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Delay-dependent stability criteria for time-varying delay neural networks in the delta domain

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ABSTRACT

In this paper, the delay-dependent stability criterion for time-varying delay neural networks in the delta domain is investigated. The unified neural networks, which can be used in both continues-time space and discrete-time space, takes advantage with a high sampling frequency. In the framework of the newly proposed neural networks, the delay-dependent stability criteria is derived in terms of linear matrix inequality by constructing the Lyapunov-Krasovskii function in the delta domain. A numerical simulation is given to show the effectiveness and superiority of the proposed approach.

1. Introduction

Neural networks has attracted great attentions in pattern recognition, image processing, associative memories, fixed-point computations and many other fields. In its implementation, time delays which is often caused by the limitation of the amplifier switching speed and signal propagation speed, often leads to the instability and oscillation of the neural networks. Thus, the factor of time delays in neural networks cannot be neglected and many works have been done on it [1–10]. To the best of our knowledge, the works on neural networks with time-delays can be classified into two categories: the delay-independent stability of neural networks [1–3] and the delay-dependent stability of neural networks [4–6]. The delay-independent stability criterion does not need the specific information of the time-delays and can be applied in neural networks with unknown time delays. However, it does not take use of the upper and lower bound of the time delays which are often available in practical situations, leading to conservation as a result. Therefore, many researchers have focused on the delay-dependent stability of neural networks with time varying delays, no matter in the continues-time domain [1–6] or the discrete-time domain [7–10]. It is noteworthy that the common ground of [7–10] is to use the shift operator for discretization. But shift operator performs poor with a small sampling interval and high frequency sampling system is universal in industry. So in this paper, we propose the time-varying delay neural networks in the delta domain which takes advantages in the fast sampling system and unifies both the continues time domain and the discrete time domain.

Delta operator systems (DOSs) is firstly proposed by Middleton and Goodwin [11] and has aroused great attentions since then, mainly because control systems and communication receivers require processing of the fast-sampled signals in industries extensively now. Different from the shift operator, the delta operator is a limiting case of the differential operator. Therefore, it performs better with high-speed sampling while the shift operator can lead to numerical instability under the same condition. More importantly, the delta operator unifies some previous related results of the continuous and discrete systems into the frame work of the DOSs, which is also the reason that the DOSs is usually called unified systems. Due to the superiority mentioned above, we introduce the delta operator into the modeling of neural networks with time varying delays and develop Lyapunuv functions in the delta domain to derive its stability criterion.

The rest of the paper is organized as follows. In Section 2, a brief introduction of the delta operator is given. In Section 3, the time-varying delay neural networks in the delta domain is introduced and the stability criterion is derived in terms of linear matrix inequality(LMI). The numerical simulation is given in Section 4. Section 5 gives the conclusion of this paper.

Notation: In this paper, the superscript T denotes the matrix transportation; the matrix inequality $X>Y$ means that $X-Y$ is positive definite. I denotes the identity matrix with proper dimensions and $diag(A)$ represents diagonal matrix A. Finally, $*$ denotes the symmetric item in block matrices.

2. Delta operator

In this section, we introduce some basic concepts about delta operator and show the advantage of it. Firstly, consider the

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definition of the delta operator

$$\delta x(t_k) = \begin{cases} dx(t)/dt & T=0 \\ \{x(t_k+T)-x(t_k)\}/T & T\neq 0 \end{cases} \quad (1)$$

where $t_k = kT$ and T is the sampling interval. It is evident that the delta operator is the traditional shift operator as $T=1$. When T approaches 0, the delta operator is equivalent to the differential operator which is numerical stable. On the contrary, the shift operator leads to numerical instability as $T\to 0$ by causing poles clustering at 1 as the sample rate increases. Hence, the delta operator not only unifies the continues domain and the discrete domain but also improves the numerical stability as $T\to 0$. Following the definitions of delta operator mentioned above, we obtain the relations which are important basis to derive time-varying delay neural networks in the delta domain:

$$z = e^{sT}, \quad \delta = \frac{z-1}{T} = \frac{e^{sT}-1}{T} \quad (2)$$

where z, s and δ are in the z domain, s domain and δ domain, respectively. In order to study the stability of DOSs, we introduce the Lyapunov function in the delta domain.

Definition 1. Let $V(x(t_k))$ be the Lyapunov function in the delta domain. The DOSs is stable if the following conditions are satisfied.

1. $V(x(t_k)) \geq 0$ with the equality if and only if $x(t_k)=0$
2. $\delta V(x(t_k)) = [V(x(t_k+T))-V(x(t_k))]/T < 0$

Remark 1. When $T=1$, there exists

$$\delta V(x(t_k))|_{T=1} = [V(x(k+1))-V(x(k))]/1 = \Delta V(x(k)) < 0$$

When T approaches 0, the following holds:

$$\lim_{T\to 0}\delta V(x(t_k)) = \lim_{T\to 0}\frac{V(x(t_k+T))-V(x(t_k))}{T} = \frac{dV(x(t))}{dt} < 0$$

Therefore, it is easy to see that the Lyapunov function in the delta domain can be transformed into the traditional Lyapunov functions in the continues or discrete domain when T approaches 0 or set 1.

