Neurocomputing 125 (2014) 68–71

Contents lists available at ScienceDirect

Neurocomputing

journal homepage: www.elsevier.com/locate/neucom

Stabilization of nonlinear systems with time-varying delays via impulsive control

Juan Yu, Cheng Hu, Haijun Jiang *, Zhidong Teng

College of Mathematics and System Sciences, Xinjiang University, Urumqi 830046, Xinjiang, PR China

ARTICLE INFO

Available online 26 February 2013

Keywords:
Nonlinear system
Time-delay
Impulsive control
Exponential stability

ABSTRACT

In this paper, the stabilization for a class of nonlinear systems with time-varying delays is proposed via impulsive control. Using some analysis techniques such as reduction to absurdity, some new and useful criteria for global exponential stability are established. Furthermore, an example and some numerical simulations are presented to verify the effectiveness of the theoretical results.

1. Introduction

In the past decades, analysis and synthesis for nonlinear systems with time-delays has been one of the most active research areas. As we all know, in many practical systems, the system plants contain severe nonlinear properties. Moreover, the stability and stabilization problems of dynamical systems subject to nonlinearities are of interest due to the fact that such systems, especially time-delay systems, include a wide variety of practical systems and devices, like servo systems, flexible systems, etc. Indeed, smooth and non-smooth nonlinearities often occur in a real control process, due to physical, technological, safety constraints, even inherent characteristic of considered systems [1].

During the past three decades, the controlling problem concerning nonlinear systems especially chaotic systems (see [2,3]) has been one of the extensive research subjects and many useful controlling methods are proposed, such as observer-based control [4], adaptive control [5–8], fuzzy control [9], intermittent control [10,11], impulsive control [12–16], switching control [7,17,18] and so on.

Impulsive control, as an important control means, in the past several years, has been widely used to stabilize and synchronize nonlinear systems and chaotic systems. The main idea of impulsive control is to change the states of a system whenever some conditions are satisfied. Moreover, using the impulsive control method, the driven network receives the signals from the driving system only in discrete times and the amount of conveyed information is decreased. This is predominant in practice due to reduced control cost. In view of those merits, impulsive control has been extensively applied to investigate the stabilization and synchronization for different nonlinear models in recent years [12–17,19–21].

It is easy to see that the present results focus on the stabilization and synchronization of non-time-delayed dynamical systems [19,20] and nonlinear systems with constant delays [13,14], there are few reports dealing with nonlinear dynamical systems with time variable delays by using impulsive control. Moreover, the linearity of impulsive control is required in many previous results, such as in [14–16,19–21]. Motivated by the above discussion, the purpose of this paper is to investigate the stabilization of nonlinear systems with time-varying delays under impulsive control.

The main contribution of this paper lies in the following aspects. Firstly, a nonlinear dynamical system and corresponding control scheme are presented by virtue of Dirac impulsive function. Next, some new criteria are given to ensure the exponential stability of the origin of the addressed system. Especially, reduction to absurdity and induction principle are utilized in this paper to propose the time-varying delays, which are different from the traditional Lyapunov functional technique. Besides, it is noted that the constructed impulsive functions in this paper can be nonlinear, which extend the previous results to some extent such as [14–16,19–21], in which the linear impulsive function is required.

This paper is organized as follows. In Section 2, a class of nonlinear systems with time-varying delay is formulated and some preliminaries are provided. In Section 3, the main results on global exponential stability of the addressed system are stated and proved. In Section 4, some numerical simulations are presented to verify the theoretical results.

2. Model description and preliminaries

Throughout this paper, we always use P^T, $\lambda_M(P)$ and $\lambda_m(P)$ to denote the transpose, maximum and minimum eigenvalues of a

* Corresponding author.
E-mail address: jianghai@xju.edu.cn (H. Jiang).

0925-2312/$ - see front matter
http://dx.doi.org/10.1016/j.neucom.2012.06.052

symmetrical real matrix P, respectively. The vector (or matrix) norm is taken to be Euclidean, denoted by $\|\cdot\|$. We use $P>0$ ($<0, \le 0, \ge 0$) to denote a positive (negative, semi-negative, semi-positive) definite matrix P.

