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Stochastic stability of Markovian jump BAM neural networks with leakage delays and impulse control ☆

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ABSTRACT

This paper deals with the globally exponential stability of impulsive bidirectional associative memory (BAM) neural networks with both Markovian jump parameters and mixed time delays. The jumping parameters are determined by a continuous-time, discrete-state Markov chain. Different from the previous literature, the mixed time delays considered here comprise discrete, distributed and leakage time-varying delays. By using the Lyapunov–Krasovskii functional having triple integral terms and model transformation technique, some novel sufficient delay-dependent conditions are derived to ensure the globally exponential stability in the mean square of the suggested system. Moreover, the derivatives of time delays are not necessarily zero or smaller than one since several free matrices are introduced in our results. Finally, a numerical example and its simulations are provided to demonstrate the effectiveness of the theoretical results.

1. Introduction

As is well known, bidirectional associative memory (BAM) neural networks belong to a class of two-layer hetero-associative networks because they generalize the single-layer auto-associative Hebbian correlator to a two-layer pattern-matched hetero-associative circuit. In fact, they are composed of neurons arranged in two layers, the U-layer and the V-layer. Moreover, the neurons in one layer are fully interconnected to the neurons in the other layer, but there may be no interconnection among neurons in the same layer. As a consequence, the addressable memories or patterns of BAM neural networks can be stored with a two-way associative search. These advantages have drawn a great deal of attention. Indeed, since Kosko first proposed BAM neural networks in [1–3], this class of neural systems has been extensively studied, and subsequently applied to many areas such as pattern recognition, signal and imagine processing, automatic control, associative memory and artificial intelligence [4–9]. However, these applications heavily depend on the stability of the equilibrium point of BAM neural networks since the stability is the first requirement in modern control theories. Therefore, it is important to discuss the stability issue of BAM neural networks.

In practical situations, time delays are often encountered and inevitable in biological and artificial neural networks because of the finite switching speed of amplifiers. For example, one can find the circuit diagram and connection pattern implementing for the delayed BAM neural networks. As we know, the existence of time delays may cause an oscillation or instability in neural networks, which is harmful to the applications of neural networks. Thus, there is a need of stability analysis for neural networks with time delays, which are usually called delayed neural networks. Recently, various sufficient conditions for the stability of delayed neural networks have been proposed, either delay-independent or delay-dependent. Generally speaking, delay-independent criteria are more conservative than delay-dependent criteria, especially when the size of delay is small, and so much attention has been paid to the latter ones.

However, besides delay effects, impulsive perturbations widely exist in many fields, such as biology and medicine, economics, electronics and telecommunications [10–12]. Especially in real neural networks, impulsive perturbations are likely to emerge since the states of neural networks are changed abruptly at certain moments of time in the above fields. Thus, impulsive perturbations should be taken into account when investigating the stability of neural networks. It is worth pointing out that an impulsive neural network model

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belongs to a new category of dynamical systems, which is neither purely continuous-time nor purely discrete-time. This leads to a great difficulty of the stability analysis for a class of impulsive neural networks.

On the other hand, systems with Markovian jumping parameters can be described by a set of linear systems with the transitions between models resolved by a continuous-time discrete state homogeneous Markov process [13]. Usually, neural networks in real life have a phenomenon of information latching, and the abrupt phenomena such as random failures or repairs of the components, sudden environmental changes, changing subsystem interconnections, etc. To cope with this situation, neural networks with Markovian jumping parameters, which are also called Markovian jump neural networks, have been widely used to model the above complex systems. Generally speaking, a Markovian jump neural network is a hybrid system with a state vector that has two components $x(t)$ and $r(t)$, whereas the first component $x(t)$ is referred to as the state, and the second component $r(t)$ is a continuous-time Markov chain with a finite state space $S=\{1,2,\dots,N\}$, which is usually regarded as the mode. In its operation, this class of neural networks will switch from one mode to another in a random way, which is determined by a continuous-time Markov chain $r(t)$. Hence, it is interesting and challenging to study Markovian jump neural networks.

Recently, there have been a large number of results on the stability of Markovian jump neural networks reported in the literature, for instance, see [14–27] and references therein. In [19], the authors studied the stability analysis for Markovian jump BAM neural networks with impulse control and mixed time delays without leakage term. But the leakage term has a great impact on the dynamical behavior of neural networks. In fact, leakage delays have a tendency to destabilize the neural networks and they are usually not easy to handle. So it is necessary to consider the effect of leakage delays when studying the stability of neural networks. It is inspiring that the effect of leakage delays in neural networks has led to a new research topic in recent years [28–35]. For example, Gopalsamy [30] studied the dynamics of bidirectional associative memory (BAM) network by using contraction mapping theorem and suitable degenerate Lyapunov–Krasovskii functional together with some differential inequalities and linear matrix inequality (LMI) technique. In [32], the authors investigated the stability of BAM fuzzy neural networks with delays in the leakage terms. Peng [33] discussed the globally attractive periodic solutions of BAM neural networks with continuously distributed delays in the leakage terms. In [34], the authors studied the stability analysis of recurrent neural networks with time delay in the leakage term under impulsive perturbations by using the Lyapunov–Krasovskii functional and LMI technique. However, to the best of our knowledge, there was no published papers on the stability analysis for a class of Markovian jump neural networks with leakage delays. This leads to our present research.

