Neurocomputing 129 (2014) 401–408

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Improved delay-dependent stability criteria for recurrent neural networks with time-varying delays

Xiangbing Zhou [a], Junkang Tian [b,*], Hongjiang Ma [a], Shouming Zhong [b]

[a] Department of Computer Science, Aba Teachers College, WenChuan, Sichuan 623002, China
[b] School of Mathematical Sciences, University of Electronic Science and Technology of China, Chengdu, Sichuan 611731, China

ARTICLE INFO

Article history:
Received 28 January 2013
Received in revised form
2 June 2013
Accepted 19 September 2013
Communicated by S. Arik
Available online 22 October 2013

Keywords:
Delay-dependent stability
Neural networks
Time-varying delays
Linear matrix inequality (LMI)

ABSTRACT

This paper is concerned with the problem of delay-dependent stability criteria for recurrent neural networks with time-varying delays. A new class of Lyapunov functional is introduced by decomposing the delays in all integral terms. By exploiting all possible information in various delay intervals and using reciprocally convex approach, some less conservative stability criteria are obtained in terms of linear matrix inequalities (LMIs). Finally, two numerical examples are given to illustrate the effectiveness of the derived results.

1. Introduction

In the past few decades, stability analysis for recurrent neural networks has been extensively investigated for their successful applications in various fields, such as pattern recognition, image processing, and associative memories. Time delays are unavoidably encountered in neural networks, and often lead to instability and oscillation. Thus, considerable attention has been focused on recurrent delayed neural networks. Many interesting stability conditions, including delay-independent results [1–3], and delay-dependent results [4–42], have been proposed for the stability of delayed neural networks. Generally speaking, delay-dependent stability criteria, which include information concerning the time delays are usually less conservative than delay-independent ones, especially the time delays are small. For the delay-dependent case, some criteria have been derived by using Lyapunov–Krasovskii functional (LKF). Construction of an appropriate LKF is crucial for obtaining less conservative stability conditions. In recent years, some new improved methods have been developed for reducing conservativeness, such as free-weighting matrix method [5–8], augmented LKF method [9], complete delay decomposing approach [18], delay-slope-dependent method [19] and so on. Obviously, it is hard to obtain less conservative results by employing the identical Lyapunov–Krasovskii functional. Thus, some new Lyapunov–Krasovskii functionals based on decomposing the time delay interval were introduced to investigate the stability of delayed neural networks, which significantly reduced the conservativeness of the obtained stability criteria [12–18]. Recently, a novel method was proposed in [12] for delayedHopfield neural networks, which divides the delay interval $[0,h]$ into subintervals with the same size. This method considering much information about the delay interval $[0,h]$ can reduce the conservativeness. Very recently, a novel Lyapunov–Krasovskii functional decomposing the delay in all integral terms is introduced in [18]. By considering independent upper bounds of the delay derivative in various delay interval, some new delay dependent stability criteria are obtained in terms of linear matrix inequalities. The derived results proved to be less conservative than most of the previous results, and the conservatism can be notably reduced by thinning the delay partitioning. However, these methods suffer two common shortcomings. First, the relationship between time-varying delay and each subinterval is completely neglected when the delay is time-varying. For example, for any integer $m \geq 1$, let $h = d/m$, then the delay interval $[0,d]$ has been divided into m segments, ie., $[0,d] = \cup_{i=1}^{m} [(i-1)h, ih]$. It is well known that the time-varying delay $\tau(t)$ can appear in each subinterval. When dealing with the $\dot{V}(z(t))$, [13–15] only consider the information $0 \leq \tau(t) \leq d$, but the information $(k-1)h \leq \tau(t) \leq kh$, $k = 1, 2, \ldots, m$ is ignored, which may lead to some conservatism. Second, the information of neuron activation functions are not adequately considered, which may lead to some conservatism. Thus, there is room for further investigation.

* Corresponding author. Tel.: +86 28 61831290; fax: +86 28 61831299.
E-mail address: junkangtian2010@163.com (J. Tian).

