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Design and analysis of associative memories based on external inputs of delayed recurrent neural networks

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ABSTRACT

This paper presents two new design procedures for synthesizing autoassociative memory and hetero-associative memory based on recurrent neural networks with different external inputs and mixed delays. Sufficient criteria are established to guarantee the global exponential stability for recurrent neural networks with mixed delays. The design procedures are dependent on external inputs, which ensure to restore accurate memories. The stored patterns are associated with their retrieval probes internally in a robust and fault tolerant way. The obtained results improve and extend some related works. Two illustrative examples are given to verify the effectiveness of the theoretical results.

1. Introduction

An associative memory is a system which stores mappings from specific input patterns to specific output patterns. In the past few decades, researchers have done extensive works on design procedures of associative memories [1–11]. As implementation tools, continuous-time recurrent neural networks and discrete-time recurrent neural networks with time delays have been studied extensively [12–17].

Generally, there exist two methods to design associative memories based on recurrent neural networks. The first one is dependent on the initial values of neural network states. In this method, existence of many equilibria of neural networks is a necessary feature. Multistability of recurrent neural networks has received much attention, for example, see [18–30], and the references therein. However, there is a defect in this method. The neural network system may yield many spurious equilibria due to dependence on initial states. The second one is dependent on the external inputs of neural network models. In this method, each trajectory globally converges to a unique equilibrium point dependent on the external input. And they would not produce spurious equilibria. Many design procedures have been proposed in this method by researchers, for example, see [7–11].

In electronic implementation of analog neural networks, time delays always exist due to the transmission of signal and the finite switching speed of amplifiers. The existence of time delays may lead to instability and oscillation in a neural network. Extensive works have been done on the stability analysis of recurrent neural networks with time delays and their variants [31–35]. In simple circuits having a small number of cells, discrete delays can serve as a good approximation. As for the presence of an amount of parallel pathways of a variety of axon sizes and lengths, it is desirable to introduce distributed delays. Therefore, the stability of neural networks with discrete delays and distributed delays has become a topic of great theoretical and practical significance.

In the multi-equilibrium associative memories, accurate pattern cannot be guaranteed because each neuron state converges to one of the locally stable equilibria. In order to overcome this defect, the authors designed associative memory procedures by assuring each neuron state converged to a unique equilibrium point in [7–11]. However, there exist some shortages in these literature. In [7,8], globally asymptotically stable criteria were established. In [10], the self-inhibition is restricted to be equal to 1. In [11], the initial values of neuron states were fixed to zero. It is well known that the results will be less conservative if the neuron states are globally exponentially stable. In [29,30], the authors derived the relationship between convergence of neuron states and external inputs of neural networks. This provides us an inspiration whether we can use the relationship to design an associative memory based on recurrent neural networks. To the best of the authors' knowledge, recurrent neural networks with

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mixed delays are seldom considered as associative memory. Based on the above discussion, this paper proposes two new design procedures of autoassociative memory and heteroassociative memory based on recurrent neural networks with mixed delays. Globally exponentially stable criteria are established for recurrent neural networks with mixed delays. A remarkable feature of our methodology is that the memorized patterns depend on external inputs of recurrent neural networks, which ensures to restore accurate memory. And bias vectors are chosen in large ranges. This makes the associative memory more robust. Moreover, the design methods relax the conservation of the relationship of parameters of recurrent neural networks.

The remainder of this paper consists of the following sections. Section 2 describes some preliminaries and problem formulations. The main results are stated in Section 3. In Section 4, two examples are used to show the effectiveness of the obtained results. Finally, in Section 5, the conclusion is drawn.