Motivated by the above discussion, in this paper we study the stability issue of Markovian jump BAM neural networks with mixed time delays and impulse control. Different from the previous literature, the mixed time delays considered here comprise discrete, distributed and leakage time-varying delays. Some novel sufficient conditions of globally exponential stability in the mean square are obtained by using the Lyapunov–Krasovskii functional having triple integral terms and model transformation technique. In particular, our conditions are delay-dependent and expressed in terms of LMIs, which can be calculated by Matlab LMI toolbox [36]. Finally, we use a numerical example and its simulations to illustrate the effectiveness of the theoretical results.

The rest of this paper is organized as follows. In Section 2, we introduce a new class of Markovian jump BAM neural networks with both impulsive perturbations and leakage time varying delays, and give some necessary assumptions and preliminaries. Our main results are presented in Section 3. In Section 4, a numerical example and its simulations are given to show the effectiveness of the obtained results. Finally, in Section 5, the paper is concluded with some general remarks.

Notations: Throughout this paper, $\mathbb{R}^n$ and $\mathbb{R}^{n\times n}$ denote the n-dimensional Euclidean space and the set of all $n\times n$ real matrices, respectively. The superscript "T" denotes the transpose of a matrix or vector. $\mathrm{Tr}(\cdot)$ denotes the trace of the corresponding matrix. For square matrices X and Y, the notation $X>(\geq,<,\leq)Y$ denotes $X-Y$ is a positive-definite (positive-semi-definite, negative, negative-semi-definite) matrix. Let $(\Omega,\mathcal{F},P)$ be a complete probability space with a natural filtration $\{\mathcal{F}_t\}_{t\geq 0}$ and $\mathbb{E}[\cdot]$ stand for the correspondent expectation operator with respect to the given probability measure P. Also, let $\tau>0$, $\delta>0$ and $C^2_{\mathcal{F}_0}([-\tau,0]);\mathbb{R}^n)$, $C^2_{\mathcal{F}_0}([-\delta,0]);\mathbb{R}^n)$ denote the family of continuously differentiable functions ϕ from $[-\tau,0]$ to $\mathbb{R}^n$ and ψ from $[-\delta,0]$ to $\mathbb{R}^n$ with the uniform norms $\|\phi\|=\sup_{-\delta\leq\theta\leq 0}|\phi(\theta)|$, $\|\psi\|=\sup_{-\tau\leq\theta\leq} 0|\psi(\theta)|$, respectively.

2. Problem description and preliminaries

Consider the following class of Markovian jump BAM neural networks with both impulsive perturbations and leakage time varying delays:

$$\begin{cases} \dot{u}(t)=-C(r(t))u(t-\sigma_1(t))+A(r(t))\tilde{f}^{(1)}(v(t))+B(r(t))\tilde{f}^{(2)}(v(t-\tau(t)))+I, & t\neq t_k \\ \Delta u(t_k)=u(t_k)-u(t_k^-)=-D_k(r(t))\left\{u(t_k^-)-C(r(t))\int_{t_k-\sigma_1(t_k)}^{t_k}u(s)\,ds\right\}, & k\in\mathbb{Z}_+, \\ \dot{v}(t)=-E(r(t))v(t-\sigma_2(t))+F(r(t))\tilde{g}^{(1)}(u(t))+G(r(t))\tilde{g}^{(2)}(u(t-\delta(t)))+J, & t\neq t_k \\ \Delta v(t_k)=v(t_k)-v(t_k^-)=-H_k(r(t))\{v(t_k^-)-E(r(t))\int_{t_k-\sigma_2(t_k)}^{t_k}v(s)\,ds\}, & k\in\mathbb{Z}_+, \end{cases} \tag{1}$$

for $t>0$ and $k=1,2,\dots$, where $u(t)=[u_1(t),u_2(t),\dots,u_n(t)]^T$ and $v(t)=[v_1(t),v_2(t),\dots,v_n(t)]^T$ are state vectors associated with the n neurons. $I=[I_1,I_2,\dots,I_n]$ and $J=[J_1,J_2,\dots,J_n]$ denote the constant external inputs.

The diagonal matrices $C(r(t))=\mathrm{diag}(c_1(r(t)),c_2(r(t)),\dots,c_n(r(t)))$ and $E(r(t))=\mathrm{diag}(e_1(r(t)),e_2(r(t)),\dots,e_n(r(t)))$ have positive entries $c_i(r(t))>0$, $e_i(r(t))>0$ $(i=1,2,\dots,n)$, respectively; the matrices $A(r(t))=(a_{ij}(r(t)))_{n\times n}$, $F(r(t))=(f_{ij}(r(t)))_{n\times n}$, $B(r(t))=(b_{ij}(r(t)))_{n\times n}$ and $G(r(t))=(g_{ij}(r(t)))_{n\times n}$ are the time varying delay connection weight matrix, and the distributed delay connection weight matrix, respectively; f_{ij} and g_{ij} denote the signal functions of the ith neuron and the jth neuron at time t, respectively; and $\tau(t)$, $\delta(t)$ and $\sigma_1(t)$, $\sigma_2(t)$ are time varying delays and time varying leakage delays, respectively; $\tilde{f}(\cdot)$ and $\tilde{g}(\cdot)$ are the activation functions; let $\{r(t),t\geq 0\}$ be a right-continuous Markov chain on a complete probability space $(\Omega,\mathcal{F},P)$ taking values in a finite state space $S=\{1,2,\dots,N\}$ with generator