0925-2312/$ - see front matter © 2013 Elsevier B.V. All rights reserved.
http://dx.doi.org/10.1016/j.neucom.2013.09.019

In this paper, similar to some existing results, for any integer $m \geq 1$, let $h = d/m$, then the delay interval $[0, d]$ has been divided into m segments, ie., $[0, d] = \cup_{i=1}^{m} [(i-1)h, ih]$. But different from [13–15], when handling $\dot{V}(z_t)$, not only the relationship between time-varying delay $\tau(t)$ and the delay interval $[0, d]$ is considered, but also the relationship between time-varying delay $\tau(t)$ and each subinterval $[(k-1)h, kh], k = 1, 2, \ldots, m$ is considered, which may lead to less conservative results. By taking more information of states and activation functions as augmented vectors, an augmented Lyapunov–Krasovskii functional is introduced. Then, inspired by the results of [44], a less conservative result is obtained to guarantee asymptotically stable neural networks with time- varying delays. Finally, two numerical examples are given to indicate significant improvements over some existing results.

2. Problem formulation

Consider the following recurrent neural networks with time-varying delays:

$$\dot{x}(t) = -Cx(t) + Ag(x(t)) + Bg(x(t-\tau(t))) + \mu \tag{1}$$

where $x(t) = [x_1(t), x_2(t), \ldots, x_n(t)]^T \in \mathcal{R}^n$ is the neuron state vector, $g(x(\cdot)) = [g_1(x_1(\cdot)), g_2(x_2(\cdot)), \ldots, g_n(x_n(\cdot))]^T \in \mathcal{R}^n$ denotes the neuron activation function, $\mu = (\mu_1, \mu_2, \ldots, \mu_n)^T \in \mathcal{R}^n$ is a constant input vector. $A \in \mathcal{R}^{n \times n}$ is the connection weight matrix and $B \in \mathcal{R}^{n \times n}$ is the delayed connection weight matrix. $C = \text{diag}(C_1, C_2, \ldots, C_n)$ is a diagonal matrix with $C_i > 0, i = 1, 2, \ldots, n$. $\tau(t)$ is time-varying continuous function that satisfies $0 \leq \tau(t) \leq d$, $\dot{\tau}(t) \leq u$, where d and u are constants. In addition, it is assumed that each neuron activation function $g_i(\cdot), i = 1, 2, \ldots, n$, satisfies the following condition:

$$k_i^- \leq \frac{g_i(x) - g_i(y)}{x - y} \leq k_i^+, \quad \forall x,\ y \in R,\ x \neq y,\ i = 1, 2, \ldots, n \tag{2}$$

where $k_i^-, k_i^+, i = 1, 2, \ldots, n$ are constants.

Assuming that $x^* = [x_1^*, x_2^*, \ldots, x_n^*]^T$ is the equilibrium point of system (1) whose uniqueness has been proven in [21]. Using the transformation $z(\cdot) = x(\cdot) - x^*$, system (1) is converted to the following system:

$$\dot{z}(t) = -Cz(t) + Af(z(t)) + Bf(z(t-\tau(t))) \tag{3}$$

where $z(t) = [z_1(t), z_2(t), \ldots, z_n(t)]^T$, $f(z(\cdot)) = [f_1(z_1(\cdot)), f_2(z_2(\cdot)), \ldots, f_n(z_n(\cdot))]^T$ and $f_i(z_i(\cdot)) = g_i(z_i(\cdot) + x_i^*) - g_i(x_i^*), i = 1, 2, \ldots, n$. From the inequality (2), we obtain

$$k_i^- \leq \frac{f_i(z_i(t))}{z_i(t)} \leq k_i^+ \quad f_i(0) = 0, \quad i = 1, 2, \ldots, n \tag{4}$$