2. Problem description and preliminaries

In this paper, we will propose two design procedures for binary autoassociative memory and heteroassociative memory based on recurrent neural networks. Denote $\{-1,1\}^n$ as the set of n-dimensional bipolar vectors, i.e., $\{-1,1\}^n=\{x\in\mathbb{R}^n,\ x=(x_1,x_2,\ldots,x_n)^T, x_i=1 \text{ or } -1, i=1,2,\ldots,n\}$. Denote $\{-1,1\}^n\times\{-1,1\}^{\overline{n}}$ as the product of the set of n-dimensional and $\overline{n}$-dimensional bipolar vectors, i.e., $\{-1,1\}^n\times\{-1,1\}^{\overline{n}}=\{(x,y)\in\mathbb{R}^{n\times\overline{n}},\ x=(x_1,x_2,\ldots,x_n)^T,\ x_i=1\text{or } -1,\ i=1,2,\cdots,n,\ y=(y_1,y_2,\ldots,y_{\overline{n}})^T,\ y_j=1 \text{ or } -1,\ j=1,2,\ldots,\overline{n}\}$. The design problem can be depicted as follows.

Design problem: Given $m(m\le\min\{2^n,2^{\overline{n}}\})$ pairwise vectors $(p^{(1)},u^{(1)})$, $(p^{(2)},u^{(2)})$, …, $(p^{(m)},u^{(m)})$, where $(p^{(k)},u^{(k)})\in\{-1,1\}^n\times\{-1,1\}^{\overline{n}}$, $k\in\{1,2,\ldots,m\}$, design an associative memory based on recurrent neural networks such that if $(u_1^{(k)},u_2^{(k)},\ldots,u_{\overline{n}}^{(k)})^T$ is fed to the associative memory from its input as a probe, then the output vector of neural network converges to a corresponding pattern $(p_1^{(k)},p_2^{(k)},\ldots,p_n^{(k)})^T$, $k\in\{1,2,\ldots,m\}$. When $p^{(k)}=u^{(k)}$, $p^{(k)}$ is said to be autoassociatively memorized with $u^{(k)}$ in the associative memory. Otherwise, $p^{(k)}$ is said to be heteroassociatively memorized with $u^{(k)}$.

For the case of autoassociative memory, we consider the following continuous-time recurrent neural networks with both discrete and distributed delays:

$$\dot{x}_i(t)=-d_ix_i(t)+\sum_{j=1}^{n}a_{ij}f_j(x_j(t))+\sum_{j=1}^{n}b_{ij}f_j(x_j(t-\tau_{ij}))+\sum_{j=1}^{n}c_{ij}\int_{-\infty}^{t}K_{ij}(t-\theta)f_j(x_j(\theta))\,d\theta+u_i, \tag{1}$$

where $i=1,2,\ldots,n$; $x(t)=(x_1(t),x_2(t),\ldots,x_n(t))^T\in\mathbb{R}^n$ is the neuron state vector; n denotes the number of neurons; $D=\text{diag}(d_1,d_2,\ldots,d_n)$ is a diagonal matrix, $0<d_i\le d$ $(d=\max_{1\le i\le n}\{d_i\})$ represents the self-inhibition with which the i-th neuron will reset its potential to the resting state in isolations when disconnected from the network; $A=(a_{ij})_{n\times n}$, $B=(b_{ij})_{n\times n}$ and $C=(c_{ij})_{n\times n}$ are the connection weight matrices that are not assumed to be symmetric; τ_{ij} is the constant delay with $0<\tau_{ij}\le\tau$ $(\tau=\max_{1\le i,j\le n}\{\tau_{ij}\})$; $f(x(t))=(f_1(x_1(t)),f_2(x_2(t)),\ldots,f_n(x_n(t)))^T$ denotes the neuron activation function; the delay kernel $K_{ij}(\theta):[0,+\infty)\to[0,+\infty)$ is bounded, piecewise continuous and satisfies

$$\int_0^{+\infty}K_{ij}(\theta)\,d\theta=1,\ \int_0^{+\infty}K_{ij}(\theta)e^{\mu\theta}\,d\theta<+\infty,$$

for some positive constant μ and $i,j=1,2,\ldots,n$; $u=(u_1,u_2,\ldots,u_n)^T$ stands for the external constant input.

