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Sensitivity study of Binary Feedforward Neural Networks

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ABSTRACT

This paper presents a novel and effective approach for establishing a quantified output sensitivity of Binary Feedforward Neural Networks to weight and input perturbations. Firstly, analytical formulae are derived for computing a neuron's sensitivity by means of matrix and probability theories. Then, based on the neuron's sensitivity and the network's architecture feature, a bottom-up strategy is followed to compute the entire network's sensitivity. The proposed approach has the obvious advantages of higher generality, lower computational complexity, and yet much higher accuracy. Experimental results verify the correctness and effectiveness of the approach.

1. Introduction

Generally, a neural network can realize a mapping between its input and output spaces by a set of connection weights. Therefore, the investigation of a neural network's output sensitivity to its weight perturbation is very fundamental to both theoretical researches and practical applications. For example, in the development of training algorithms, a key issue is to know the effects of small weight adaption on a neural network's output during training, and conceptually this is what the sensitivity analysis deals with. Similarly, in the hardware design of a neural network, understanding the impact of small noise or even truncature inaccuracy to weights on the network's output is another important consideration. A literature survey shows that sensitivity analysis has drawn more and more researchers' attention, and a lot of results [1–16] have been achieved. There also appear many applications of the sensitivity for various purposes, such as improving error tolerance [5,17–19] and generalization ability [18,20–22], enhancing learning performance [21,23], constructing proper architecture [22,24–28]. This paper focuses on the sensitivity study of Binary Feedforward Neural Networks (BFNNs) also called Adaline Networks or Madalines.

A Madaline is a kind of discrete feedforward neural network with a supervised learning mechanism. Although a discrete neural network can be viewed as a special case of a continuous one, there are many inherently discrete situations to which the continuous networks are not naturally and efficiently applicable, such as logical calculation, signal processing, and classification. Conceptually, Madalines can be directly applied to tackle discrete problems with the following advantages: (1) be easy to compute, understand and interpret, (2) facilitate hardware implementation, and (3) reduce computational complexity and implementation cost. However, since the continuous techniques are no longer suitable for Madalines due to their discrete feature, their learning mechanism has not yet been well established by now. It is hopeful that the sensitivity study of Madalines could pave the way to improving Madalines' learning ability.

In the early research, Hoff [1] firstly made use of hypersphere as a mathematical model to theoretically study the sensitivity of an Adaline (threshold logic units with discrete binary inputs) based on the fact that all its n-dimensional inputs are uniformly distributed on an n-dimensional hypersphere. He defined the sensitivity as the probability of erroneous output of Madalines and derived an approximate solution from the model on the assumption that the possibility of any input falling into a region on the hypersphere is equal to the ratio of that region's surface area to the whole hypersphere's. Later on, Glanz [2] extended Hoff's model by treating an Adaline's bias as a special input, and thus established a mathematical model with $n+1$ dimensional semi-hypersphere. Based on the extended model, an Adaline with a bias can be easily treated as the one without bias. However, although their creative research introduced a theoretical way to the study, there still exist some problems under the model. Firstly, the Hoff's assumption only considers a region's area on a hypersphere and ignores the position where the region locates on the

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hypersphere. Actually, different regions with an equal area will generally contain different number of inputs due to the input discrete feature. Secondly, when the input dimension of an Adaline becomes larger, the input vertexes on the hypersphere become sparser, and this will cause the assumption to become further inaccurate. Following Hoff and Glanz's approach, Stevenson et al. [3] made further study on the sensitivity of both Adalines and Madalines. They used the surface of a hypersphere with the radius of $\sqrt{n}$ to approximate the input space of an Adaline and considered the angle θ between the two hyperplanes determined by the unperturbed and perturbed weights as a random variable. Based on that approximation and consideration, they derived some approximate formulae under the conditions that the input and weight perturbations are small and the input dimension of Adalines is sufficiently large. Unfortunately, as discussed above, the input space they approximated is not accurate even if the conditions are satisfied. Further, the restrictions on the formulae are also too strong to real applications. For Madalines' sensitivity, they adopted a bottom-up approach, which computed the sensitivity layer by layer, from the first to the last. However, compared with an isolated Adaline, Adalines in a Madaline may not only suffer from the weight perturbation but also the input perturbation in possibility, which will cause a complex combinational computation. For solving this problem, they approximated the real input perturbations of a layer by the mean of the input perturbations, and programmed an algorithm to compute the Madalines' sensitivity. Different from the hypersphere model, Piché [5] defined the sensitivity as the ratio of the variance of the output error to the variance of the output and proposed a stochastic model to study the sensitivity of Adalines by assuming that the inputs and weights as well as perturbations are all independent, identically distributed with mean zero. Under such a stochastic model and on the condition that the perturbations are small enough, he derived an analytical expression to compute the sensitivity. However, his approach is only applicable to a population of networks, but not to any individual ones because of its too general assumptions.

