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Distributed cooperative tracking of uncertain nonlinear multi-agent systems with fast learning

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ABSTRACT

In this paper, a distributed adaptive control architecture is proposed for cooperative tracking of uncertain dynamical multi-agent systems over an undirected network. Adaptive cooperative tracking controllers with both static and adaptive coupling gains are developed using local information obtained from neighboring agents. Neural networks together with filtering adaptive laws are employed to enable fast learning in the presence of unmodeled uncertainties and external disturbances. Moreover, this result is extended to the output feedback case in which only partial state information of each agent can be measured. Observer-based adaptive cooperative tracking controllers with static and adaptive coupling are developed, and a parameter dependent Riccati equation is employed to derive the stability of the overall multi-agent systems. Compared with the existing results, a distinct feature of the developed controllers enables fast learning using large adaptive gains with guaranteed low frequency control signals. Two illustrative examples are given to validate the efficacy of the proposed approaches.

1. Introduction

Recent years have witnessed increasing interest in cooperative control of multi-agent systems due to its widespread applications in engineering. The ubiquitous feature of a cooperative system is the distribution of information. Due to the fact a large lumber of agents are usually involved in the network, centralized controllers based on the information gathered by all agents are generally impractically to implement. Therefore, distributed control strategies based on local information have been widely investigated [1–13].

In the literature, a great deal of effort has been dedicated to two control problems of multi-agent systems. One is *cooperative regulation* problem where controllers based on local information are pursued to drive the state of each agent to a common value. The other is *cooperative tracking* problem where there exists one leader who acts as a trajectory generator for the entire group to follow. The key challenge different from the traditional control is that only a fraction of agents in the group have access to the trajectory information of the leader. In the past few years, many elegant results on these two topics have been obtained. Readers are referred to [3–13] and references therein.

Cooperative tracking of linear systems with first-order, second-order, high-order and general linear dynamics is presented in [10–13]. In [10], a distributed observer is proposed to solve coordinated tracking problem of first-order linear dynamics. In [11], a coordinated tracking algorithm is developed for first-order systems with a time-varying leader. In [12], consensus algorithms are devised for second-order linear system. In [13], a model reference consensus controller is proposed for high-order linear system. Cooperative tracking control of general linear system is discussed in [16–18]. Note that the all aforementioned works are dedicated to linear systems. However, most real-world agents are governed by nonlinear systems [14,15], and cooperative tracking control of nonlinear systems is more challenging.

Cooperative tracking control of first-order nonlinear systems on strongly connected graphs is studied in [19]. This result is extended to second-order nonlinear systems [20] and further to high-order nonlinear systems [21]. It is worthwhile to mention that in these studies, standard adaptive control methods have been used to achieve the cooperative control without excessive knowledge of system model. However, standard adaptive control methods have a limitation, that is, the using of large adaptive gains to achieve system performance may result in high-frequency oscillations in control channels as pointed out in [22].

In this paper, we focus on the cooperative tracking problem of uncertain multi-agent systems with general nonlinear dynamics over an undirected network motivated by the previous works [18,21,22]. The follower agents are governed by high-order non-Brunovsky form nonlinear dynamics, and only a fraction of follower agents have access to the state information of the leader. We first consider the state feedback case by assuming that all state information can be measured. Two neural cooperative tracking

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controllers, with static and adaptive coupling gains, respectively, are developed, and uniform ultimate boundedness of cooperative tracking errors are guaranteed by rigorous Lyapunov design. Then, we consider the output feedback case by assuming that only partial state information or just the output information can be measured. A local observer is designed to recover the unmeasurable state. We show that the stability of the overall multi-agent systems can be derived based on a parameter dependent Riccati equation. For both cases, filtering adaptive laws are employed to enable fast learning in the presence of unmodeled uncertainties and disturbances.

Comparisons with existing results are as follows: Firstly, compared with the agent dynamics considered in [19–23], the agent model discussed in this paper are more general which contains first-order, second-order and high-order nonlinear systems as special cases. Furthermore, the time-varying disturbances, unmodeled uncertainties and unknown input of leader are treated as one form to be approximated. In this regard, the developed controller can be applicable to a larger class of nonlinear systems. Secondly, in contrast to the adaptive cooperative tracking controllers proposed in [19–23], the cooperative controllers in this paper permit fast learning without generating high-frequency oscillations in control signals. Finally, filtering adaptive laws are employed to design the cooperative state and output feedback tracking controllers which result in new distributed adaptive control architectures with guaranteed low frequency control signals.

This paper is organized as follows. Section 2 formulates the control problem and introduces some preliminaries. Section 3 presents the state feedback design and gives the stability analysis results. The above result is extended to the output feedback case in Section 4. Examples are given in Section 5 for illustrations. Conclusions are drawn in Section 6.

Throughout the paper, the Euclidean norm, Frobenius norm, minimum singular value, maximum singular value, and trace of a given matrix are denoted by $\|\cdot\|$, $\|\cdot\|_F$, $\underline{\sigma}(\cdot)$, $\overline{\sigma}(\cdot)$, and $\mathrm{tr}\{\cdot\}$, respectively. A diagonal matrix is represented by $\mathrm{diag}\{\lambda_1,\ldots,\lambda_N\}$ with λ_i being the ith diagonal element. An identity matrix of dimension N is denoted by I_N. The Kronecker product is denoted by $\otimes$.