Thus, under this assumption, the following inequality holds for any diagonal matrix $R > 0$:

$$z^T(t)KRKz(t) - f^T(z(t))Rf(z(t)) \geq 0 \tag{5}$$

where $K = \text{diag}(k_1, k_2, \ldots, k_n)$, $k_i = \max(|k_i^-|, |k_i^+|)$

Lemma 1 (*See Boyd et al. [43]*). *For any constant matrix $Z \in \mathcal{R}^{n \times n}, Z = Z^T > 0$, scalars $h_2 > h_1 > 0$, then*

$$-(h_2 - h_1) \int_{t-h_2}^{t-h_1} x^T(s) Z x(s)\, ds \leq -\int_{t-h_2}^{t-h_1} x^T(s)\, ds\, Z \int_{t-h_2}^{t-h_1} x(s)\, ds \tag{6}$$

3. Main results

In this section, a new Lyapunov functional is proposed and a less conservative delay-dependent stability criterion is obtained.

Theorem 1. *Given scalars $d \geq 0, u$, diagonal matrices $K_1 = \text{diag}(k_1^-, k_2^-, \ldots, k_n^-)$, $K_2 = \text{diag}(k_1^+, k_2^+, \ldots, k_n^+)$, for positive integer $m \geq 1$, $h = d/m$, the system (3) is globally asymptotically stable if there exist symmetric positive matrices $X = [X_{ij}]_{m \times m}$, $Q = [Q_{ij}]_{2 \times 2}$, $Y^{(k)} = [Y_{ij}^{(k)}]_{2 \times 2}$, $R_k (k = 1, 2, \ldots, m)$, $\hat{R}_k (k = 1, 2, \ldots, m)$, positive diagonal matrices $T_1, T_2, R, \Delta = \text{diag}(\delta_1, \delta_2, \ldots, \delta_n), \Lambda = \text{diag}(\lambda_1, \lambda_2, \ldots, \lambda_n)$, and any matrices $N_{k1}, N_{k2}, M_{k1}, M_{k2}, S_k (k = 1, 2, \ldots, m)$ with appropriate dimensions, such that for $k = 1, 2, \ldots, m$*

$$\Omega^{(k)} = \begin{bmatrix} \Omega_{11}^{(k)} & \mathcal{A}^T \hat{R} \\ * & -\hat{R} \end{bmatrix} < 0 \tag{7}$$

$$\Xi_1^{(k)} = \begin{bmatrix} Y_{11}^{(k)} & Y_{12}^{(k)} & N_{k1} \\ * & Y_{22}^{(k)} & N_{k2} \\ * & * & \hat{R}_k \end{bmatrix} > 0 \tag{8}$$

$$\Xi_2^{(k)} = \begin{bmatrix} Y_{11}^{(k)} & Y_{12}^{(k)} & M_{k1} \\ * & Y_{22}^{(k)} & M_{k2} \\ * & * & \hat{R}_k \end{bmatrix} > 0 \tag{9}$$

$$\Psi^{(k)} = \begin{bmatrix} R_k & S_k \\ * & R_k \end{bmatrix} > 0 \tag{10}$$

where

$$\Omega_{11}^{(k)} = \Sigma + \Phi_{11} + \overline{\Phi}_{11}^{(k)} + \tilde{\Phi}_{11}^{(k)}$$

$$\Sigma = \begin{bmatrix} \Sigma_{11} & X_{12} & \cdots & X_{1m} & 0 & 0 & \Sigma_{1(m+3)} & \Sigma_{1(m+4)} \\ * & X_{22} - X_{11} & \cdots & X_{2m} - X_{1(m-1)} & -X_{1m} & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ * & * & \cdots & X_{mm} - X_{(m-1)(m-1)} & -X_{(m-1)m} & 0 & 0 & 0 \\ * & * & \cdots & * & -X_{mm} & 0 & 0 & 0 \\ * & * & \cdots & * & * & \Sigma_{(m+2)(m+2)} & 0 & \Sigma_{(m+2)(m+4)} \\ * & * & \cdots & * & * & * & \Sigma_{(m+3)(m+3)} & (\Lambda - \Delta)B \\ * & * & \cdots & * & * & * & * & \Sigma_{(m+4)(m+4)} \end{bmatrix}$$

