau(t)$ and $\sigma(t)$ denote the time varying delay and time varying leakage delay, respectively, and satisfy

$$0 \leq \tau_1 \leq \tau(t) \leq \tau_2, \quad \dot{\tau}(t) \leq \tau_\mu, \quad 0 \leq \sigma(t) \leq \sigma, \quad |\dot{\sigma}(t)| \leq \sigma_\mu$$

where $\tau_1, \tau_2, \tau_\mu, \sigma, \sigma_\mu$ are constants.

Assumption 2. There exist four diagonal matrices $L^{(1)} = diag(L_1^{(1)}, L_2^{(1)}, \ldots, L_n^{(1)})$, $L^{(2)} = diag(L_1^{(2)}, L_2^{(2)}, \ldots, L_n^{(2)})$, $U^{(1)} = diag(U_1^{(1)}, U_2^{(1)}, \ldots, U_n^{(1)})$, and $U^{(2)} = diag(U_1^{(2)}, U_2^{(2)}, \ldots, U_n^{(2)})$ satisfying

$$L_i^{(1)} \leq \frac{f_i^{(1)}(\alpha) - f_i^{(1)}(\beta)}{\alpha - \beta} \leq U_i^{(1)}, \quad L_i^{(2)} \leq \frac{f_i^{(2)}(\alpha) - f_i^{(2)}(\beta)}{\alpha - \beta} \leq U_i^{(2)}$$

for all $\alpha, \beta\in\mathbb{R}$, $\alpha\neq\beta$, $i = 1, 2, \ldots, n$.

The main purpose of this paper is to establish stability of the system (1). Let $x^* = (x_1^*, x_2^*, \ldots, x_n^*)$ be the equilibrium point of neural networks (1). Let $y(t) = x(t) - x^*$. The system (1) can be written as

$$\begin{cases} \dot{y}(t) = -C(r(t))y(t-\sigma(t)) + A(r(t))g^{(1)}(y(t)) + B(r(t))g^{(2)}(y(t-\tau(t))), & t\neq t_k \\ y(t_k) - y(t_k^-) = -D_k(r(t))\Big\{y(t_k^-) - C(r(t))\int_{t_k-\sigma(t_k)}^{t_k} y(s)\,ds\Big\}, & k\in\mathbb{Z}_+ \end{cases} \tag{2}$$

where $g^{(1)}(y(t)) = [g_1^{(1)}(y_1(t)), g_2^{(1)}(y_2(t)), \ldots, g_n^{(1)}(y_n(t))]^T$, $g^{(2)}(y(t)) = [g_1^{(2)}(y_1(t)), g_2^{(2)}(y_2(t)), \ldots, g_n^{(2)}(y_n(t))]^T$ and $g_i^{(1)}(y(t)) = f_i^{(1)}(y(t) + x^*) - f_i^{(1)}(x^*)$, $g_i^{(2)}(y(t)) = f_i^{(2)}(y(t) + x^*) - f_i^{(2)}(x^*) (i = 1, 2, \ldots, n)$.

It is noted that $g^{(1)}(0) = 0$, $g^{(2)}(0) = 0$, and the trivial solution of the system (2) exists. From Assumption 2, we have the following:

$$L_i^{(1)} \leq \frac{g_i^{(1)}(\alpha) - g_i^{(1)}(\beta)}{\alpha - \beta} \leq U_i^{(1)}, \quad L_i^{(2)} \leq \frac{g_i^{(2)}(\alpha) - g_i^{(2)}(\beta)}{\alpha - \beta} \leq U_i^{(2)}. \tag{3}$$

Let $y(t;\xi)$ denote the state trajectory from the initial data $y(\theta) = \xi(\theta)$ on $-\tau^\star \leq \theta \leq 0$ in $L^2_{\mathcal{F}_t}([-\tau^\star, 0]; \mathbb{R}^n)$ where $\tau^\star = \max\{\tau_2, \sigma\}$.

Now we give the definition of the exponential stability for the system (2).

Definition 2.1 (*Zhu and Cao [45]*). Let $a_1 = \max_{-\tau^\star\leq\theta\leq 0}\mathbf{E}|\xi(\theta)|^2$ and $a_2 = \max_{-\tau^\star\leq\theta\leq 0}\mathbf{E}|\xi'(\theta)|^2$. Then the equilibrium point of Eq. (1) is said to be exponentially stable in the mean square if for every $\xi\in L^2_{\mathcal{F}_0}([-\tau^\star, 0]; \mathbb{R}^n)$, there exist scalars $\alpha > 0$ and $\beta > 0$ such that the following inequality holds:

$$\mathbb{E}|y(t;\xi)|^2 \leq \alpha e^{-\beta t}\max\{a_1, a_2\}.$$