Recently, based on the same definition as that of Hoff's, Zeng et al. [12] proposed an approach to accurately compute the sensitivity of Adalines by setting up a mathematical model with hypercube and making use of analytical geometry techniques. However, although this approach is accurate and has relatively low average complexity, its worst complexity is still on exponential order with respect to the input dimension, and thus not applicable to the Adalines with high input dimension. In order to overcome the deficiency, Wang et al. [13] established another stochastic model, in which only the Adalines with bias being zero and input dimension being large enough are considered, and the summation of weighted inputs is regarded as a random variable. Based on the model, they derived a formula to approximately compute the Adaline's sensitivity. Unfortunately, the formula is not general enough to tackle Adalines having a bias. Actually the formula is equivalent to Stevenson's [3] formula under the condition of bias being zero and weight perturbation being very small although they are derived from different perspectives and expressed in different forms. As to the sensitivity of Madalines, they employed the same way as Stevenson's [3], and designed an algorithm to approximately compute Madalines' sensitivity based on the Adaline's formula.

From the above discussions, one could find that the sensitivity studies of Adalines and Madalines still have several weaknesses and need further exploring, such as improving computational accuracy while reducing complexity and relaxing restricted assumptions for gaining high generality and applicability. In this paper, a novel and effective approach for quantifying the sensitivity of Adalines and Madalines is proposed, which is defined as the probability of output inversion due to its weight perturbation and offers certain advantages over the foregoing ones. For example, it does not demand the weight perturbation to be very small and the perturbation ratio to be the same for all Adalines in a Madaline as Stevenson's [3] does; its stochastic model has fewer assumptions on inputs, weights, and their perturbations and is more direct and exact than that of Piché [5] (because the probability employed by our approach reflects the Madaline's output inversion more directly and exactly than the ratio of variances by Piché's); it does not demand the bias of Adalines be zero as Wang's [13] does, which is just a special case of this proposed approach; its computational complexity is less than Stevenson's [3] and Wang's [13] for computing the Madaline sensitivity, yet its computational accuracy is much higher than theirs. In sum, the contributions of this paper are:

(1) It gracefully applies matrix and probability theories to approximately compute the Adaline sensitivity with high accuracy.
(2) It rationally transforms a summation of complicated discrete combinations to an easy continuous integration to approximately compute the Madaline sensitivity with low computational complexity.
(3) The proposed approach is more general, extendable, and applicable than other existing approaches.

The rest of this paper is arranged as follows. The Madaline model and some preliminary knowledge are briefly described in Section 2. The definition and computation of the Adaline sensitivity and the Madaline sensitivity are discussed in Sections 3 and 4 respectively. In Section 5, experimental results are presented to demonstrate the accuracy of the quantified sensitivity. Finally, Section 6 concludes this paper.

2. Madaline model and preliminaries

In the following discussion, it is assumed that a scalar is denoted by a small letter, a vector or matrix by a capital letter, a random scalar variable by a small letter in bold, and a random vector or matrix variable by a capital letter in bold.

2.1. Model

A Madaline is a typical kind of binary feedforward neural networks consisting of several layers of neurons called Adalines. An Adaline is in structure composed of input, weight, bias, activation function and output. In this paper, $X=(x_1,x_2,\ldots,x_n)^T \in \{-1,1\}^n$ denotes an Adaline's input, $W=(w_1,w_2,\ \ldots,w_n)^T \in R^n$ denotes its weight, w_0 is its bias, its activation function is $f(v)=\begin{cases}-1, & v<0\\ 1, & v\geq 0\end{cases}$, and its output is $y\in\{-1,1\}$. Fig. 1 illustrates the structure of an Adaline with n inputs, and a mapping between the input and output can be established as follows:

$$y=f(W^TX+w_0)=f\left(\sum_{j=1}^{n} w_jx_j+w_0\right)=\begin{cases}-1, & \sum_{j=1}^{n} w_jx_j+w_0<0\\ 1, & \sum_{j=1}^{n} w_jx_j+w_0\geq 0\end{cases} \tag{2.1}$$

A Madaline is a layered network by linking Adalines together, in which links only exist between Adalines of two adjacent layers, and there is no link between Adalines in the same layer and in any two non-adjacent layers. All Adalines in a layer are fully linked from every Adaline in the immediately preceding layer and are