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Neural-networks-based distributed output regulation of multi-agent systems with nonlinear dynamics

Jia Liu [a,b], Zengqiang Chen [a,*], Xinghui Zhang [b], Zhongxin Liu [a]

[a] Department of Automation, Nankai University, Tianjin 300071, China
[b] Tianjin University of Technology and Education, Tianjin 300222, China

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ABSTRACT

This paper deals with the output regulation problem of the nonlinear multi-agent systems based on dynamic neural networks. Assume that the models of following agents in the considered systems are unknown, and the state of the leader agent is not completely measurable for each follower. By employing Lyapunov approach, a dynamic neural network is established to approximate the systems of the following agents. Based on the dynamic neural network, a state feedback control law is designed guaranteeing the following agents can asymptotically track the reference generated by an exosystem. The exosystem is regarded as the active leaders in the multi-agent systems. A numerical simulation example is provided to demonstrate the effectiveness of the obtained results.

1. Introduction

During the past decades, consensus control has become an important and fundamental problems of the coordination control for multi-agent systems. Via different approaches, a great number of results for the consensus problem have been derived in [1–6]. It is well known that the leader-following problem in consensus control of multi-agent systems is an intensive research topic [7–9]. The model of the leader has been in many references, was usually assumed to be simple or immobile. But in practical situation, the leader agents are dynamic, and their position or speed states cannot be completely measurable. For example, the position and speed states of the leaders usually constantly change in the pursuing and formation control problem. With regard to this case, several results have been investigated, see, e.g., [10,11]. In [10], Fax and Murray presented distributed control concerning observer design for multi-agent systems, and first tackled this problem with fixed interaction topologies. In [11], Hong et al. designed the distributed controllers and observers for the second-order follower agents.

In addition, the output regulation problem of controlled system is that of controlling a plant to track (or reject) reference (or disturbance) signals, which are produced by the exosystem. It has a profound theoretical background and practical values in engineering application. So many researchers have studied this problem in [12–14]. In view of these advantages, the output regulation problem for the multi-agent systems has been studied in recent years (see, for example [15–18] and the references therein). In [15], the formation control was considered for nonlinear output servomechanism problem. But the designed controller is not distributed. The distributed output regulation problem of linear multi-agent systems was presented in [16], while the authors designed a distributed controller in [17] to solve the robust output regulation problem of a networked linear system with uncertainties. The two references have proposed a general design procedure for distributed output regulation of multi-agent systems. But the model of the multi-agent systems considered in [16,17] is the linear systems. For the more complex nonlinear multi-agent systems, in [18], the nonlinear distributed coordinative controller were designed to solve the output regulation problem of a multi-agent system with general nonlinear dynamics.

However, the approaches as reported in [18] have a fundamental limitation for nonlinear multi-agent systems output regulation problem. According to the results in [18], the regulator equations must be solved. This requires the precise knowledge of the multi-agent systems. But in practical applications, various uncertainties are acting on the agents due to external disturbances, modeling error, etc., so we cannot obtain the precise model of multi-agent systems usually. Therefore, it is difficult or even impossible to solve these regulator equations by internal model theorem. It is well known that neural networks have the capability to approximate any smooth functions over a compact set to arbitrary accuracy. The neural network techniques are very effective to identify a wide class of nonlinear systems [19,20]. So the output regulation with dynamic neural networks for

* Corresponding author. Tel.: +86 22 23508547.
E-mail address: chenzq@nankai.edu.cn (Z. Chen).

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nonlinear multi-agent systems overcomes the drawbacks of the classical output regulation theory, and it is a novel and interesting approach to solve the output regulation problem of multi-agent systems with general nonlinear dynamics.

In this paper, we focus on the output regulation problem of a nonlinear multi-agent systems based on dynamic neural networks. The motivation in our work is to solve the consensus control problem of the multi-agent systems with general nonlinear dynamics via the output regulation theory and the universal approximation ability of neural networks. Assume that the information of the active leader is not completely measured by all the followers. The inaccurate models of following agents are approximated by establishing a dynamic neural network, then a distributed feedback control law is designed to make the following agents asymptotically track the reference generated by the active leader. This approach avoids the difficulty and complexity of solving the output regulator equation in the traditional output regulation problem, and it is applicable to a broader range in practice. Finally, a numerical simulation example is presented to demonstrate the effectiveness of the main results.

Notations: Throughout this paper, I denotes an identity matrix of appropriate dimension. $Df(\cdot)$ denotes the Jacobian matrix of $f(\cdot)$. The superscript 'T' represents the transpose, the notation '$\otimes$' denotes the Kronecker product. We use $diag(\cdot)$ to denote a diagonal matrix. '$\mathbf{1}$' denotes the column vector with appropriate dimension whose elements are all ones. $\mathcal{C}^k$ represents a function set on which functions are continuous and k times continuously differentiable. The notation $\|x\|$ denotes a vector norm defined by $\|x\| = (\sum_{i=1}^{n} x_i^2)^{1/2}$ when x is a vector. Matrices, if their dimensions are not explicitly stated, are assumed to have compatible dimensions for algebraic operations.

2. Problem statement and preliminaries

In this section, we introduce the preliminary knowledge and the considered multi-agent model.