2. Preliminaries and problem formulation

2.1. Problem formulation

Consider a class of nonlinear multi-agent systems consisting of N follower agents and a leader agent. The dynamics of the follower agents are given by

$$\dot{x}_i = Ax_i + B[u_i + f_i(x_i,t)],\\ y_i = Cx_i,\quad i=1,\ldots,N, \tag{1}$$

where $x_i=[x_{i1},\ldots,x_{in}]^T\in\mathbb{R}^n$ represents the system state; $u_i\in\mathbb{R}^m$ is the control input; $f_i(x_i,t)\in\mathbb{R}^m$ is an unknown matched uncertain dynamics which contains bounded exogenous disturbances and unmodeled uncertainties; $A\in\mathbb{R}^{n\times n}, B\in\mathbb{R}^{n\times m}$, and $C\in\mathbb{R}^{n\times m}$ are known matrices, and the triple (A,B,C) is assumed to be stabilizable and detectable.

The dynamics of the leader agent is described as

$$\dot{x}_0 = Ax_0 + Br(t), \tag{2}$$

where $x_0\in\mathbb{R}^n$ denote the leader state, and $r(t)\in\mathbb{R}^m$ represents a bounded input. Assume that for any initial conditions, the solution x_0 exists for all $t\ge 0$.

The *control objective* of this paper is to design a distributed control law u_i for each follower (1) to track the leader (2) such that the state of each follower node synchronizes to that of the leader node.

2.2. Preliminaries

2.2.1. Graph theory

Consider a network of multi-agent systems consisting of N agents and one leader. If each agent is considered as a node, the neighbor relation can be described by a *graph* $\mathcal{G}=\{\mathcal{V},\mathcal{E}\}$, where $\mathcal{V}=\{n_1,\ldots,n_N\}$ is a node set and $\mathcal{E}=\{(n_i,n_j)\in\mathcal{V}\times\mathcal{V}\}$ is an edge set with the element (n_i,n_j) that describes the communication from node i to node j. The neighbor set of the node i is denoted by $\mathcal{N}_i=\{j|(n_j,n_i)\in\mathcal{E}\}$. Define an adjacency matrix $\mathcal{A}=[a_{ij}]\in\mathbb{R}^{N\times N}$ with $a_{ij}=1$, if $(n_j,n_i)\in\mathcal{E}$, and $a_{ij}=0$, otherwise. Define the in-degree matrix as $\mathcal{D}=\mathrm{diag}\{d_1,\ldots,d_N\}$ with $d_i=\sum_{j\in\mathcal{N}_i}a_{ij}$. The Laplacian matrix $L=[l_{ij}]$ associated with the graph $\mathcal{G}$ is defined as $L=\mathcal{D}-\mathcal{A}$. If $a_{ij}=a_{ji}$, for $i,j=1,\ldots,N$, then the graph $\mathcal{G}$ is *undirected*. A *path* in a graph is an ordered sequence of nodes such that any two consecutive nodes in the sequence are an edge of the graph. An undirected graph is *connected* if there is a path between every pair of nodes. Finally, define a leader adjacency matrix as $\mathcal{A}_0=\mathrm{diag}\{a_{10},\ldots,a_{N0}\}$, where $a_{i0}>0$ if and only if the ith agent has access to the leader information; otherwise, $a_{i0}=0$. For simplicity, denote $L+\mathcal{A}_0$ by H.

Lemma 1 (*Hong et al. [10]*). *Suppose that the graph $\mathcal{G}$ is undirected and connected, and at least one agent has access to the leader. Then, H is positive definite.*

2.2.2. Projection operator

Definition 1. Let $\phi:\mathbb{R}^n\to\mathbb{R}$ be a continuously differentiable convex function given by

$$\phi(\theta)=\frac{\theta^T\theta-\theta_M^2}{\varepsilon_\theta\theta_M^2}, \tag{3}$$

where $\theta_M\in\mathbb{R}$ is a projection norm bound imposed on $\theta\in\mathbb{R}^n$ and $\varepsilon_\theta>0$ is a projection tolerance bound. Then, the projection operator $\mathrm{Proj}:\mathbb{R}^n\times\mathbb{R}^n\to\mathbb{R}^n$ is defined as

$$\mathrm{Proj}(\theta,y)\triangleq\begin{cases} y-\dfrac{\phi'(\theta)\phi'^T(\theta)y}{\|\phi'(\theta)\|^2}\phi(\theta) & \text{if } \phi(\theta)\ge 0 \text{ and } \phi'(\theta)y<0,\\ y & \text{else,}\end{cases} \tag{4}$$

where $\phi'(\theta)=d\phi/d\theta$.

One important property of the projection operator follows:

$$(\theta-\theta^*)^T[\mathrm{Proj}(\theta,y)-y]\le 0,\quad \theta^*\in\mathbb{R}^n, \tag{5}$$

where θ^* denotes the true value of the parameter θ. Moreover, the definition of the projection operator can be generalized to matrices as $\mathrm{Proj}(\Theta,Y)$, where $\Theta\in\mathbb{R}^{n\times m}$ and $Y\in\mathbb{R}^{n\times m}$. In this case, it follows from the property (5) that

$$\mathrm{tr}[(\Theta-\Theta^*)^T(\mathrm{Proj}(\Theta,Y)-Y)]\le 0,\quad \Theta^*\in\mathbb{R}^{n\times m}, \tag{6}$$

where Θ^* denotes the true value of Θ.

3. State feedback design

In the following, we start with the state feedback case by assuming that the state information of each agent can be measured. Then, we extend to consider the output feedback case where only partial state information of each agent is available. For both cases, adaptive cooperative tracking protocols are developed to enable fast learning against uncertain dynamics and disturbances.