For the case of heteroassociative memory, we consider the following continuous-time recurrent neural networks with both discrete and distributed delays:

$$\dot{x}_i(t)=-d_ix_i(t)+\sum_{j=1}^{n}a_{ij}f_j(x_j(t))+\sum_{j=1}^{n}b_{ij}f_j(x_j(t-\tau_{ij}))+\sum_{j=1}^{n}c_{ij}\int_{-\infty}^{t}K_{ij}(t-\theta)f_j(x_j(\theta))\,d\theta+\sum_{j=1}^{\overline{n}}e_{ij}\overline{u}_j+v_i, \tag{2}$$

where $E=(e_{ij})_{n\times\overline{n}}$ is the input matrix; $\overline{u}=(\overline{u}_1,\overline{u}_2,\ldots,\overline{u}_{\overline{n}})^T$ stands for the external constant input; $v=(v_1,v_2,\ldots,v_n)^T$ is a bias used to compensate the parameter perturbations or random disturbance on inputs.

Consider the following standard activation function:

$$f_j(\xi)=\tfrac{1}{2}(|\xi+1|-|\xi-1|),\quad j=1,2,\ldots,n. \tag{3}$$

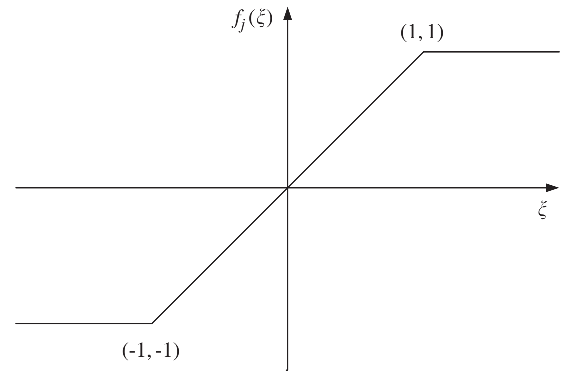

Fig. 1. Typical configuration of the activation function (3).

Typical configuration of the activation function is depicted in Fig. 1.

System (1) is equivalent to

$$\dot{x}_i(t)=-d_ix_i(t)+\sum_{j=1}^{n}a_{ij}f_j(x_j(t))+\sum_{j=1}^{n}b_{ij}f_j(x_j(t-\tau_{ij}))+\sum_{j=1}^{n}c_{ij}\int_0^{+\infty}K_{ij}(\theta)f_j(x_j(t-\theta))\,d\theta+u_i, \tag{4}$$

Initial condition associated with (1) (and (2)) is assumed to be

$$\phi(\theta)=(\phi_1(\theta),\phi_2(\theta),\ldots,\phi_n(\theta))^T,\quad \theta\in(-\infty,0]. \tag{5}$$

Denote the norm of ϕ as

$$\|\phi\|=\max_{1\le i\le n}\left\{\sup_{\theta\in(-\infty,0]}|\phi_i(\theta)|\right\}.$$

The definition of the exponential stability is now given.

Definition 1. The equilibrium point x^* of system (1) is said to be globally exponentially stable, if there exist constants $\alpha>0$, $\beta>0$ such that for any $t\ge0$

$$\|x(t;\phi)-x^*\|\le\beta\|\phi-x^*\|e^{-\alpha t},$$

where $x(t;\phi)$ is the solution of system (1) with any initial condition $\phi(\theta)$, $\theta\in(-\infty,0]$.

In order to obtain the main results, we need the following lemma.

Lemma 1 (*see [36]*). *Let $\mathbb{D}$ be a bounded and closed set in $\mathbb{R}^n$, and H be a mapping on complete matric space $(\mathbb{D},\|\cdot\|)$, where $\|x-y\|=\max_{1\le i\le n}\{|x_i-y_i|\}$ is measurement in $\mathbb{D}$, for any $x,y\in\mathbb{D}$. If $H(\mathbb{D})\subset\mathbb{D}$ and there exists a positive constant $\alpha<1$ such that for*