Lemma 1 (*Li et al. [12]*). *The basic operational role of delta operator: for any $x(t_k)$ and $y(t_k)$*

$$\delta(x(t_k)y(t_k)) = \delta(x(t_k))y(t_k)+\delta(y(t_k))x(t_k)+T\delta(x(t_k))\delta(y(t_k))$$

Lemma 2 (*Yang et al. [13]*). *There exist a constant positive semi-definite matrix W and two positive integers r and r_0 satisfying $r \geq r_0 \geq 1$ such that the following inequality holds:*

$$\left(\sum_{i=r_0}^{r} x(i)\right)^T W \left(\sum_{i=r_0}^{r} x(i)\right) \leq (r-r_0+1)\sum_{i=r_0}^{r} x^T(i)Wx(i)$$

3. Primaries

In this section, we present the time-varying delay neural networks in the delta domain and develop criterion based on it. Firstly, let us introduce the continues time neural networks with interval time-varying delays as follows:

$$u'(t) = -A_s u(t)+W_{0s}g(u(t))+W_{1s}g(u(t-\tau(t)))+J_s \quad (3)$$

where $u(t)=[u_1(t)\ u_2(t)\ \cdots\ u_n(t)]^T$ denotes the neural state vector. $A_s = diag(a_{1s},a_{2s}\cdots a_{ns})$ is the diagonal matrix. W_{0s} and W_{1s} are constant matrices. $g(u(t)) = [g_1(u_1(t))\ g_2(u_2(t))\ \cdots\ g_n(u_n(t))]^T$ represents the neural activation function and J_s is the constant external input vector. $\tau(t)$ is the time-varying function. In this paper, the following assumption is made for the neural activation function.

Assumption 1. For $i=1,2\ldots n$ and $k\in R$, the following sector conditions are satisfied with the neural activation function continues and bounded.

$$0 \leq \frac{g_i(x)-g_i(y)}{x-y} \leq k \quad \forall x,y\in R \quad (4)$$

According to the Brouwer's fixed point theorem [14], there exists at least one equilibrium point $u^*(t) = [u_1^*(t)\ u_2^*(t)\ \cdots\ u_n^*(t)]^T$ in system (3) to make the following establish.

$$A_s u^*(t) = W_{0s}g(u^*(t))+W_{1s}g(u^*(t-\tau(t)))+J_s \quad (5)$$

Subtract (5) from (3), the following is derived by denoting $x(t)=u(t)-u^*(t)$:

$$x'(t) = -A_s x(t)+W_{0s}f(x(t))+W_{1s}f(x(t-\tau(t))) \quad (6)$$

where $f(x(t)) = [f_1(x_1(t))\ f_2(x_2(t))\ \cdots\ f_n(x_n(t))]^T$ and $f_i(x_i(t)) = g_i(x_i(t)+u_i^*(t))-g_i(u_i^*(t))$. The stability of (3) with the equilibrium $u^*(t)$ can be verified via proving the stability of (6). According to Assumption 1, we can obtain

$$0 \leq \frac{f_i(x_i(t))}{x_i(t)} \leq k \quad \forall x_i(t)\neq 0,\ i=1,2\ldots n \quad (7)$$

Note that (7) can be rewritten as

$$f_i(x_i(t))(f_i(x_i(t))-kx_i(t)) \leq 0 \quad \forall x_i(t)\neq 0,\ i=1,2\ldots n$$

which implies the following:

$$f_i(x_i(t_k))(f_i(x_i(t_k))-kx_i(t_k)) \leq 0 \quad \forall x_i(t_k)\neq 0,\ i=1,2\ldots n \quad (8)$$

From (2), the corresponding neural networks with interval time-varying delays in the delta domain is

$$\delta x(t_k) = Ax(t_k)+W_0 f(x(t_k))+W_1 f(x(t_k-\tau(t_k))) \quad (9)$$

where $A=(e^{-A_sT}-I)/T$, $W_0 = \int_0^T e^{-A_s(T-s)}W_{0s}\,ds/T$ and $W_1 = \int_0^T e^{-A_s(T-s)}W_{1s}\,ds/T$. Let $\tau(t_k) = \lceil\tau(t)\rceil$ where $\lceil\tau(t)\rceil$ denote the nearest integer around $\tau(t)$. $\tau(t_k)$ satisfies the inequalities $0\leq\tau_m\leq\tau(t_k)\leq\tau_M$, with $\tau_m = n_mT$ and $\tau_M = n_MT$. n_m and n_M are both known and finite integers. The following theorem is derived via the Lyapunov method by exploiting the newly proposed neural networks in the delta domain.

Theorem 1. *The time-varying delay neural networks in the delta domain described by (9) is stable if there exist positive symmetric matrices P, P_1, W_1, W_0, R, Q and positive diagonal matrices T_1, S_1 such that the following LMI holds:*

$$\Sigma_1 = \begin{bmatrix} \Sigma_1(1,1) & P_1A & P_1W_0 & P_1W_1 & 0 & 0 \\ * & \Sigma_1(2,2) & PW_0+S_1 & PW_1 & 0 & R/\tau_M \\ * & * & -2h^{-1}S_1 & 0 & 0 & 0 \\ * & * & * & -2h^{-1}T_1 & T_1 & 0 \\ * & * & * & * & -Q & 0 \\ * & * & * & * & * & -S-R/\tau_M \end{bmatrix} < 0 \quad (10)$$

where h is a positive constant value. $\Sigma_1(1,1) = TP+\tau_MR-2P_1$ $\Sigma_1(2,2) = A^TP+PA+Q+S+(\tau_M-\tau_m+T)Q-R/\tau_M$.

Proof. The following functions are chosen to construct the Lyapunov function in the delta domain.

$$V_1(t_k) = x^T(t_k)Px(t_k) \quad V_2 = T\sum_{i=1}^{n} x^T(t_k-iT)Qx(t_k-iT)$$

$$V_3(t_k) = T\sum_{i=1}^{n_M} x^T(t_k-iT)Sx(t_k-iT)$$