$$\Phi_{11} = \begin{bmatrix} \varphi_1 + \hat{\varphi}_1 & R_1 + \hat{R}_1 & \cdots & 0 & 0 & 0 & 0 & 0 \\ * & \varphi_2 + \hat{\varphi}_2 & \cdots & 0 & 0 & 0 & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots & \vdots & \vdots & \vdots \\ * & * & \cdots & \varphi_m + \hat{\varphi}_m & R_m + \hat{R}_m & 0 & 0 & 0 \\ * & * & \cdots & * & \varphi_{m+1} + \hat{\varphi}_{m+1} & 0 & 0 & 0 \\ * & * & \cdots & * & * & 0 & 0 & 0 \\ * & * & \cdots & * & * & * & 0 & 0 \\ * & * & \cdots & * & * & * & * & 0 \end{bmatrix}$$

$$\overline{\Phi}_{11}^{(k)} = (\overline{\Psi}_{ij}^{(k)})_{(m+4)\times(m+4)} + (\overline{\Psi}_{ij}^{(k)})^T_{(m+4)\times(m+4)} + (\hat{\Psi}_{ij}^{(k)})_{(m+4)\times(m+4)} + (\hat{\Psi}_{ij}^{(k)})^T_{(m+4)\times(m+4)}$$

$$\tilde{\Phi}_{11}^{(k)} = (\tilde{\Psi}_{ij}^{(k)})_{(m+4)\times(m+4)} + (\tilde{\Psi}_{ij}^{(k)})^T_{(m+4)\times(m+4)}$$

with

$$\mathcal{A} = [-C\ \ 0\ \ \cdots\ \ 0\ \ 0\ \ 0\ \ A\ \ B]$$

$$\hat{R} = \sum_{i=1}^{m} h^2 R_i + \sum_{i=1}^{m} h \hat{R}_i$$

$$\Sigma_{11} = -PC - CP - (K_2\Delta - K_1\Lambda)C - C(K_2\Delta - K_1\Lambda) + KRK + Q_{11} - K_2 Q_{12}^T - Q_{12}K_2 + K_2 Q_{22} K_2 - 2K_2 T_1 K_1 + hY_{11}^{(k)}$$

$$\Sigma_{1k} = N_{k1}, \quad \Sigma_{1(k+1)} = -M_{k1}, \quad \Sigma_{1(m+2)} = -N_{k1} + M_{k1} + hY_{12}^{(k)},$$

$$\Sigma_{k(m+2)} = N_{k2}^T, \quad \Sigma_{(k+1)(m+2)} = -M_{k2}^T$$

$$\Sigma_{1(m+3)} = PA - C(\Lambda - \Delta) + (K_2\Delta - K_1\Lambda)A + Q_{12} - K_2 Q_{22} + K_2 T_1 + K_1 T_1$$

$$\Sigma_{1(m+4)} = PB + (K_2\Delta - K_1\Lambda)B$$

$$\Sigma_{(m+2)(m+2)} = -(1-u)KRK - (1-u)Q_{11} + (1-u)Q_{12}K_2 + (1-u)K_2 Q_{12}^T - (1-u)K_2 Q_{22} K_2 - 2K_2 T_2 K_1 - N_{k2} - N_{k2}^T + M_{k2} + M_{k2}^T + hY_{22}^{(k)}$$

$$\Sigma_{(m+2)(m+4)} = -(1-u)Q_{12} + (1-u)K_2 Q_{22} + (K_1 + K_2)T_2$$

$$\Sigma_{(m+3)(m+3)} = (\Lambda - \Delta)A + A^T(\Lambda - \Delta) - R + Q_{22} - 2T_1$$

$$\Sigma_{(m+4)(m+4)} = (1-u)R - (1-u)Q_{22} - 2T_2$$