Consider a class of nonlinear systems

$$\dot{x}(t)=Ax(t)+f(x(t))+g(x(t-\tau(t))), \tag{1}$$

where $t\in R^+=[0,+\infty)$, $x\in R^n$ is the state variable, A is an $n\times n$ constant matrix, $f(\cdot)$, $g(\cdot):R^n\to R^n$ are the continuous vector-valued functions, $\tau(t)$ denotes the time-delay and satisfies $0\le\tau(t)\le\tau$ for all $t\in R^+$ for some constant $\tau\ge 0$.

Assume that system (1) satisfies the following initial condition:

$$x(t)=\phi(t) \quad \text{for all } t\in[-\tau,0],$$

where $\phi(t)\in C([-\tau,0],R^n)$, here $C([-\tau,0],R^n)$ denotes the Banach space of all continuous vector-valued functions $\phi(t)=(\phi_1(t),\phi_2(t),\dots,\phi_n(t))^T:[-\tau,0]\to R^n$ with the norm $\|\phi\|=\sup_{-\tau\le t\le 0}\sum_i=1^n|\phi_i(t)|$.

In this paper, we introduce the following assumption.

(H_1) $f(0)=g(0)=0$ and there exist two positive definite matrixes $L=(l_{ij})_{n\times n}$ and $M=(m_{ij})_{n\times n}$ such that for all $x,y\in R^n$

$$\|f(x)-f(y)\|^2\le(x-y)^TL(x-y), \quad \|g(x)-g(y)\|^2\le(x-y)^TM(x-y).$$

Remark 1. Assumption (H_1) implies that both functions $f(\cdot)$ and $g(\cdot)$ satisfy the global Lipschitz condition. In fact, from assumption (H_1) the following inequalities are obtained:

$$\|f(x)-f(y)\|\le\sqrt{\lambda_M(L)}\|x-y\|, \quad \|g(x)-g(y)\|\le\sqrt{\lambda_M(M)}\|x-y\|$$

for all $x,y\in R^n$.

Remark 2. In this paper $f(0)=g(0)=0$ are required for convenience, which guarantee that the origin is an equilibrium of system (1). In fact, if system (1) has an equilibrium $x^*\ne 0$, then let $y=x-x^*$, from system (1), we have

$$\dot{y}(t)=Ay(t)+\overline{f}(y(t))+\overline{g}(y(t-\tau(t))), \tag{2}$$

where $\overline{f}(y(t))=f(x(t))-f(x^*)$, $\overline{g}(y(t-\tau(t)))=g(x(t-\tau(t)))-g(x^*)$. For system (2), we see that $\overline{f}(0)=\overline{g}(0)=0$ and the origin is an equilibrium.

To stabilize the origin of system (1), we introduce a control input $u(t,x(t))$ into system (1), and further establish the following control system:

$$\dot{x}(t)=Ax(t)+f(x(t))+g(x(t-\tau(t)))+u(t,x(t)), \tag{3}$$

where $u(t,x(t))=u_1(t,x(t))+u_2(t,x(t))$ and

$$\begin{cases} u_1(t,x(t))=\sum_{k=1}^{\infty}l_k(t)Fx(t), \\ u_2(t,x(t))=\sum_{k=1}^{\infty}\delta(t-t_k)(P_k(x(t))-x(t)), \end{cases} \tag{4}$$

here F is an $n\times n$ constant matrix, $P_k(\cdot):R^n\to R^n$ is a vector-valued function for each $k\in Z^+=\{1,2,\dots\}$, $\delta(\cdot)$ denotes the Dirac function, and $l_k(t)$ is given by

$$l_k(t)=\begin{cases}1, & t_{k-1}<t\le t_k,\\ 0 & \text{otherwise},\end{cases}$$

with $0<t_1<t_2<\cdots<t_k<\cdots$ and $\lim_{k\to\infty}t_k=\infty$.