2.1. Algebraic graph theory

Let $\mathcal{G} = \{\mathcal{V}, \varepsilon, \mathcal{A}\}$ denotes a directed graph, where $\mathcal{V} = \{\nu_1, \nu_2, \dots, \nu_n\}$ is a finite nonempty node set; $\varepsilon \subseteq \mathcal{V} \times \mathcal{V}$ is a directed edge set, each edge is labeled by $e_{ij} = (\nu_i, \nu_j)$, we refer to ν_i and ν_j as the tail and head of the edge (ν_i, ν_j), respectively, where $i,j \in \mathcal{I}(\mathcal{I} = 1,2,\dots,n)$. $\mathcal{A} = [a_{ij}] \in \mathbf{R}^{n\times n}$ is an adjacency matrix of the directed graph, a_{ij} is a positive weight if $(\nu_i, \nu_j) \in \varepsilon$, while $a_{ij} = 0$ if $(\nu_i, \nu_j) \notin \varepsilon$. Similarly, a undirected graph is composed of a finite nonempty node set, a undirected edge set and an adjacency matrix of the undirected graph. In undirected edge set, (ν_i, ν_j) means (ν_j, ν_i), so a undirected graph is the special case of a undirected graph.

Define in-degree and out-degree of node ν_i as

$$deg_{in}(\nu_i) = \sum_{j=1}^{n} a_{ij}, \quad deg_{out}(\nu_i) = \sum_{j=1}^{n} a_{ji},$$

then the Laplacian matrix is defined as

$$\mathcal{L} = \mathcal{D} - \mathcal{A},$$

where $\mathcal{D} = diag(deg_{in}(\nu_1), deg_{in}(\nu_2), \dots, deg_{in}(\nu_n))$, is the in-degree matrix of the graph $\mathcal{G}$. For an undirected graph, $\mathcal{L}$ is symmetrical and is called the Laplacian matrix. However, for a directed graph, $\mathcal{L}$ is not necessarily symmetrical and is called the nonsymmetrical Laplacian matrix or the directed Laplacian matrix [21].

2.2. Problem statement

Suppose that the considered multi-agent system consists of a leader agent and N following agents, and they are regarded as the nodes in $\mathcal{V} = \{0,1,2,\dots,N\}$, where 0 represents the leader, and $1,2,\dots,N$ represent the following agents. The relationships between N agents are shown as a directed graph $\mathcal{G}$, and (i,j) denotes one of the graph's edges when agent i and agent j are neighbors. $A = [a_{ij}]_{(N+1)\times(N+1)} \in \mathbf{R}^{(N+1)\times(N+1)}, i,j = 0,1,\dots,N$ is the adjacency matrix of graph $\mathcal{G}$, the corresponding Laplacian matrix $L = \tilde{D} - A$, where

$$\tilde{D} = diag\left(\sum_{j=0}^{n} a_{0j}, \sum_{j=0}^{n} a_{1j}, \dots, \sum_{j=0}^{n} a_{Nj}\right).$$

In this paper, consider the network system is divided into two type subsystems:

(I) The models of N following agents are described as

$$\dot{x}_i = f_i(x_i, u_i), \quad i = 1,2,\dots,N, \tag{1}$$

where $x_i \in \mathbf{R}^n$ and $u_i \in \mathbf{R}^n$ represent the position state and control input of each agent, respectively. $f_i(x_i, u_i) \in \mathbf{R}^n$ is an unknown smooth function, and represents the uncertain dynamics of the i-th following agent.

(II) The leader dynamics is considered as an exogenous dynamics system (or simply, exosystem), it has the following form:

$$\begin{aligned} \dot{w} &= s(w), \\ y_0 &= q(w), \end{aligned} \tag{2}$$

where $w \in \mathbf{R}^m$ is the state of the leader, $y_0 \in \mathbf{R}^q$ is measured output and reference signal, $s(w)$ and $q(w)$ are smooth mappings. Assume that $f_i(0,0) = 0$, $h_i(0) = 0$, $s(0) = 0$ and $q(0) = 0$.

Define a regulation output of the i-th agent as

$$e_i = h_i(x_i) - y_0, \quad i = 1,2,\dots,N, \tag{3}$$

$h_i(x_i), i = 1,2,\dots,N$ is a known and smooth function with $h_i(0) = 0$. Our control target is

$$\lim_{t\to+\infty} e_i(t) = 0, \quad i = 1,2,\dots,N.$$

Now consider the observer of each agent, it receives the state measurements relative to its neighbors or the leader as follows:

$$z_i = \sum_{j=1, j\neq i}^{N} a_{ij}(x_i - x_j) + a_{0i}(x_i - y_0), \quad i = 1,2,\dots,N. \tag{4}$$

Base on the observer, we design the distributed control law u_i as follows:

$$u_i = \vartheta_i(x_i, z_i), \tag{5}$$

where $\vartheta_i(x_i, z_i)$ is a $\mathcal{C}^k (k \geq 2)$ mapping.

Denote

$$x = \begin{pmatrix} x_1 \\ x_2 \\ \vdots \\ x_N \end{pmatrix}, \quad u = \begin{pmatrix} u_1 \\ u_2 \\ \vdots \\ u_N \end{pmatrix}, \quad e = \begin{pmatrix} e_1 \\ e_2 \\ \vdots \\ e_N \end{pmatrix}, \quad z = \begin{pmatrix} z_1 \\ z_2 \\ \vdots \\ z_N \end{pmatrix},$$

$$f(\cdot) = \begin{pmatrix} f_1(\cdot) \\ f_2(\cdot) \\ \vdots \\ f_N(\cdot) \end{pmatrix}, \quad h(x) = \begin{pmatrix} h_i(x_1) \\ h_i(x_2) \\ \vdots \\ h_i(x_N) \end{pmatrix}, \quad \vartheta(x,z) = \begin{pmatrix} \vartheta_1(x_1,z_1) \\ \vartheta_2(x_2,z_2) \\ \vdots \\ \vartheta_N(x_N,z_N) \end{pmatrix}.$$

Then the systems (1)–(3) can be rewritten as

$$\begin{aligned} \dot{x} &= f(x,u), \\ \dot{w} &= s(w), \\ e &= h(x) - \mathbf{1} \otimes q(w) \end{aligned} \tag{6}$$