It is clear from (4) that

$$u_1(t,x(t))=Fx(t), \quad t\in(t_{k-1},t_k],\ k\in Z^+. \tag{5}$$

From (3) and (4), we see that $u_2(t,x(t))=0$ at $t\ne t_k$ and for any small enough constant $h>0$

$$\begin{aligned} x(t_k+h)-x(t_k)&=\int_{t_k}^{t_k+h}[Ax(s)+f(x(s))+g(x(s-\tau(s))) \\ &\quad +u_1(s,x(s))+u_2(s,x(s))]\,ds \\ &=\int_{t_k}^{t_k+h}[Ax(s)+f(x(s))+g(x(s-\tau(s)))]\,ds \\ &\quad +\int_{t_k}^{t_k+h}u_1(s,x(s))\,ds+P_k(x_k(t_k))-x(t_k). \end{aligned} \tag{6}$$

Let $h\to 0^+$, then we can obtain from (6)

$$x(t_k^+)=P_k(x(t_k)). \tag{7}$$

According to (5) and (7), system (3) can be rewritten as

$$\begin{cases} \dot{x}(t)=Ax(t)+f(x(t))+g(x(t-\tau(t)))+Fx(t), & t\ne t_k,\\ x(t_k^+)=P_k(x(t_k)), & k\in Z^+. \end{cases} \tag{8}$$

For each function $P_k(\cdot)$ ($k\in Z^+$), the following assumption is introduced.

(H_2) $P_k(0)=0$ and there is a positive definite matrix $\overline{P}_k=(\overline{p}_{ij}^k)_{n\times n}$ such that

$$P_k^T(x)P_k(x)\le x^T\overline{P}_kx \quad \text{for all } x\in R^n.$$

Lemma 1 (*Zhang et al.* [22]). *Let P be an $n\times n$ positive definite matrix, Q be an $n\times n$ symmetrical matrix, then for any $x\in R^n$*

$$\lambda_m(P^{-1}Q)x^TPx\le x^TQx\le\lambda_M(P^{-1}Q)x^TPx.$$

3. Main results

In this section, we investigate the stability of the origin of system (1) in virtue of the control (4).

Theorem 1. *Under assumptions (H_1) and (H_2), if there exist $n\times n$ matrix $Q>0$, constants $p_2>0$ and $\varepsilon_i>0$ $(i=1,2)$ such that the following conditions hold*

(a) $QA+A^TQ+QF+F^TQ+\varepsilon_1Q^2+\varepsilon_1^{-1}L+\varepsilon_2Q^2+p_2Q\le 0$;
(b) $4\delta-p_2+(\lambda_M(Q^{-1}M)/\varepsilon_2)\exp(4\delta\tau)<0$, *where*

$$\delta=\sup_{k\in Z^+}\left\{\frac{\ln\delta_k}{t_k-t_{k-1}}\right\}, \quad \delta_k=\max\{1,\lambda_M(Q)\lambda_M(Q^{-1}\overline{P}_k)\}.$$

Then the origin of system (1) is globally exponentially stable under controller (4).

Proof. Consider the following auxiliary function:

$$V(t)=x^T(t)Qx(t),$$

which implies that

$$\lambda_m(Q)\|x(t)\|^2\le V(t)\le\lambda_M(Q)\|x(t)\|^2. \tag{9}$$

When $t\ne t_k$, from condition (a), the Dini right derivative of $V(t)$ with respect to time t along solution $x(t)$ of system (8) is calculated and estimated as follows:

$$\begin{aligned} D^+V(t)&\le x^T(t)(QA+A^TQ+QF+F^TQ)x(t)+\varepsilon_1x^T(t)Q^2x(t) \\ &\quad+\varepsilon_1^{-1}x^T(t)Lx(t)+\varepsilon_2x^T(t)Q^2x(t) \\ &\quad+\varepsilon_2^{-1}x^T(t-\tau(t))Mx(t-\tau(t)) \\ &\le -p_2V(t)+\frac{\lambda_M(Q^{-1}M)}{\varepsilon_2}V(t-\tau(t)). \end{aligned} \tag{10}$$

When $t=t_k$, from assumption (H_2) and (9) we further obtain

$$V(t_k^+)\le\lambda_M(Q)x^T(t_k)\overline{P}_kx(t_k)\le\lambda_M(Q)\lambda_M(Q^{-1}\overline{P}_k)V(t_k). \tag{11}$$

From condition (b), there exists a constant $\gamma>2\delta$ such that

$$2\gamma-p_2+\frac{\lambda_M(Q^{-1}M)}{\varepsilon_2}e^{2\gamma\tau}<0.$$